The Space-Time Metric Inside a Black Hole.

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A spherically symmetric uncharged black hole with mass M is usually described (1) by the space-time metric

$$ds^2 = (1 - R_S/r)\,c^2\,dt^2 - (1 - R_S)^{-1}\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,, \tag{1}$$

where $R_S = 2GM/c^2$ is the Schwarzschild radius and r is a radial co-ordinate chosen to make the surface area of a sphere of radius r equal to $4\pi r^2$, as in Minkowski space. Equation (1) has the well-known singularities at $r = R_S$ and $r = 0$.

Solution (1) gives the static isotropic metric for the empty space-time disturbed by the gravitational field of an *outside* body with mass M. It appears then that one should not use a solution of the field vacuum equation $R_{\mu\nu} = 0$ to describe the space-time metric *inside* a so massive object as a black hole where one must indeed use the full Einstein equation (2) $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$, which in standard form reads (3)

$$(8\pi G/c^4)\,T_1^1 = \exp[-\lambda](\nu'/r + 1/r^2) - 1/r^2\,, \tag{2}$$

$$(8\pi G/c^4)\,T_0^0 = \exp[-\lambda](1/r^2 - \lambda'/r) - 1/r^2\,, \tag{3}$$

$$(8\pi G/c^4)\,T_0^1 = \exp[-\lambda]\,\lambda'/r\,, \tag{4}$$

where the co-ordinates r, θ, φ, ct have been, respectively, denoted by x^1, x^2, x^3, x^0, so that $-g_{00} = g^{11} = \exp[-\lambda] = \exp[\nu]$.

If we assume that the vacuum Schwarzschild solution (1) is no longer valid inside a black hole, we need another different solution there which, in turn, should also become no longer valid for vacuum. Thus the event horizon should be viewed as the space-time surface separating two different space-time metrics. Therefore, since the space-time region occupied by a black hole should not be merely a given definite part of the indefinite vacuum space-time, but the part of another indefinite space-time which is realized

(1) P. C. W. DAVIES: *Rep. Prog. Phys.*, **41**, 1313 (1978).
(2) S. WEINBERG: *Gravitation and Cosmology* (New York, N. Y., 1971), p. 207.
(3) L. D. LANDAU and E. M. LIFSHITZ: *Teoría Clásica de Campos* (Barcelona, 1966), p. 376.

physically, the mathematical translation of the above conceptual scheme is simply to remove the integration limits in the formal integration of eqs. (2) and (3) for nonzero values of the energy-momentum tensor components and impose after that, for $r > R_S$, the so-obtained solution becomes no longer valid, such as the Schwarzschild solution is here assumed to do for $r < R_S$.

Thus, for an uncharged black hole with mass M about equal to a solar mass, the Hawking's temperature [4,5] and, thereby, the average kinetic energy of the matter particles making up the black hole should be very small, so that $T_0^0 \simeq -\varepsilon = -Mc^2(4\pi R_S^2/3)^{-1}$ and $T_1^1 \simeq p$ (a pressure parameter that we allow to be determined later). Then, from eq. (3), it follows that

$$\exp[-\lambda] = 1 - (8\pi G/c^4 r)\int \varepsilon r^2 \, \mathrm{d}r = 1 - (\alpha r^2 + \beta/r)\,, \tag{5}$$

where $\alpha = R_S^{-2}$ and β is an integration constant. The value of β is simply obtained by considering that, in order to mach interior and exterior solutions at event horizon, $\exp[-\lambda] = 0$ for $r = R_S$, so that $\beta = 0$.

Now, from eqs. (2) and (5), one obtains that

$$\nu' = [(8\pi G/c^4)pr + R_S^{-2} r](1 - R_S^{-2} r^2)^{-1}\,; \tag{6}$$

in order to reproduce the singularity of metric (1) at $r = R_S$, *i.e.* $\exp[\nu] = \exp[-\lambda]$, the following unusual state equation in then required:

$$p = -\varepsilon \tag{7}$$

(which will be discussed later). We finally obtain

$$\mathrm{d}s_{bh}^2 = (1 - R_S^{-2} r^2) c^2 \mathrm{d}t^2 - (1 - R_S^{-2} r^2)^{-1} \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2)\,. \tag{8}$$

Equation (8) is our main result. It becomes the same as eq. (1) at $r = R_S$. Moreover, in the same way as eq. (1) reduces to the Galilean metric for $r = \infty$, eq. (8) reduces also to the flat-space metric at $r = 0$.

The metric potentials in eq. (8) possess two properties: i) they give rise to an infra-red divergence for $r = R_S$ (matter confinement inside the black hole) and ii) they have an ultraviolet free-field asymptotic behaviour for $r = 0$, which are just the two most dramatic features of strong interactions. Property i) is simply an alternative form for defining the black-hole event horizon [1], while property ii) is at least compatible with the feature that only a theory with non-Abelian gauge field, such as the gravitational theory in [6], can be asymptotically free [7]. Is this a further argument in favour of the unification between black holes and elementary particles [8]?

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(4) S. W. HAWKING: *Nature (The Hague)*, **248**, 30 (1974).
(5) S. W. HAWKING: *Commun. Math. Phys.*, **43**, 199 (1975).
(6) F. W. HEHL, P. VON DER HEYDE and G. D. KERLICK: *Rev. Mod. Phys.*, **48**, 393 (1976).
(7) D. J. GROSS and F. WILCZEK: *Phys. Rev. Lett.*, **30**, 1343 (1973); H. D. POLITZER: *Phys. Rev. Lett.*, **30**, 1346 (1973).
(8) The analogy between black holes and hadrons has been recently noted from a different point of view by P. F. GONZÁLEZ-DÍAZ: *Lett. Nuovo Cimento*, **31**, 39 (1981).

As to interpreting eq. (7), it should be thought that $p \geqslant 0$ for matter interacting under Coulomb-Newton–type potentials $\varphi = K/r$ (*i.e.* the potentials of all known macroscopic interactions); but this is not the case for the potential inside a black hole which depends on r^2. We discover then that $p \leqslant 0$ for potentials $K'r^2$. In this way, our most general state equation would read (9)

$$-4\varepsilon \leqslant T_{ii}(= -\varepsilon + 3p) \leqslant 0 . \tag{9}$$

An extension of these ideas will be soon published. The author is indebted to F. CORTÉS-GUILLÉN and C. SIGÜENZA for useful discussions.

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(9) L. D. LANDAU and E. M. LIFSHITZ: *Teoría Clásica de Campos* (Barcelona, 1966), p. 111.