Geo-spatial Information Science
Vol. 4, No. 1, p. 14~18 Mar. 2001

DETERMINATION OF WEIGHTED MEAN TROPOSPHERIC TEMPERATURE USING GROUND METEOROLOGICAL MEASUREMENTS

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KEY WORDS weighted mean tropospheric temperature; conversion parameter; sequential regression analysis

ABSTRACT The weighted mean tropospheric temperature is a critical parameter in the conversion of wet zenith delay to precipitable water vapor in GPS Meteorology. This parameter can not be calculated from the radiosonde data in real time through the conventional methods. In this study, we first discuss the admissible error of weighted mean temperature to enable the accuracy of the conversion better than 1 mm, then summarize the performance of some of the existing methods. An empirical formula is established that satisfies the real-time requirement in GPS meteorology using Sequential Regression Analysis method. It is shown that this real-time formula as compared with other empirical methods is more accurate for local applications.

1 Introduction

In ground-based GPS Meteorology, the precipitable water vapor is converted from the wet zenith delay of the GPS signal. Qualitatively, the Precipitable Water Vapor (PWV) can be related to the Wet Zenith Delay (WZD) by

$$\mathrm{PWV} = F \cdot \mathrm{WZD}$$

$$F = \frac{10^6}{\rho_v \cdot R_v \cdot \left[\frac{k_3}{T_m} + k_2\right]} \quad (1)$$

where the mapping scale factor F is a dimensionless parameter (Askne and Nordius, 1987; Bevis *et al.*, 1994; Chen, 1998). In the above expression, ρ_v is the density of the liquid water; R_v is the specific gas constant for water vapor ; k_3 and k_2 are the atmospheric refractivity constants. The weighted mean temperature of troposphere T_m is defined as follows (Davis *et al.*, 1985).

$$T_m = \int (e/T) \cdot \mathrm{d}h / \left(\int (e/T^2) \cdot \mathrm{d}h\right) \quad (2)$$

where e and T are water vapor pressure and absolute temperature along the zenith direction. The magnitude of T_m varies in different locations and times due to the spatial and temporal irregularity of water vapor pressure and temperature. Therefore, the mapping scale factor F also varies with T_m because the other parameters in Eq. (1) are constants.

In order to satisfy the real-time requirement of precipitable water vapor in meteorological prediction activities, the mapping scale factor F should be determined in real time. Some existing methods have been proposed for this purpose (Askne and Nordius, 1987; Bevis *et al.*, 1994; Bevis *et al.*, 1996; Ingold and Kampfer, 1998). However, they are not accurate enough for GPS meteorological activities in real time. In this paper, we first derive

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Project supported by the Hong Kong Polytechnic University Grant Work Program (GV366)

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the admissible error of the weighted mean tropospheric temperature needed in the determination of mapping scale factor. We then summarize some of the existing methods for determining the T_m. An empirical formula is proposed to calculate the T_m, using the upper air and surface meteorological measurements in Hong Kong, which satisfy the real-time requirement in the Hong Kong region.

2 Existing methods for calculating weighted mean temperature T_m

2.1 An approximate closed form

Although there is no exact closed form to obtain the weighted mean temperature T_m, an approximate formula was given by Askne and Nordius in 1987 as below.

$$T_m = T_0\left(1 - \frac{\alpha \cdot R}{(\lambda + 1) \cdot g}\right) \tag{3}$$

where the parameters α and λ are time and location dependent and must be determined in advance.

2.2 Use of a constant T_m value

Some researchers (Baker *et al.*, 1996) simply treat the mapping scale factor F as a constant of 1/6.5. This means $T_m = 269.7$ K. To investigate the magnitude of uncertainty in this assumption, we assume F to be known and WZD = 500 mm. Then the weighted mean tropospheric temperature T_m varies from 230 K to 310 K. The difference between the actual mapping scale factor and the constant above, together with the corresponding PWV error, are up to ± 0.03 in the mapping scale factor and 10 mm in PWV.

2.3 Use of a trend model

Most water vapor is in the lower 2~3 km of the atmosphere. Hence, T_m should be correlated with surface temperature T_0. Bevis *et al.* investigated this correlation by analyzing a large number of radiosonde data in the United States and found that

$$T_B = 70.2 + 0.72 T_0 \tag{4}$$

with RMS scattering about 4.7 K. In Eq. (4), the T_B denotes the linear estimate of T_m (Bevis *et al.*, 1994).

2.4 Numerical Integration

A closed form for Eq. (2) is not possible due to the irregular variation of water vapor content in the troposphere. Approximate methods are available to meet this need. The most accurate approach is based on approximating Eq. (2) by the following numerical integration

$$T_m = \frac{\sum \frac{e}{T}(h_{i+1} - h_i)}{\sum \frac{e}{T^2}(h_{i+1} - h_i)} \tag{5}$$

where h_i and h_{i+1} denote the height of two sequential observations; e and T are the average of water vapor pressure and absolute temperature, respectively, in the layer defined by h_i and h_{i+1}.

3 The required accuracy for weighted mean temperature

In Eq. (1), ρ_v and R_v are well determined. The atmospheric refractivity parameters are experimentally determined. Of course, the uncertainties of the atmospheric refractivity parameter affect the accuracy of the mapping scale factor F.

Let σ_1, σ_2 and σ_T denote the uncertainties of the refractivity constants k_2, k_3 and the weighted mean temperature T_m, respectively. The differential relationship between k_2, k_3, T_m and F can be derived from Eq. (1).

$$\mathrm{d}F = \frac{10^6}{\rho_v \cdot R_v \cdot [k_3 + T_m \cdot k_2]^2} \cdot (-T_m^2 \cdot \mathrm{d}k_2 - T_m \cdot \mathrm{d}k_3 + k_3 \cdot \mathrm{d}T_m) \tag{6}$$

Assuming there is no correlation among these parameters and applying the variance propagation law to Eq. (6), we can estimate the uncertainty of the parameter F with following equation.

$$\sigma_F = \frac{10^6}{\rho_v \cdot R_v \cdot [k_3 + T_m \cdot k_2]^2} \cdot [(T_m^2 \cdot \sigma_1)^2 + (T_m \cdot \sigma_2)^2 + (k_3 \cdot \sigma_T)^2]^{\frac{1}{2}} \tag{7}$$

To discuss the maximum mapping error Δ_F, the T_m should set an appropriate value in Eq. (7). The value of Δ_F is increasing with the increasing T_m, thus the largest T_m can provide the largest Δ_F. The maximum T_m is 300 K in Hong Kong, we obtain the relationship between Δ_F and σ_T as

$$\Delta_F = 0.5595 \times 10^{-3}\sqrt{1.2075 + \sigma_T^2} \tag{8}$$

To budget the PWV error, let WZD = 500 mm, which is the extreme wet zenith delay that can be observed on earth. The corresponding PWV errors are shown in the last row in Table 1. Apparently, the accuracy of T_m dominates the accuracy of mapping scale factor. To guarantee the conversion accuracy within 1 mm for the PWV in Eq. (2), the error of the weighted mean tropospheric temperature should be less than 3.4 K.

Table 1 The relationship between Δ_F and σ_T

σ_T/K	Δ_F	PWV error/mm
1	0.000 8	0.4
2	0.001 2	0.6
3	0.001 8	0.9
4	0.002 3	1.15
5	0.002 9	1.45

4 Sequential regression analysis

It is very difficult to know the parameters α and λ in Eq. (3), because these parameters are temporally and spatially variable. Moreover, the three assumptions in Eq. (3) are hard to be satisfied in the practice.

As mentioned above, when the T_m ranges from 230 K to 310 K, the mapping error varies from −0.022 to 0.023 , and corresponding PWV error exceeds 5 mm on the assumption of constant mapping scale 1/6.5. This is unacceptable in the GPS meteorological application.

The approximation of numerical integration is the most accurate way to calculate the T_m (Bevis *et al.*, 1992; 1994; Duan *et al.*, 1996). The effect of the observation error and the approximation error is smaller than 1 K (Liu, 1999). However, radiosonde data is not always available at every GPS station at any time. The radiosonde balloon is only launched a few times a day because the lauching costs much. Hence, the corresponding T_m is only sampled a few times a day, not in real time. To get a real-time T_m, an extrapolation method (forecast) must be used. Bevis *et al.* (1996) proposed using the output for T_m from the United States National Meteorological Center ' s Nested Grid Model (NGM).

Noting that in Eq. (4), T_m with an error 4.7 K will induce an 0.003 uncertainty in F. The corresponding PWV error will reach to 1.5 mm. This accuracy of T_m is not acceptable in GPS Meteorology. In addition, Eq. (4) may not be suitable for the Hong Kong region due to the strong dependence of T_m on location.

To check whether Eq. (4) is suitable for Hong Kong region, we compared the estimated T_B from Eq. (5) with the actual T_m from the radiosonde in Hong Kong, and 13-month radiosonde data were used to calculate T_m with Eq. (5). Radiosonde data and surface weather records in Hong Kong were recorded from September 1st of 1996 to September 30th of 1997. The T_m calculated from radiosonde data is shown in Fig. 1. The errors of the T_m caused by Bevis method are showed in Fig. 2, which are usually larger than 3 K. Evidently, Eq. (4) does not satisfy the accuracy requirement that it should be less than 3.4 K.

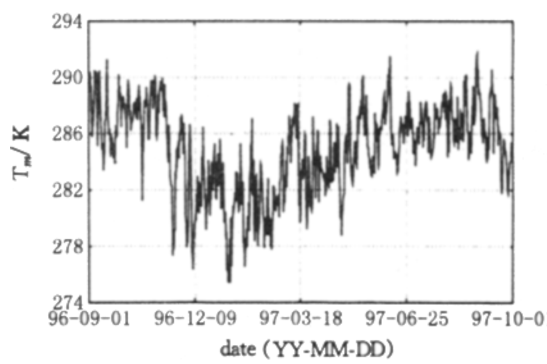

Fig. 1 The T_m calculated from radiosonde data

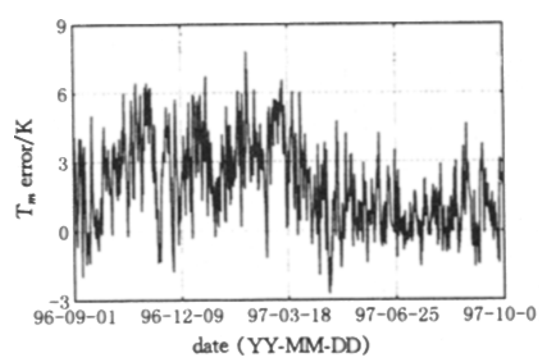

Fig. 2 The T_m error caused by Bevis method

The weighted mean temperature is correlated with the surface temperature. It may also be correlated with other weather elements. The correlation coefficients, between T_m and the surface temperature t_o in degree Celsius or T_o in Kelvin, water vapor pressure e_0 and total pressure P_0, are also computed and shown in Table 2. It is apparent that all these parameters are strongly correlated with T_m.

Table 2 The correlation coefficients between T_m and meteorological parameters

$T_m \propto t_0$	$T_m \propto P_0$	$T_m \propto e_0$	$T_m \propto e_0/T_0$	$T_m \propto e_0/T_0^2$	$T_m \propto P_0/T_0$
0.830	-0.653	0.763	0.759	0.755	-0.814

To obtain a simple and suitable expression to calculate the T_m in real-time, the following model is postulated

$$T_m = b_0 + b_1 \cdot t_0 + b_2 \cdot P_0 + b_3 \cdot e_0 + b_4 \cdot \frac{e_0}{T_0} + b_5 \cdot \frac{e_0}{T_0^2} + b_6 \cdot \frac{P_0}{T_0} \qquad (9)$$

In order to determine the most significant parameters, as the optimal model, we used the well known sequential analysis approach to test the statistical significance of each coefficient b_i. In this process a new model is constructed each time by removing one of coefficients from Eq. (9) and testing the contribution of each removed coefficient. We found that the optimal regression equation is

$$T_S = 272.4 + 0.556 t_0 \qquad (10)$$

where T_S represents the estimated T_m, and the estimated standard deviation σ is 1.7.

5 Analysis and conclusion

To verify the performance of Eq. (10), the predicted T_S is calculated from another two months, and compared with the radiosonde-based T_m from September 1 to October 30 in 1998. Their differences are shown in Fig. 3.

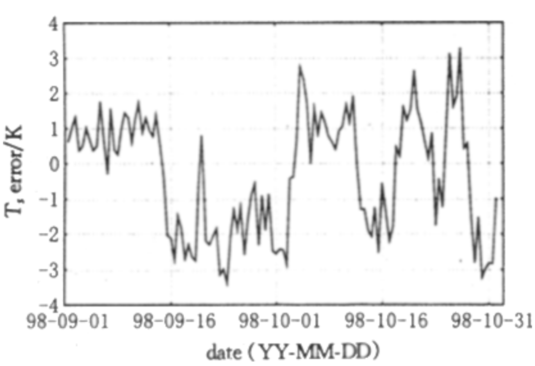

Fig. 3 The differences between the predicted T_S and the actual T_m

The differences are smaller than 4 K, with more than 95 percent smaller than 3 K. The average is -0.33 ± 1.68 K and the RMS is 2.9 K in this period.

Let us compare T_B with T_S. Fig. 4 lists the difference between the predicted T_B and T_S. The variation of T_B is similar to that of T_S. Both of them exhibit a linear trend with respect to surface temperature t_0, but the bias of $T_B - T_m$ is in the range from -5 to 2 K. The average bias for $T_B - T_m$ is -1.63 ± 1.67 K and the RMS is 5.4 K. On the other hand, the average bias between T_S and T_m is -0.33 ± 1.67 K. Fig. 4 shows the difference between T_S and T_B. We can see that T_B is smaller than T_S. Their average bias is 1.3 ± 0.28 K.

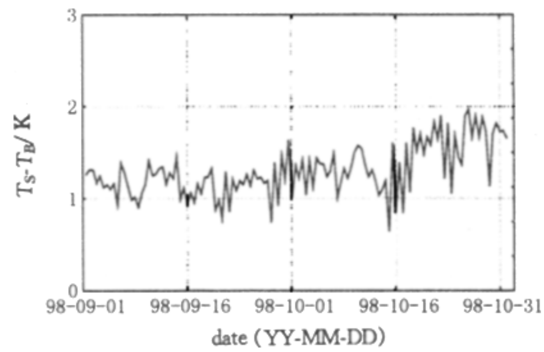

Fig. 4 The differences between T_S and T_B

The above analyses suggest that a tailored relationship is better for the prediction of the weighted mean temperature in Hong Kong. The estimated model given by Eq. (10) works well for the precipitable water vapor conversion in this region.

Acknowledgements

The Hong Kong Polytechnic University Grant Work Program, GV366, supported this study. We are thankful to the Hong Kong Observatory for providing us the upper air and surface meteorological data. Modifications provided by Dr. H. Baki. Iz are gratefully acknowledged.

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