IL NUOVO CIMENTO VOL. LVII A, N. 1 1° Settembre 1968

Consistency Condition for the Inelastic Process $\pi^- + p \to \eta + n$.

A. B. SAXENA, C. WATAL, B. K. AGARWAL and V. P. SETH

Physics Department, Allahabad University - Allahabad

(ricevuto il 6 Maggio 1968)

Summary. — A consistency condition has been derived for the inelastic process $\pi^- + p \to \eta + n$. The nonnull condition is found to be violated when numerically tested.

1. – Introduction.

ADLER (1) has shown that PCAC leads to a consistency condition involving the strong interactions alone. Such conditions have been derived and verified for $\Delta S = 0$ and $\Delta S = 1$ currents in elastic processes (1-5). ADLER (5) has further exploited the PCAC hypothesis to relate the matrix element for any strong-interaction process with the matrix element for the corresponding process in which an additional massless, zero-energy pion is involved. In this paper we will derive a consistency condition for the inelastic process $\pi^- + p \to \eta + n$ and test it following essentially the method of Adler (1).

2. – Derivation of the consistency condition.

By PCAC we mean the hypothesis that

$$\partial_\mu J^A_\mu = C\varphi_\pi = \frac{-i\sqrt{2}\, m_\mathcal{N} g_A(0)\, m_\pi^2}{g_{\pi\mathcal{N}\mathcal{N}} K_{\pi\mathcal{N}\mathcal{N}}(0)}\, \varphi_\pi\,, \tag{1}$$

(1) S. L. ADLER: *Phys. Rev.*, **137**, B 1022 (1965).
(2) B. R. MARTIN: *Nuovo Cimento*, **43** A, 629 (1966); *Nucl. Phys.*, **87**, 177 (1966).
(3) M. A. AHMED: *Nuovo Cimento*, **46** A, 569 (1966).
(4) C. WATAL and B. K. AGARWAL: *Nuovo Cimento* (in press).
(5) S. L. ADLER: *Phys. Rev.*, **139**, B 1638 (1965).

where $m_{\mathcal{N}}$ is the nucleon mass, m_π is the pion mass, $g_A(0)$ is the β-decay axial vector coupling constant, $g_{\pi\mathcal{N}\mathcal{N}}$ is the renormalized pion-nucleon coupling constant, $K_{\pi\mathcal{N}\mathcal{N}}(0)$ is the pionic form factor of the nucleon evaluated at zero pion mass, and φ_π is the renormalized field operator which creates the pion. As the kinematics of $\pi^- p \to \eta + n$ is identical with the kinematics of $\pi + \mathcal{N} \to \to \pi + \mathcal{N}$ the consistency condition in the η case is easily obtained from the condition in the π case by replacing $g_{\mathcal{N}\mathcal{N}\pi}$ by $g_{\eta\mathcal{N}\mathcal{N}}$. We get, following ADLER [1-3],

$$A^{\pi^- - p}(0, 0, 0) = \frac{g_{\mathcal{N}\mathcal{N}\pi} g_{\eta\mathcal{N}\mathcal{N}} K_{\mathcal{N}\mathcal{N}\pi}(0)}{m_{\mathcal{N}}} . \tag{2}$$

3. – Test of the consistency condition.

The fixed-t dispersion relation for the invariant amplitude describing the π^--p scattering is

$$A(0, t) = \frac{1}{\pi} \int_0^\infty ds' \, \mathrm{Im}\, A(s', t) \left[\frac{1}{s' - s} - \frac{1}{s' - u}\right].$$

The integral has been evaluated by taking contributions from the low-lying resonant states $\mathcal{N}^*(1470)$ and $\mathcal{N}^*(1550)$. Taking contribution of the $\mathcal{N}^*(1470)$ state below the threshold from the Born term we get

$$A(s, t) = \frac{1}{\pi} \int_{s_0}^\infty ds' \, \mathrm{Im}\, A(s', t) \left[\frac{1}{s' - s} - \frac{1}{s' - u}\right] + \\ + (M^* - m_{\mathcal{N}}) g_{\mathcal{N}^*\eta\mathcal{N}} g_{\pi\mathcal{N}\mathcal{N}^*} \left[\frac{1}{M^{*2} - s} + \frac{1}{M^{*2} - u}\right]. \tag{3}$$

The contributions from resonances above the threshold are obtained by using the Breit-Wigner formula. We use the relations

$$s = m_{\mathcal{N}}^2 - 2m_{\mathcal{N}} \nu_B + 2m_{\mathcal{N}} \nu , \tag{4}$$

$$u = m_{\mathcal{N}}^2 - 2m_{\mathcal{N}} \nu_B - 2m_{\mathcal{N}} \nu \tag{5}$$

to rewrite eq. (3) as

$$A(s, t) = \frac{1}{\pi} \int_{\bar{\nu}}^\infty d\nu' \, \mathrm{Im}\, A(\nu', \nu_B, K^2 = -1) \left[\frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu}\right] + \\ + (M^* - m) \sqrt{2} g_{\pi\mathcal{N}\mathcal{N}^*} g_{\eta\mathcal{N}^*\mathcal{N}} \left[\frac{1}{M^{*2} - s} + \frac{1}{M^{*2} - u}\right], \tag{6}$$

where $\bar{\nu}=\nu_0+\nu_B$, $\nu_0=m_\eta+m_\eta^2/2m_{\mathcal{N}}$ and M^* is the mass of $\mathcal{N}^*(1470)$. The high-energy behaviour of A is given by $\nu^{\alpha(0)}$, where α is the Regge parameter of the leading trajectory. The only quantum numbers which may be exchanged in the t-channel of $\pi^-+p\to\eta+n$ are $P=(-1)^J$, $G=-1$, $I=1$ and $A=0$, *i.e.* the quantum numbers of the A_2. For A_2 we have $\alpha(0)\simeq 0.35$. Therefore, the integral does not converge and a subtraction is necessary. If we make this subtraction at $\nu=\bar{\nu}$ and then fix $\nu=\nu_B=0$ in the resulting equation, we have

$$(7)\qquad A(0,0,-1)=A(\nu_0,0,-1)-\frac{2\nu_0^2}{\pi}\int\limits_{\nu_0}^{\infty}\mathrm{d}\nu'\,\frac{\operatorname{Im}A(\nu',0,-1)}{\nu'(\nu'^2-\nu_0^2)}+$$
$$+(M^*-m_{\mathcal{N}})\sqrt{2}\,g_{\pi\mathcal{N}\mathcal{N}^*}g_{\eta\mathcal{N}^*\mathcal{N}}\left[\left(\frac{1}{M^{*2}-s}+\frac{1}{M^{*2}-u}\right)_{\nu=\nu_B=0}-\right.$$
$$\left.-\left(\frac{1}{M^{*2}-s}+\frac{1}{M^{*2}-u}\right)_{\substack{\nu=\nu_0\\ \nu_B=0}}\right].$$

The subtraction constant in (7) is expressible in terms of scattering length $\mathscr{A}_\pm$ on using the equations

$$\frac{A(\nu,0)}{4\pi}=\frac{2(W+m_{\mathcal{N}})}{[(p_{i0}+m_{\mathcal{N}})(p_{f0}+m_{\mathcal{N}})]^{\frac{1}{2}}}f_1-\frac{2(W-m_{\mathcal{N}})}{[(p_{i0}-m_{\mathcal{N}})(p_{f0}-m_{\mathcal{N}})]^{\frac{1}{2}}}f_2$$

and

$$f_1=\sum_{l=0}^{\infty}\mathscr{A}_{l+}\frac{(l+1)[2(l+1)]!}{2^{l+1}[(l+1)!]^2}m_K^{2l},$$

$$f_2/q^2=\sum_{l=2}^{\infty}(\mathscr{A}_{l-}-\mathscr{A}_{l+})\frac{l(2l!)}{2(l!)^2}(m_K)^{2(l-1)}.$$

CAMPBELL and LOGAN [6] found that an s-wave resonance accounts for the enhancement of the reaction near the threshold. They also found disagreement among the various solutions as to whether or not P_{11} is resonant. Moreover, since both π and η are pseudoscalar the lowest angular momentum state is an s-state. Therefore, by the partial-wave threshold behaviour, we expect ηn production near the threshold (1488 MeV) to occur in s-waves. As η has zero isospin, the total isospin is $\frac{1}{2}$. Thus the quantum numbers of low-energy η-production are S_{11}. In view of this, in our calculations we keep only the s-wave scattering length. We find that $\mathscr{A}_0=0.163$ and $A(\nu_0,0,-1)=5.15$.

Finally we shall consider the rescattering integral in (7). Since only s-wave is dominating, we shall calculate the contributions to $\operatorname{Im}A(\nu',0,-1)$ from

([6]) F. UCHIYAMA-CAMPBELL and R. K. LOGAN: *Phys. Rev.*, **149**, 1220 (1966).

the $\mathcal{N}^*(1550)$ pole only. The value of integral comes out to be 1.16. The contribution of the last term in eq. (7) is equal to 12.06. Combining all these terms we have

$$A(0, 0, -1) = 5.15 - 1.16 + 12.06 = 16.05 .$$

The right-hand side of the consistency condition comes out to be 8.41 using $g^2_{\mathcal{N}\mathcal{N}\pi}/4\pi = 14.5$, $q_{\eta\mathcal{N}\mathcal{N}} = 4.69$ and (by the Goldberger-Treiman relation) $K_{\mathcal{N}\mathcal{N}\pi}(0) = 0.87$. Therefore, the consistency condition does not appear to hold good for the inelastic process considered within the approximation of our calculations. The positive off-mass-shell correction cannot alter this conclusion. Therefore, the use of the consistency condition to evaluate the off-mass-shell amplitude should be resorted to with much care.

* * *

A. B. SAXENA, C. WATAL and V. P. SETH are grateful to C.S.I.R., New Delhi, for financial assistance.

RIASSUNTO (*)

Si è dedotta la condizione di consistenza per il processo anelastico $\pi^- + p \to \eta + n$. Risulta che, ove sia controllata numericamente, la condizione di non nullità è violata.

(*) *Traduzione a cura della Redazione.*

Условие согласованности для неупругого процесса $\pi^+ + p \to \eta + n$.

Резюме (*). — Было выведено условие согласованности для неупругого процесса $\pi^- + p \to \eta + n$. Было найдено, что ненулевое условие нарушается, при проведении численных расчетов.

(*) *Переведено редакцией.*