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Hadron Decay Processes and the Quark Model (*).

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Summary. — The quark model is used for the calculation of the decay rates of baryons and mesons. According to this model the 56 states of baryons contained in the fundamental octet and decuplet are S-states of a three-quark system and the 36 states of mesons in the scalar and vector octets are S-states of a quark-antiquark system. The states differ from each other in the relative orientation of spin and unitary spin of the quarks and not much in the spatial wave functions. The rates are calculated for transitions between these states which are: *A*) by weak interaction—emission of lepton pairs, *B*) by electromagnetic interaction—emission of γ-rays or electron pairs, and *C*) by strong interaction—emission of pions. These rates are calculated with the same method for all transitions. In all cases the interaction is transmitted by a spin-flip only (mechanical and/or isotopic). In addition, the same methods lend themselves to a calculation of the annihilation of the quark and the antiquark in mesons, accompanied by either a lepton pair emission (weak or electromagnetic interaction) or a pair of γ-rays. These methods allow the calculations of the rates of all decay processes where the number of hadrons remains unchanged, without any assumptions regarding the quark interactions. All that enters are the known weak, electromagnetic and strong coupling constants and the proton magnetic moment. The processes where a meson disappears depend also upon the value $|\psi(0)|^2$, which is the square of the wave function of the quark-antiquark system at zero distance. From comparison with the observed rates that magnitude turns out to be of the order fm^{-3} and proportional to the meson

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mass. The following predictions can be made on the basis of our model for the partial width of decays not yet measured: $\Sigma^0 \to \Lambda + \gamma$ 8.3 keV, $\omega \to e^+e^-$ 0.63 keV, $\rho \to \eta + \gamma$ 50 keV, $\varphi \to e^+e^-$ 1.0 keV, $\omega \to \eta + \gamma$ 6.3 keV, $\eta \to 2\gamma$ 0.45 keV (*), $\varphi \to \eta + \gamma$ 0.34 MeV, $\eta \to \pi^0 2\gamma$ 0.53 eV, $\eta \to 2\pi\gamma$ 0.16 keV. We find also: $\rho \to e^+e^-$ 5.8 keV, $\pi^0 \to 2\gamma$ 12 eV, $\rho \to 2\pi$ 185 MeV, $\omega \to 3\pi$ 14 MeV which agree rather well with the experimental data.

1. – Introduction.

In spite of the problematic nature of the quark hypothesis (1) of baryon and meson structure, this hypothesis presents for the first time a single and consistent way of calculating a large number of decay processes. Very few assumptions are needed for these calculations. Little needs to be assumed about the quantum state of the quark system or about the forces which keep the quarks together. The results of these calculations agree surprisingly well with the facts. They are mostly within 30% of the observed values.

It must be said, however, that this agreement should not be interpreted as an indication that quarks really exist. It must be emphasized that quarks never have been observed, and that serious problems arise when one intends literally to interpret the quark model. We mention the problem of statistics of the quarks; Fermi statistics would require a ground state of the baryons antisymmetric in its position co-ordinates, a most unattractive feature. Another problem is the fact that only triplets of quarks and quark-antiquark pairs seem to form bound states. The binding particles seemingly are such as to allow only bound states in which the fractional charges add up to integer ones.

Moreover, in order to get agreement with facts we are forced to make the following assumption regarding the probability $|\psi(0)|^2$ to find the quark and the antiquark at the same place within a meson ($\psi(\boldsymbol{r})$ is the internal wave function of the quark-antiquark system). We are forced to assume that $|\psi(0)|^2$ is proportional to the mass of the meson. This is a rather unexpected result in a theory which should be approximately invariant to SU_3 and SU_6.

We will not dwell further upon the mysterious properties of quarks, but we will apply the model in its most primitive form to the determination of baryon and meson decays. Many results, if not all, which we will obtain by explicit use of the quark model, can also be obtained without ever using this

(*) We have neglected here η-η' mixing. An estimate of this effect would give $\eta \to 2\gamma$ $(0.9 \div 1.1)$ keV (see ref. (13)).

(1) M. GELL-MANN: *Phys. Lett.*, **8**, 274 (1964); G. ZWEIG: Cern preprint 8479/TH 472 (1964).

model, for example by clever application of symmetry properties or current algebras of which the quark model is a special example. We do not consider the quark-model calculations the only way of obtaining our results. We consider them as a simple and physically intuitive way, however.

Most of the calculations in this paper have been performed by some authors (2). It is intended to collect all of them and present them in a unified treatment.

2. – Description of the quark model.

We use the following simple quark model for our calculations. We assume that baryons are bound systems of three quarks, and mesons are bound systems of a quark and an antiquark. The binding is supposed to be such that the motion of quarks in these systems is approximately nonrelativistic.

In this paper, we consider only the 56 baryons of the fundamental octet and decuplet, and the 36 mesons of the pseudoscalar and the vector octet and singlet. These states are combinations of the three fundamental quark states, n, p, λ and its two spin directions ↑, ↓, which we denote as $\overset{\uparrow}{\mathrm{p}}, \overset{\uparrow}{\mathrm{n}}, \overset{\uparrow}{\lambda}; \overset{\downarrow}{\mathrm{p}}, \overset{\downarrow}{\mathrm{n}}, \overset{\downarrow}{\lambda}$. The Tables I and II of Appendix III present a list of the combinations which make up the 56 baryons and 36 mesons.

We intend to make as few assumptions as possible about the binding forces. Our calculations will not contain any factors which depend on these forces with the exception of the value of $|\psi(0)|^2$ for the internal wave function in mesons. It is important, nevertheless, to discuss the nature of these forces in a qualitative way, because their nature has a bearing upon our assumptions regarding the effective magnetic moment of quarks and upon the stringency of our condition of nonrelativistic motion in the bound systems of quarks.

Let us describe the binding forces approximately by introducing a potential well $V(r)$ which acts on one quark and is caused by the other quarks of the bound system; r is the distance from the center of mass. This well has a depth V_0 and a width b. The depth of the well should be such as to produce the observed mass of the bound system, and the width b should be large enough to allow the quark to move with nonrelativistic velocities within the well.

In order to estimate the order of magnitude of these parameters, we will first simplify the situation by assuming that the binding forces are SU_6 invariant. Under these conditions all baryons have the same mass M_B which we assume

(2) Excellent reviews are: R. H. DALITZ: *Quark Models for the Elementary Particles*, Lectures given at Les Houches (1965); R. H. DALITZ: Invited paper presented at the *Oxford International Conference* (Oxford, Sept. 1965); H. THIRRING: *Acta Phys. Austriaca, Suppl.* III, **3**, 294 (1966).

to be the weighted average of the 56 baryon states: $M_B = 1300$ MeV. All 36 meson states will then have the same mass M_m assumed to be the weighted average of the observed meson masses: $M_m = 744$ MeV. Let us call M_q the mass of a free quark, which we expect to be much larger: $M_q \gg M_B$. Then, the potential depth V_0 for the potential $V(r)$ in mesons or in baryons must be so deep that the resulting mass of the quark is $M_m/2$ or $M_B/3$ respectively:

$$V_0^{(M)} = M_q - \frac{M_m}{2} \qquad \text{for mesons},$$

$$V_0^{(B)} = M_q - \frac{M_B}{3} \qquad \text{for baryons}.$$

There we neglect any kinetic energy the quarks may have in the bound state. The two depths are not very different since $M_q \gg M_B$ and $M_m \sim \frac{2}{3} M_B$.

The mass differences between the 56 baryons and 36 mesons are brought about by an additional SU_6-violating potential $V'(r)$ whose depth will be small compared to $V_0^{(M)}$ and $V_0^{(B)}$.

We can make two assumptions regarding the nature of the binding potential V: it may be the fourth component of a four-vector, like the Coulomb potential, or it may be a scalar potential. The two alternatives give rise to different conditions, which can be easily understood under the simplifying assumption that the effect of a given quark on the other quark or quarks can be described by a potential well of given depth V_0 and width b. Inside the well, the potential has the constant value $-V_0$, and the motion of the quark can be described by a Dirac equation:

$$\left(\alpha_i \frac{\partial}{\partial x_i} + \beta M_q\right) \Psi = (E + V_0)\Psi$$

or

$$\left(\alpha_i \frac{\partial}{\partial x_i} + \beta(M_q - V_0)\right) \Psi = E\Psi.$$

The first equation holds if V is the fourth component of a vector; the second holds if V is a scalar.

In the scalar case the particle behaves, in the well, exactly like a particle of mass $M_{\text{eff}} = M_q - M_0$. This has two consequences. First, in order to assure nonrelativistic behavior, the width b of the potential must be large compared to $(M_q - V_0)^{-1}$, that is, large compared to the Compton wavelength of a mass $M_m/2$ or $M_B/3$. Hence, with our choices of M_m and M_B, the width b must be

(3) N. N. Bogoliubov, *et al.*: Dubna Report D-1968 (1965), P-2569 (1966); H. J. Lipkin and A. Tavkhelidse: *Phys. Lett.*, **17**, 331 (1965).

chosen quite large, say of the order of 1 or 2 fm. Second, the Dirac magnetic moment [3] of a bound quark would be $e/2M_{\text{eff}}$, that is about 3 proton magnetons. This is about the moment which one ascribes to a quark, in order to obtain the observed moments of the nucleons in the quark model. In this case the quark would have no or a small anomalous magnetic moment.

In the case of a vector potential, the nonrelativistic behavior is guaranteed for $b \gg M_{\text{q}}^{-1}$; this is a much less stringent condition on b than in the scalar case. The Dirac magnetic moment, however, would be the one of a particle with a mass M_{q}. This moment would be very much smaller than the magnetic moment $e/2M_{\text{eff}}$ ascribed to the quark. Hence, in this case, one must assume that most of the magnetic moment of the quark is anomalous.

It is tempting, therefore, to assume that the SU_6-invariant part of the binding potential is of scalar character. The « actual » magnetic moment of the quark is then reasonably well reproduced by the simple assumption that it is equal to the Dirac moment with no or a small anomalous contribution. In this case, however, one must assume that the SU_6-violating potential, which separates the pion mass from M_{m}, must be of vector character so that $M_{\text{eff}} = M_{\text{m}}/2$ also for the pion. This would ensure the nonrelativistic motion of quarks within the pion for values $b \sim 1$ or 2 fm, and leaves the magnetic moment of the quarks the same in pions as in other hadrons, an assumption which will turn out necessary for reaching reasonable agreement with the data. Later on we will find some more indications in favor of no or a weak anomalous magnetic moment of the quarks, and therefore in favor of a mainly scalar binding potential.

It should be emphasized again that neither the depth nor the width, nor the special form of these forces between quarks will ever enter any calculations. The foregoing discussions serve the sole purpose of convincing the reader that models are possible in which the magnetic moment of the quarks is about equal to $e'/2M_{\text{eff}}$, with e' being the quark charge, and with $M_{\text{eff}} \sim M_{\text{B}}/3 \sim M_{\text{m}}/2$ and that the motion of the bound quarks can be assumed to be approximately nonrelativistic.

Similar considerations are brought forward by BOGOLIUBOV, STRUMINSKI, TAVKHELIDSE and LIPKIN [3]; the problem of relativistic bound states was discussed by CIAFALONI and MENOTTI [4].

3. – The interactions of quarks with different fields.

In what follows, we will calculate transition probabilities of quark systems from one quantum state to another, induced by the interaction of quarks with

[4] M. CIAFALONI and P. MENOTTI: preprint June 1963, Scuola Normale Superiore, Pisa.

certain fields. In short, we study transitions between quantum states of quark systems with the emission or absorption of field quanta.

The main idea is here that baryons and mesons interact with fields via the quarks of which they consist. This is in complete analogy with the theory of nuclear structure. The interaction of a nucleus with the lepton field or with the electromagnetic field is expressed by the interaction of the nuclear constituents with these fields.

3'1. *Weak interactions.* – We assume that there is a weak interaction between a quark and the lepton field of the type

$$\int d^3x\,\bar{q}(x)(G'_v\gamma_\mu + G'_A\gamma_\mu\gamma_5)(\cos\theta\,\tau^+ + \sin\theta\,\sigma^+)q(x)\bar{\ell}(x)\gamma_\mu(1+\gamma_5)\nu(x) + \text{h.c.}\,, \tag{1}$$

where G'_v and G'_A are the renormalized interaction constants of quarks for vector- and axial-vector interaction; the prime distinguishes them from the corresponding constants for nucleons; $q(x)$ signifies the state of the quark, $\ell(x)$ the state of the charged lepton (electron or muon), $\nu(x)$ the state of the corresponding neutrino, θ is the Cabibbo angle, $\tau^\pm$ are the isotopic-spin raising and lowering operators, $\sigma^\pm$ are the corresponding operators of the σ-spin which changes the λ-like quark state into the protonlike state.

We will consider two kinds of transitions: in the first kind (I), the quarks change their spin, isospin or strangeness; in the second (II), a quark and an antiquark annihilate.

Examples of the two kinds are:

$$\text{(I)}\qquad \left\{\begin{aligned} &\mathcal{N} \to P + e^+ + \nu\,,\\ &\Lambda \to P + e^- + \bar\nu\,,\\ &\pi^+ \to \pi^0 + e^+ + \nu\,,\\ &K^+ \to \pi^0 + e^+ + \nu\,,\\ &\Sigma^- \to \Sigma^0 + e^- + \bar\nu\,,\\ &\Sigma^- \to \Lambda + e^- + \bar\nu\,;\end{aligned}\right.$$

$$\text{(II)}\qquad \left\{\begin{aligned} &\pi \to \mu + \nu\,,\\ &K \to \mu + \bar\nu\,.\end{aligned}\right.$$

We now take advantage of the fact that the quarks move nonrelativistically. Hence, for the processes of the first kind we may write the interaction in its

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nonrelativistic form with respect to the quark states:

$$(2)\qquad G'_v\int \mathrm{d}^3x\,\bar{q}(x)(\cos\theta\tau^+ + \sin\theta\sigma^+)q(x)\,\bar{\ell}(x)\gamma_0(1+\gamma_5)\nu(x) + \\ + G'_A\int \mathrm{d}^3x\,\bar{q}(x)\sigma_k(\cos\theta\tau^+ + \sin\theta\sigma^+)q(x)\,\bar{\ell}(x)\gamma_k(1+\gamma_5)\nu(x)\,,$$

where the index k is summed over the three spatial co-ordinates. The constants G'_v and G'_A can be determined by calculating the nucleon-lepton interaction resulting from (2) and adjusting these constants. Assuming that the spatial parts of the quark wave functions are the same for all members of an octet, and neglecting form factors, we get for the matrix elements of any of the transitions (I) the following expression:

$$(3)\qquad G'_v\langle S_F|\cos\theta\sum_i \tau_i^+ + \sin\theta\sum_i \sigma_i^+|S_I\rangle\bar{\ell}(0)\,\gamma_0(1+\gamma_5)\,\nu(0) + \\ + G'_A\langle S_F|\cos\theta\sum_i \boldsymbol{\sigma}_i\tau_i^+ + \sin\theta\sum_i \boldsymbol{\sigma}_i\sigma_i|S_I\rangle\bar{\ell}(0)\,\boldsymbol{\gamma}\,(1+\gamma_5)\nu(0)\,.$$

Here S_F, S_I are the spin wave functions (unitary and ordinary) of the initial and final hadron. The sums are taken over the quarks (or antiquarks) contained in the hadrons. The subscript i of the spin operators σ, τ, refers to the quark (or antiquark) i.

If this expression is applied to the proton-neutron transition, we see that the vector part is simple, since $\sum_i \tau_i^+$ is equal to the total isospin operator: $\langle P|\sum_i \tau_i^+|N\rangle = 1$. Thus $G'_V = G_V$ where G_V is the vector coupling constant for nucleons. The axial part gives rise to the matrix element $\langle P|\sum_i \sigma_i\tau_i^+|N\rangle = \frac{5}{3}\langle P|\sigma|N\rangle$. Hence we get

$$(4)\qquad G_A = \tfrac{5}{3}\,G'_A\,.$$

We obtain from the experimental result $G_A = 1.2\,G_V$:

$$(5)\qquad G'_V = G_V\,,\qquad G'_A = 0.7\,G_V\,.$$

These values allow us to calculate the reactions of the type (I) and related reactions. The results are identical with the ones obtained from the application of SU_6 symmetry and the conserved-vector-current hypothesis.

As far as the branching ratio $(\Sigma^-\to\Sigma^0)/(\Sigma^-\to\Lambda)$ is concerned, one obtains the following ratios of the squares of the matrix elements: 1:0 for the vector part and $1:\sqrt{3}/2$ for the axial vector part. This result is in agreement with SU_6 symmetry and corresponds to a so-called F/D ratio of 2/3.

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The results concerning the $K \to \pi + e$, $\mu + \nu$ reaction correspond to:

TABLE I.

Process	Matrix element	Width (MeV)	Experimental width (MeV)
$K \to \pi + e + \nu$	$(G_V/2)\sin\theta$	$2.98 \cdot 10^{-15}$	$2.62 \cdot 10^{-15}$
$K \to \pi + \mu + \nu$	$(G_V/2)\sin\theta$	$1.93 \cdot 10^{-15}$	$1.82 \cdot 10^{-15}$

We now turn to the reactions of kind (II) where mesons disappear and transform into lepton pairs. Similar calculations were carried out by MATVEJEV, STRUMINSKI and TAVKHELIDSE [5]. One faces here a quark-antiquark annihilation from the point of view of the quark picture. This annihilation is not strictly a nonrelativistic process, but the nonrelativistic character of the quark and antiquark motion within the meson simplifies the matrix elements for annihilation. The interaction Hamiltonian (3) contributes only from its axial-vector part for mesons with zero spin. These matrix elements are of the form (cf. Appendix I)

$$A = G'_A \psi_m(0)\sqrt{2}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\bar{u}_l(\boldsymbol{p})\gamma_0(1+\gamma_5)u_\nu(-\boldsymbol{p})\,. \tag{6}$$

Here $\cos\theta$ stands for strangeness conserving, $\sin\theta$ for strangeness changing transitions. $\psi_m(\boldsymbol{r})$ is the value of the nonrelativistic wave function of the quark-antiquark system of the meson m at the relative distance r. The transition probability is then proportional to $|\psi_m(0)|^2$ as expected for an annihilation process. No form factors for the annihilation of quarks are introduced.

It is interesting to compare this matrix element with the phenomenological one which is usually used for these reactions. One writes

$$A = G_v f_M \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\frac{p_\alpha}{\sqrt{2p_0}}\bar{u}_l(\boldsymbol{p})\gamma_\alpha(1+\gamma_5)u_\nu(-\boldsymbol{p})\,; \tag{7}$$

where p_α is the momentum four-vector of the meson, and f_M is a constant with the dimensions of mass. This is so on the grounds that the hadronic part must transform like a four-vector, and the only four-vector available is p_α. For a meson at rest only $p_0 = m_\pi$ contributes. Comparison with (6) allows a deter-

(5) V. A. MATVEJEV, *et al.*: Dubna Report P-2524 (1965).

mination of the constant f_M on the basis of our quark model:

$$f_M = \frac{G'_A}{G'_v} 2 \frac{\psi_m(0)}{m_\pi^{\frac{1}{2}}} = 1.4 \frac{\psi_m(0)}{m_\pi^{\frac{1}{2}}} . \tag{8}$$

Experimentally it is known that the value of f_M is the same for the decay of π- and K-mesons and is approximately: $f_\pi = f_K = m_\pi$. It should be emphasized that this result depends upon our knowledge of the Cabibbo angle which can be determined from other decays. Here we meet an interesting point which casts a certain shadow upon the quark model: the above result implies from (8) that $|\psi_\pi(0)|^2$ is equal to $\sim \frac{1}{2} m_\pi^3$ for π-mesons, a result which indicates a plausible meson « size » of the order of $\hbar/m_\pi c$, but also implies that $|\psi_m(0)|^2$ is proportional to the meson mass. It should be four times larger for K-mesons than for π-mesons, which is rather puzzling.

In more detail we find

$$\begin{cases} \Gamma_{\pi^+\to\mu^+\nu} = \left(\frac{1}{2\pi}\right) |\psi_\pi(0)|^2 G_A'^2 \cos^2\theta\, m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 , \\ \Gamma_{K^+\to\mu^+\nu} = \left(\frac{1}{2\pi}\right) |\psi_K(0)|^2 G_A'^2 \sin^2\theta\, m_\mu^2 \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 . \end{cases} \tag{9}$$

From the experimental values [4]

$$\Gamma_{\pi^+\to\mu^+\nu} = 2.6\cdot 10^{-14}\ \text{MeV} ,$$

$$\Gamma_{K^+\to\mu^+\nu} = 3.4\cdot 10^{-14}\ \text{MeV}$$

we easily get

$$|\psi_\pi(0)|^2 = 1.4\cdot 10^6\ (\text{MeV})^3,$$

$$|\psi_K(0)|^2 = 5.1\cdot 10^6\ (\text{MeV})^3$$

$$(\text{compare with } m_\pi^3 = 2.7\cdot 10^6\ (\text{MeV})^3) .$$

This means that we have almost exactly

$$\frac{|\psi_K(0)|^2}{|\psi_\pi(0)|^2} = \frac{m_K}{m_\pi} . \tag{10}$$

More support for the proportionality of $|\psi(0)|^2$ with the meson mass will be found in electromagnetic decays.

The quark model is based upon the approximate validity of SU_3 symmetry, and this symmetry would require a similar wave function for the quark-antiquark system in all mesons of the same multiplet.

How could the symmetry-breaking forces cause this difference in the values of $|\psi(0)|^2$? At first, one would think that stronger binding should lead to a smaller quark-antiquark system and therefore to larger values of $|\psi(0)|^2$, in contrast to our findings. The simple potential-well model shows that this is not necessarily so. For example, $|\psi(0)|$ would be unchanged (apart from very small surface effects) if the symmetry-breaking force changes the potential depth only and not the width b of the well. If the width b is increased, $|\psi(0)|$ will decrease, but the binding energy will not be very sensitive to changes in b since the motion of the quarks is supposed to be nonrelativistic; changes of b therefore cause only energy differences small compared to M_{eff}. Hence it is possible to construct symmetry-breaking forces which give the observed mass dependence of $|\psi(0)|$: add to the potential depth the necessary amount to obtain the mass split and also change the potential width such that the ratio of the widths b_i and b_k for meson i and k meson is given by $b_i/b_k=(m_k/m_i)^{\frac{1}{3}}$, a ratio which is not too far from unity.

Although this example demonstrates a relatively simple form of symmetry breaking which gives the desired result, it seems to be amost arbitrary and artificial way of arriving at the simple ratio (10) There must be a more natural reason for it, which may be connected with the limitations of the quark model.

3·2. *Electromagnetic interactions.* – We assume that the quarks have the following electromagnetic interaction energy:

$$H=\int j_\mu(x)\,A_\mu(x)\,\mathrm{d}^3\boldsymbol{x}+\int \mathcal{M}_{\mu\nu}\,\frac{\partial}{\partial x_\mu}\,A_\nu(x)\,\mathrm{d}^3x\,, \tag{11}$$

where the current j_μ is given by

$$j_\mu(x)=e'\,\bar{\psi}_{\text{q}}(x)\,\gamma_\mu\psi_{\text{q}}(x)$$

with e' being the charge of the quark, and the magnetization $\mathcal{M}_{\mu\nu}$ is given by

$$\mathcal{M}_{\mu\nu}=\mu'_{\text{q}}\,\bar{\psi}_{\text{q}}(x)\,\sigma_{\mu\nu}\psi_{\text{q}}(x)\,,$$

where μ'_{q} is the anomalous magnetic moment of the quark. The total magnetic moment of the free quark would be

$$\mu'=\frac{e'}{2M_{\text{q}}}+\mu'_{\text{q}}\,. \tag{12}$$

If the quark is in a bound state with nonrelativistic motion, the effective magnetic moment would be

$$\mu' = \frac{e'}{2M_q^*} + \mu'_q\,, \tag{13}$$

where M_q^* is the effective quark mass as defined in Sect. **2**. As was shown in that Section, a mainly scalar potential may allow us to set the anomalous magnetic moment μ'_q equal to zero.

We again consider two kinds of electromagnetic transitions; in the first kind the quarks change their spin, in the second kind a quark and antiquark annihilate. Examples of the two kinds are:

$$(\text{III}) \qquad \left|\begin{array}{ll} \mathcal{N}^* & \to \mathcal{N} + \gamma\,, \\ \Sigma^0 & \to \Lambda + \gamma\,, \\ \omega & \to \pi^0 + \gamma\,, \\ \rho & \to \pi + \gamma\,, \\ \rho & \to \eta + \gamma\,, \\ \eta & \to \pi + 2\gamma\,; \end{array}\right.$$

$$(\text{IV}) \qquad \left|\begin{array}{ll} \rho & \to e^+e^-\,, \\ \rho & \to \mu^+\mu^-\,, \\ \omega & \to e^+e^-\,, \\ \omega & \to \mu^+\mu^-\,, \\ \pi^0 & \to 2\gamma\,, \\ \eta & \to 2\gamma\,. \end{array}\right.$$

The first kind of transition (III) is a spin-flip process; the spatial wave function of the hadron remains unchanged, all that changes is the direction of the spin of one of the quarks. We will again neglect form factors, in which case the form of the spatial wave function does not enter into the calculation. Since the motion of the quarks is nonrelativistic, the relevant matrix element is proportional to the operator $\sum_i \mu'_i \boldsymbol{\sigma}_i$ where μ'_i is the total magnetic moment (Dirac plus anomalous) and $\boldsymbol{\sigma}_i$ is the spin operator of the i-th quark. For these calculations, it is important to know what fraction of the total quark moment is anomalous.

The calculation of the emission probability of a γ-ray (or the cross-section for the inverse process) in the case of the first two reactions in list (III) is trivial and gives the following results:

$\mathcal{N}^* \to \mathcal{N} + \gamma$ ([6])

$$M_z = \langle \mathcal{N}(\tfrac{1}{2}, \tfrac{1}{2}) | \mu_z | \mathcal{N}^*(\tfrac{3}{2}, \tfrac{1}{2}) \rangle = \langle \mathcal{N}(\tfrac{1}{2}, \tfrac{1}{2}) | \sum_i \mu_i' \sigma_{z_i} | \mathcal{N}^*(\tfrac{3}{2}, \tfrac{1}{2}) \rangle = \frac{2\sqrt{2}}{3} \mu_p$$

where μ_p is the total proton magnetic moment: $\mu_p = 2.79(e/2M_p)$. Experimentally, from the π^0 photoproduction cross-section at resonance, one finds ([7])

$$(M_z)_{exp} = (1.28 \pm 0.02) \frac{2\sqrt{2}}{3} \mu_p .$$

$\Sigma^0 \to \Lambda + \gamma$

$$M_z = \langle \Lambda(\tfrac{1}{2}, \tfrac{1}{2}) | \mu_z | \Sigma^0(\tfrac{1}{2}, \tfrac{1}{2}) \rangle = \frac{1}{\sqrt{3}} \mu_p .$$

An easy calculation gives then

$$\Gamma_{\Sigma\Lambda\gamma} = \frac{1}{\pi} |M|^2 k^3 \frac{E_\Lambda}{M_\Sigma} = 8.28 \text{ keV}$$

or $T_{\Sigma^0} = 0.7 \cdot 10^{-17}$ s. Experimentally, one only knows

$$T_{\Sigma^0} < 1.0 \cdot 10^{-14} \text{ s} .$$

It is perhaps a little disappointing that the experimental value of the matrix element for $\mathcal{N}^* \to \mathcal{N} + \gamma$ is higher than the calculated one. One would think that form factor effects would reduce the theoretical value even further ([7]).

The calculation of the other processes of the kind (III) which involve mesons is almost as trivial. The only slight complication arises from the fact that the meson, after emission of the γ-ray, has relativistic velocity. This brings about some trivial factors originating from the normalization of the outgoing meson wave, and a less trivial correction coming from the fact that the internal wave-function of the outgoing meson is Lorentz-contracted. We have neglected it, since it should not amount to more than a 15 or 20%

([6]) C. Becchi and G. Morpurgo: *Phys. Lett.*, **17**, 352 (1965); R. H. Dalitz: in *High Energy Physics* (New York, 1965), p. 251.

([7]) R. H. Dalitz and D. G. Sutherland: *Phys. Rev.*, **146**, 1180 (1966); R. G. Moorhouse: *Phys. Rev. Lett.*, **16**, 772 (1966); A. Donnachie and G. Shaw: CERN preprint T. H. 673 (1966).

correction, together with a possibly more important effect of the form factor because of the high momentum transfer involved. The latter two corrections would depend on the shape of the wave function.

We calculate the general process $V \to P + \gamma$ [8], where V stands for the vector meson and P for the pseudoscalar meson in terms of the transition magnetic moment

$$\mu_{VP} = \langle P | \mu_P \sum_i \frac{e_i}{e} \sigma_{zi} | V(s_z = 0) \rangle . \tag{14}$$

Once we have μ_{VP} one gets

$$|M|^2 = 2\mu_{VP}^2 k^2 ,$$

where $|M|^2$ denotes the squared matrix element and

$$\Gamma_{V \to P + \gamma} = \frac{1}{(3\pi)} \mu_{VP}^2 k^3 . \tag{15}$$

The derivation of this last result for $\Gamma_{V \to P + \gamma}$ was based on a nonrelativistic calculation, *i.e.* we assumed the momentum transfer k very small. It is clear that this is in most cases a bad approximation because the outcoming pseudoscalar meson is relativistic.

There is however a rather unambiguous way to take account of the relativistic behavior of the meson in the phase-space contribution.

Suppose that we start calculating $\Gamma_{V \to P\gamma}$ from the relativistically invariant interaction

$$H = f_{VP} \, \varepsilon_{\alpha\beta\gamma\delta} \, \partial_\alpha A_\beta(x) \, \partial_\gamma V_\delta(x) \, P(x) , \tag{16}$$

where $A_\beta(x)$, $V_\delta(x)$ and $P(x)$ are the fields associated with the photon, the vector meson and the pseudoscalar meson. We can then calculate again $\Gamma_{V \to P + \gamma}$ and find $\Gamma_{V \to P + \gamma} = Ck^3$ where C is a constant. Comparing now this last result with (15), it is clear that the constant C calculated for the nonrelativistic limit remains correct also in the relativistic case.

So we take as our final result:

$$\Gamma_{V \to P + \gamma} = \frac{1}{(3\pi)} \mu_{VP}^2 k^3 . \tag{17}$$

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[8] C. Becchi and R. Morpurgo: *Phys. Rev.*, **140**, B 687 (1965); W. Thirring: *Phys. Lett.*, **16**, 335 (1965); V. V. Anisovitch, A. A. Anselm, Ya. I. Azimov, G. S. Danilov and I. T. Dyatlov: *Phys. Lett.*, **16**, 194 (1965); L. D. Soloviev: *Phys. Lett.*, **16**, 345 (1965).

We collect the results for μ_{VP} in Table II. It is useful for later calculations to use the relativistic invariant form (16). An easy calculation using (15) gives

$$f_{VP} = 2\mu_{VP} . \tag{18}$$

TABLE II.

Process	μ_{VP}	$\Gamma_{V\to P+\gamma}$ calcul.	$\Gamma_{V\to P+\gamma}$ experim.
$\rho\to\pi+\gamma$	$-\frac{1}{3}\mu_P$	0.12 MeV	<0.4 MeV
$\rho\to\eta+\gamma$	$-(1/\sqrt{3})\mu_P$	50 keV	—
$\omega\to\pi+\gamma$	$-\mu_P$	1.17 MeV	1.2 MeV
$\omega\to\eta+\gamma$	$-(1/(3\sqrt{3}))\mu_P$	6.3 keV	—
$\varphi\to\pi+\gamma$	0	0	—
$\varphi\to\eta+\gamma$	$-\frac{2}{3}\sqrt{\frac{2}{3}}\mu_P$	0.34 MeV	—

The last process in list (III) is a double process. The η-meson emits two γ-rays by going over intermediate states. We consider both steps as single nonrelativistic spin-flip processes, and consider as intermediate states only states contained in the fundamental 36-plet. The states which enter as intermediate states are vector mesons, the ω, ρ mesons (*) (Fig. 1), since the spin must flip in each emission. The result, with neglect of all form factor corrections, is as follows. We find for the invariant matrix element

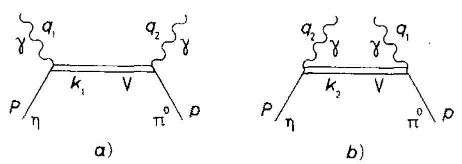

Fig. 1.

$$A = \frac{1}{\sqrt{2}}\sum_{V}(2\mu_{V\eta})(2\mu_{V\pi})(a_{1V}+a_{2V}) ,$$

where the sum goes over the vector mesons and $\mu_{V\eta}$, $\mu_{V\pi}$ are defined by (14), and

$$a_{1V} = \frac{1}{k_2^2+m_V^2}\,\varepsilon_{\alpha\beta\gamma\mu}\varepsilon_{\alpha'\beta'\gamma'\mu}q_\alpha^{(1)}\varepsilon_\beta^{(1)}k_{1\gamma}q_{\alpha'}^{(2)}\varepsilon_{\beta'}^{(2)}k_{1\gamma'} ,$$

$$a_{2V} = \frac{1}{k_2^2+m_V^2}\,\varepsilon_{\alpha\beta\gamma\mu}\varepsilon_{\alpha'\beta'\gamma'\mu}q_\alpha^{(2)}\varepsilon_\beta^{(2)}k_{2\gamma}q_{\alpha'}^{(1)}\varepsilon_{\beta'}^{(1)}k_{2\gamma'} .$$

Here the four-vectors $q^{(i)}$, $\varepsilon^{(i)}$ are the momentum and polarization of the i-th

(*) The φ meson does not contribute because the matrix element for $\varphi\to\pi^0+\gamma$ is zero.

light-quantum, $i=1, 2$; k_j is the momentum of the vector meson in the intermediate state in the two Feynman diagrams $j=1, 2$ (see Fig. 1). A straightforward calculation leads to (*)

$$\Gamma_{\eta\to\pi^0 2\gamma} = \frac{1}{64\pi^3 m_\eta} \iint \sum_{\text{pol}} |A|^2 \mathrm{d}E_1 \mathrm{d}E_2 \,,$$

where E_1 and E_2 are the photon energies and the summation goes over the photon polarizations. Numerically this leads to

$$\Gamma_{\eta\to\pi^0 2\gamma} = 5.3\cdot 10^{-7}\ \text{MeV}\,.$$

If we compare this with $\Gamma_{\eta\to 2\gamma}$ (27) calculated below, we find

$$R = \frac{\Gamma_{\eta\to\pi^0 2\gamma}}{\Gamma_{\eta\to 2\gamma}} \simeq 0.5\cdot 10^{-3}\,.$$

The experimental situation is not quite clear. DI GIUGNO *et al.* (9) find a result

$$R_{\text{exp}} \simeq 0.5$$

which is in strong disagreement with our calculations. This experimental result was contradicted by WAHLIG *et al.* (10) who found a smaller upper limit for that ratio.

The processes of kind (IV) involve annihilations. In this case it will make a difference as to whether there is an anomalous magnetic moment or not. This is best seen if we write the current operator of a quark in the form

$$(19)\qquad j_\mu = \frac{e_i}{e}\,\bar{u}_q(p')\,[f_1\gamma_\mu + f_2(p_\mu + p'_\mu)]\,u_q(p)\,,$$

where p_μ, p'_μ are the initial and final momenta. The second term does not contribute to the magnetic moment μ_{st} for $\langle p\rangle = 0$. Therefore we have

$$(20)\qquad f_1 = 1 + \left(\frac{\mu_{\text{anom}}}{\mu_{\text{Dirac}}}\right).$$

The annihilation matrix elements are proportional to f_1.

The processes $\rho\to e^+e^-$, $\mu^+\mu^-$ and $\omega\to e^+e^-$, $\mu^+\mu^-$ are single-quark annihilation processes. In fact, we can interpret them as a Rutherford scattering

(*) See also: P. MÖBIUS and H. PIETSCHMANN: *Phys. Lett.*, **22**, 684 (1966).

(9) G. DI GIUGNO, R. QUERZOLI, G. TROISE, F. VANOLI, M. GIORGI, P. SCHIAVON and V. SILVESTRINI: *Phys. Rev. Lett.*, **16**, 767 (1966).

(10) M. A. WAHLIG, E. SHIBATA and I. MANNELLI: *Phys. Rev. Lett.*, **17**, 221 (1966).

of an electron or muon by a quark if we look at the process in the crossed channel (Fig. 2).

The calculations are straightforward and give the following result for the matrix element:

$$M = f_1 \psi_V(0) \left[\sum_i a_i \frac{e_i}{e} \boldsymbol{\sigma}_i\right] \frac{\tau}{k^2} \bar{u}_\ell \boldsymbol{\gamma} v_\ell \,, \tag{21}$$

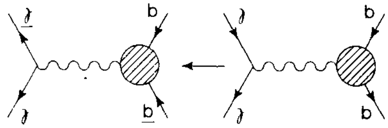

Fig. 2.

where the sum over i goes over the quarks in the meson V and k is the momentum of the virtual photon; a_i is the Clebsch-Gordan coefficient of quark i in the meson V. u_ℓ and v_ℓ are the Dirac spinors of the lepton and antilepton. The lifetime for this decay becomes

$$\Gamma_{V\to\ell^+\ell^-} = f_1^2 |\psi_V(0)|^2 \left(\frac{e^2}{4\pi}\right)^2 \frac{16\pi}{3} \left[\sum_i a_i \frac{e_i}{e}\right]^2 \cdot \tag{22}$$

$$\cdot \frac{1}{m_V^2}\left(1-\frac{4m_\ell^2}{m_V^2}\right)^{\frac{1}{2}}\left(1+\frac{2m_\ell^2}{m_V^2}\right) \simeq f_1^2 |\psi_V(0)|^2 \left(\frac{e^2}{4\pi}\right)^2 \cdot \frac{16\pi}{3} \left[\sum_i a_i \frac{e_i}{e}\right]^2 \frac{1}{m_V^2} \,.$$

The values for $\rho \to \ell^+\ell^-$ are in good agreement with the experimental values if we set $f_1 = 1$ and $|\psi_V(0)|^2 = (m_V/m_\pi)|\psi_\pi(0)|^2$ with $|\psi_\pi(0)|^2 = \frac{1}{2} m_\pi^3$ (see (10)).

We can express the obtained result in terms of the relativistic invariant V-γ coupling of the form

$$H = f_{V\gamma} V_\mu A_\mu \,.$$

A straightforward calculation gives

$$f_{V\gamma} = 2 \left[\sum_i a_i \frac{e_i}{e}\right] \psi_V(0)\, m_V^{-\frac{3}{2}} \,. \tag{23}$$

It is also useful for later calculations to introduce a new coupling constant defined by

$$f_{V\gamma} = \psi_V(0) f'_{V\gamma} \,. \tag{24}$$

We collect all the results in Table III.

It is interesting to compare the numerical value one finds for the coupling constant $f_{\rho\gamma}$(23) in the quark model with the one based on the ρ-dominance for the isovector form factor of the pion. The quark model gives $f_{\rho\gamma} = \sqrt{2} \cdot \psi_\rho(0) m_\rho^{-\frac{3}{2}} = 0.177$ and the ρ-dominance model $f_{\rho\gamma} = 1/f_{\rho\pi\pi} = 0.183$ ($f_{\rho\pi\pi}$ is determined from the ρ-width $\Gamma_{\rho\pi\pi} = (f_{\rho\pi\pi}^2/4\pi)\frac{2}{3}(p^3/m_\rho^2)$).

The calculation of the process $\pi^0 \to 2\gamma$ is similar to the calculation of the annihilation of positronium of spin zero into two light quanta. In fact the

TABLE III.

Process	$\sum_i a_i(e_i/e)$	Γ calculated (MeV)	Γ experimental (MeV)	$f'_{V\gamma}$
$\rho \to e^+e^-$ $\rho \to \mu^+\mu^-$	$1/\sqrt{2}$	$0.58\cdot 10^{-2}$	$0.6\cdot 10^{-2}$ $0.6 \pm 0.2\cdot 10^{-2}$	$\sqrt{2}\, M_\rho^{-\frac{3}{2}}$
$\omega \to e^+e^-$ $\omega \to \mu^+\mu^-$	$1/(3\sqrt{2})$	$0.63\cdot 10^{-3}$	$1.2\cdot 10^{-3}$ —	$(\sqrt{\frac{2}{3}}/3)\, M_\omega^{-\frac{3}{2}}$
$\varphi \to e^+e^-$ $\varphi \to \mu^+\mu^-$	$\frac{1}{3}$	$1.0\ \cdot 10^{-3}$	— —	$-\frac{2}{3}\, M_\varphi^{-\frac{3}{2}}$

π^0-meson can be reasonably called, « quarkonium ». The only difference comes from the fact that the intermediate state in positronium consists of an essentially free electron-positron pair, whereas here the two quarks are still strongly bound in the intermediate state.

We are simplifying the calculation by assuming that the intermediate states belong to the 36-plet, so that all that happens in the emission of the first quantum is again only a spin-flip of a quark or antiquark (Fig. 3). This calculation yields for the invariant matrix element of the emission of a pair of light quanta (*)

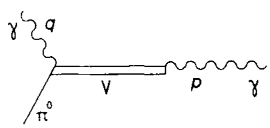

Fig. 3.

$$A = \frac{1}{\sqrt{2}} \sum_V 4e\, \psi_V(0)\, \mu_{V\pi} f'_{V\gamma} M_\pi\, \boldsymbol{q}\cdot(\varepsilon_1 \times \varepsilon_2)\,, \tag{25}$$

where ε_i are the polarization vectors, $\boldsymbol{q}$ is defined in Fig. 3, and $\mu_{V\pi}$ and $f'_{V\gamma}$ are given in Tables II and III. The summation over V, the intermediate vector mesons, includes only the ρ and the ω. This calculation leads to a theoretical lifetime of the π^0:

$$\Gamma_{\pi^0\to 2\gamma} = \frac{1}{64\pi}\frac{1}{M_\pi}\sum_{\text{pol}} |A|^2$$

and this gives

$$\Gamma_{\pi^0\to 2\gamma} = \frac{e^2}{16\pi} |\psi_V(0)|^2 \left[\frac{\sqrt{2}}{3} M_\rho^{-\frac{3}{2}} + \frac{\sqrt{2}}{3} M_\omega^{-\frac{3}{2}}\right]^2 \mu_\rho^2 M_\pi^3\,, \tag{26}$$

(*) We are much indebted to A. DAR for pointing out an error in this result.

whereas the experimental values lie between (6.3 ± 1) eV [11] and (9.2 ± 1.2) eV [12]. By comparison of these data (using the shortest experimental lifetime, *i.e.* $\Gamma_{\pi^0\to2\gamma}=9$ eV) we obtain $|\psi_V(0)|^2=1.2\,(m_V/m_\pi)(\frac{1}{2}m_\pi^3)$ in good agreement with the values we obtained from the decays $V\to\ell^+\ell^-$, if we choose $f_1=1$ in eq. (21), that means, if we assume that there is no appreciable anomalous magnetic moment of the quark (see (20)).

We can obtain a similar result for the $\eta\to2\gamma$ process, which is not yet measured. We find $\Gamma_{\eta\to2\gamma}=450$ eV. If we introduce a not unreasonable mixing angle for the η-η' of the order of $-10°$, we find [13]

$$\Gamma_{\eta\to2\gamma}=(0.9-1.1)\ \text{keV}\,. \tag{27}$$

We have here a new support for the proportionality of $|\psi(0)|^2$ with the meson mass. We also could consider these results, the good agreement of the π^0 lifetime as well as the $\rho\to\ell^++\ell^-$ decay width, as a support for the assumption $f_1=1$ (20), that is for a small or vanishing anomalous magnetic moment of the quark, which would point towards a mainly scalar binding potential. We remind the reader, here, that all our results are based upon the assumption of the same magnetic moment of the quark in all mesons. The difference in mass between mesons within the 36-plet, therefore, must be caused by potentials of the vector type, which do not change the effective Dirac magnetic moment in the bound state.

3·3. *Strong interactions.* – It is impossible to treat the strong interactions of quarks on the same basis as we treated the weak and electromagnetic ones. The main manifestation of the strong interactions of quarks are the binding effects which give rise to the existence of baryons and mesons.

It seems possible, however, to isolate one part of the strong interactions which is by its own nature a weak effect. It is the emission and absorption of « soft » pions; this means pions for which the momentum $|\boldsymbol{k}|$ is small. Let us forget—for the purpose of the argument—that all pions are quark-antiquark systems and let us assume that there exists a pseudoscalar field $\pi(x)$ which is coupled with the quarks according to the following interaction energy:

$$H_{\text{int}}=F'\int \mathrm{d}^3x\,\bar{q}(x)\gamma_5\tau_K q(x)\,\pi_K(x) \tag{28}$$

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[11] G. Von Dardel, D. Dekkers, R. Mermod, J. D. Van Putten, M. Vivargent, G. Weber and K. Winter: *Phys. Lett.*, **4**, 51 (1963).

[12] G. Belletini, C. Bemporad, P. L. Braccini and L. Foà: *Nuovo Cimento*, **40** A, 1139 (1965).

[13] R. H. Dalitz and D. G. Sutherland: *Nuovo Cimento*, **37**, 1777 (1965); **38**, 1945 (1965).

in complete analogy with the coupling of nucleons with a pion field. We propose, however, to apply this interaction only in the case of the emission or absorption of pions of low spatial momentum compared to the baryon masses. Hence we take the nonrelativistic limit of (28), which can be written in the form

(29) $$\frac{f_q}{\mu}\int d^3x\,\bar{q}_q(x)\,\boldsymbol{\sigma}\tau_K q_q(x)\nabla\pi_K(x)$$

for quark transitions with pion emission (q_q is the quark field), and

(30) $$-\frac{f_q}{\mu}\int d^3x\,\bar{q}_a(x)\,\boldsymbol{\sigma}\tau_K q_a(x)\nabla\pi_K(x)$$

for antiquark transitions with pion emission. Here $q_a(x)$ is the field of the antiquark and $f_q = (\mu/2M_{eff})F'$, where μ is the pion mass, which is introduced as a normalizing factor. The supplementary minus sign of (30) follows directly by inserting the four-component Dirac wave functions of free quarks (or antiquarks) into (28). If expression (29) is applied to a transition between those quark states which correspond to proton and neutron, one can easily see that one gets the relation

(31) $$f_q = \tfrac{3}{5}f\,,$$

where f is the well-known coupling constant of the static limit of the nucleon-nucleon-pion interaction:

(32) $$\frac{f}{\mu}\int d^3x\big(N(x)\,\boldsymbol{\sigma}\tau_k N(x)\big)\nabla\pi_k(x)$$

and has the value

(33) $$f = g_{\pi\mathcal{N}}\frac{\mu}{2M} \qquad \text{or} \qquad \frac{f^2}{4\pi} = 0.082\,,$$

where $g_{\pi\mathcal{N}}$ is the π-$\mathcal{N}$ coupling constant: $g^2_{\pi\mathcal{N}}/4\pi = 14.6$. We naturally use here the renormalized values for the coupling constants, as we have done in the case of the weak and the electromagnetic interaction.

In contrast to the other interactions, we will apply this interaction only to transitions in which the number of quarks and antiquarks remains unchanged. We will not consider annihilations, although they are contained in expression (28). The reason is that annihilations would not contain the gradient which we find in (29) and (30) which reduces these interactions to tolerably weak ones. We will, in fact, only calculate the following processes:

$$\text{(V)} \qquad \begin{cases} \mathcal{N}^* \to \mathcal{N} + \pi\,, \\ \rho \quad \to \pi + \pi\,, \\ \omega \quad \to 3\pi\,, \\ \eta \quad \to 2\pi + \gamma\,. \end{cases}$$

The calculation of the first process is straightforward; we assume again that the spatial dependence of the wave function is the same for all members of the 56-plet. We obtain [14]

$$\Gamma_{\mathcal{N}^* \to \mathcal{N} + \pi} = 80\ \text{MeV}\,, \tag{34}$$

which is somewhat low, but within 30% of the correct value. It is possible to calculate any transition which follows from $\mathcal{N}^* \to \mathcal{N} + \pi$ by an SU_3 transformation, such as $Y_1^* \to \Lambda\pi$, $\Sigma\pi$; $\Xi^* \to \Xi + \pi$. Of course, the quark model is SU_3 invariant and therefore the results are the exact SU_3 transforms of the $\mathcal{N}^* \to \mathcal{N} + \pi$ process. It is known that these transitions fulfill the SU_3-symmetric values reasonably well.

The reaction $\rho \to \pi + \pi$ can be described in our terms as a transition from a ρ-meson to a π-meson by emission of a quantum of the pion field. In other words, the ρ-meson and one of the pions is considered to be a quark-antiquark system, but the other pion is a field quantum which is emitted, when a quark in the ρ-system commits a spin-flip and changes the ρ-system into a pion quark-antiquark system.

The calculation of this process is very similar to the calculation of the $\rho \to \pi + \gamma$ process. The γ-quantum is replaced by the quantum of a pion field. Again all form factors are assumed to be unity. The result of this calculation is in rather good agreement with the experimental value. We get for the matrix element

$$M\big(\rho^+(s_3 = 0) \to \pi^+ + \pi^0\big) = \frac{f_q}{\mu}\,(k_2^{(+)} - k_2^{(0)})\,, \tag{35}$$

where $k^{(+,0)}$ are the momenta of the pions, and this gives then

$$\Gamma_{\rho \to 2\pi} = 185\ \text{MeV} \tag{36}$$

compared with the experimental value of 125 MeV. In obtaining this result we took into account the relativistic motion of the pions by applying the

(14) C. BECCHI and G. MORPURGO: *Phys. Rev.*, **149**, 1284 (1966).

same method which was used for the determination of $\Gamma_{\rho\to\pi\gamma}$ (see eq. (15) and (17)). The coupling constant $f_{\rho\pi\pi}$ defined in the conventional way by

$$H_{\text{int}} = if_{\rho\pi\pi}\,\boldsymbol{\rho}_\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})$$

(the last factor is an isotopic vector product) turns out to be

$$f_{\rho\pi\pi} = 2\,\frac{M_\varrho}{\mu}\,\frac{3}{5}\,f \qquad \text{or} \qquad f^2_{\rho\pi\pi} = 3.6\;, \tag{37}$$

whereas the experimental value gives

$$f_{\rho\pi\pi\,\text{exp}} = 1.5\,\frac{M_\varrho}{\mu}\,\frac{3}{5}\,f \qquad \text{or} \qquad \frac{f^2_{\rho\pi\pi}}{4\pi} = 2.4\,. \tag{38}$$

An intriguing question could be raised here in regard to the fact that each of the two pions could be the field quantum or the quark-antiquark system. Should one, therefore, add the amplitudes for the two alternatives (they are equal) and expect a result larger than (37) by a factor four [14]? The answer is negative. Each of the two amplitudes gives the correct answer. An intuitive argument can be found in turning the process around and having two pions collide in order to focus a ρ-meson. Clearly, in such a case, one only should be considered as a field and the other as a quark system. A more rigorous argument based upon the PCAC hypothesis is found in Appendix II.

The process $\omega\to3\pi$ can be considered as a transition of the ω quark-antiquark system by two spin flips, each accompanied by the emission of a pion-field quantum. It is quite analogous to the $\eta\to\pi+2\gamma$ process (see Fig. 1), the two light quanta being replaced by pions. (Selection rules make the $\eta\to3\pi$ process impossible without higher-order electromagnetic perturbation; the process $\omega\to\pi+2\gamma$ is forbidden by charge conjugation invariance.)

The calculation of $\omega\to3\pi$ on this basis is straightforward. We consider only intermediate states within the 36-plet, this means that only the ρ contributes, and obtain

$$\Gamma_{\omega\to3\pi} = 8.8\,\mu^2\left(\frac{f^2_{\omega\rho\pi}}{4\pi}\right)\left(\frac{f^2_{\rho\pi\pi}}{4\pi}\right), \tag{39}$$

where the number 8.8 comes from the phase space, $f_{\rho\pi\pi}$ has already been calculated (37), and $f_{\omega\rho\pi}$ is the coupling constant defined by

$$H = if_{\omega\rho\pi}\,\varepsilon_{\alpha\beta\gamma\delta}\,\partial_\alpha V^{(\omega)}_\beta(x)\,\partial_\gamma \boldsymbol{V}^{\rho}_\delta(x)\,\boldsymbol{\pi}(x) \tag{40}$$

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and can easily be calculated in the quark model using (29) and (30). We get

$$f_{\omega\rho\pi} = \frac{12}{5}\frac{f}{\mu}. \tag{41}$$

This gives then for (39) the value

$$\Gamma_{\omega\to 3\pi} \simeq 15 \text{ MeV}, \tag{42}$$

compared to the experimental value

$$(\Gamma_{\omega\to 3\pi})_{\text{exp}} \simeq (11 + 1.7) \text{ MeV}. \tag{43}$$

It is interesting to compare our results now with the calculations of the Gell-Mann–Sharp–Wagner ([15]) model. For the $\omega \to 3\pi$ decay, they use exactly the same model as we used here. However, they use a quite different model for the $\omega \to \pi+\gamma$ decay. The model they use is essentially the same as for $\omega \to 3\pi$; in $\omega \to \pi+\gamma$ they also use a vector meson as an intermediate state which couples then directly to the photon. In the quark model, the process $\omega \to \pi+\gamma$ is an elementary vertex. However, it is clear that also in the quark model we could calculate the process $\omega \to \pi+\gamma$ by vector dominance. It turns out that the two values we find in that way for $\Gamma_{\omega\to\pi+\gamma}$ are almost the same. The explanation of this agreement is to be found in the remark we made after eq. (24): there we found agreement between the values for $f_{\rho\gamma}$ (23) in the quark model and in the ρ-dominance model for the isovector form factor of the pion. This then explains why we find also agreement for the process $\omega \to \pi+\gamma$ because in this case the photon is an isovector one, and we can again assume vector dominance.

Finally, the process $\eta \to 2\pi+\gamma$ is analogous to the $\eta \to \pi+2\gamma$ process with the replacement of one light quantum by a pion-field quantum. This process is not forbidden and one obtains a transition probability

$$\Gamma_{\eta\to 2\pi+\gamma} = 1.6\cdot 10^{-4} \text{ MeV}.$$

The absolute values of the η-decay probabilities are unknown today. Our calculations give us ratios which can be compared with experiments. We obtain

$$\Gamma_{\eta\to 2\gamma} : \Gamma_{\eta\to\pi+2\gamma} : \Gamma_{\eta\to 2\pi+\gamma} = 1 : 0.5\cdot 10^{-3} : 0.12 .$$

The ratio of the first and the third processes is in reasonably good agreement

([15]) M. GELL-MANN, D. H. SHARP and W. WAGNER: *Phys. Rev. Lett.*, **8**, 261 (1962).

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with experiment:

$$\left(\frac{\Gamma_{\eta\to2\pi+\gamma}}{\Gamma_{\eta\to2\gamma}}\right)_{\text{exp}} = 0.16 \pm 0.02 \; .$$

The relative strength of the $\eta\to\pi+2\gamma$ process is still not definitely established (see Sect. **3**·2).

* * *

We would like to express our appreciation for the great help given by B. T. FELD at the earlier stages of this investigation, and for most helpful discussions with D. SUTHERLAND and with V. TELEGDI. Most of the work was done in the Theoretical Division of CERN, and we are grateful for the hospitality extended to us.

APPENDIX I

Bound-state annihilation.

We give here a short derivation to show how the factor $\psi(0)$ enters the matrix element for the annihilation of a bound state.

If a certain meson bound state M is a bound system of a quark-antiquark pair, we can generally write

$$d^*_{\text{M}}(0) = \sum_{r,s}\int \mathrm{d}^3p\, f(\boldsymbol{p})\varphi(r,s)a^*_r(\boldsymbol{p})b^*_s(-\boldsymbol{p}) \; ,$$

where $d^*_{\text{M}}(0)$ describes the creation operator for the boson M of zero momentum and $a^*_r(\boldsymbol{p})$, $b^*_s(\boldsymbol{p})$ the quark and antiquark with spin-unitary spin components r, s. $f(\boldsymbol{p})$ is the bound-state wave function and the normalization requires

$$\int \mathrm{d}^3p |f(\boldsymbol{p})|^2 = 1 \; .$$

Now $(2\pi)^{\frac{3}{2}} d^*_{\text{M}}(0)|0\rangle$ is the correct normalized state vector to calculate the decay of the boson if we require that there is one particle per unit volume. The amplitude for the annihilation is then

$$A = (2\pi)^{\frac{3}{2}}\langle 0|H_{\text{int}}d^*_{\text{M}}(0)|0\rangle = (2\pi)^{\frac{3}{2}}\sum_{r,s}\int \mathrm{d}^3p\, f(\boldsymbol{p})\varphi(r,s)\langle 0|H_{\text{int}}a^*_r(\boldsymbol{p})b^*_s(-\boldsymbol{p})|0\rangle \; .$$

If the quarks move nonrelativistically in the bound state, then we can make a series expansion of the matrix element in $\boldsymbol{p}$ and keep only the leading term, *i.e.*

$$\langle 0|H_{\text{int}} a_r^*(0) b_s^*(0)|0\rangle .$$

We then find

$$A \simeq (2\pi)^{\frac{3}{2}} \int \mathrm{d}^3 p\, f(\boldsymbol{p}) \sum_{r,s} \varphi(r,s) \langle 0|H_{\text{int}} a_r^*(0) b_s^*(0)|0\rangle .$$

From

$$\psi(\boldsymbol{r}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int f(\boldsymbol{p}) \exp[i\boldsymbol{pr}]\, \mathrm{d}^3 p$$

it follows then that

$$A \simeq \psi(0)(2\pi)^3 \sum_{r,s} \varphi(r,s) \langle 0|H_{\text{int}} a_r^*(0) b_s^*(0)|0\rangle .$$

Remark that $(2\pi)^3 a_r^*(0) b_s^*(0)|0\rangle$ is again the correct normalized state for the quark-antiquark system. Our result says then that in order to calculate the amplitude for an annihilation process, we only need to calculate the annihilation of two free quarks with zero momentum and multiply the final result by $\psi(0)$. Free is taken here in the sense of moving with an effective mass M_{eff} within the potential well. Remark moreover that all our results are independent of M_{eff}.

APPENDIX II

PCAC and the quark model.

We have seen in Sect. **3·3** how we could calculate strong-interaction vertices in the quark model. However, we should remember that these calculations are in a sense outside the quark model because we treat the strongly interacting outcoming pion in an asymmetric way with respect to the other strongly interacting particles. We saw that one of the pions was described by a field and this means that we forget about the quark structure of this pion.

In the strict quark model, it is clearly impossible to calculate the strong coupling of say $\mathcal{N} \to \mathcal{N} + \pi$, because in terms of quarks this is essentially a many-body problem. In order to be able to connect such a strong vertex to a nonrelativistic transition matrix element, we need a supplementary dynamical assumption. A possible one was considered in Sect. **3·3**.

In this Appendix, we will show how the PCAC assumption leads in a quite natural way to a calculation of strong vertices; the final results are the same as those found before. Moreover, this method shows in a clear way how we can solve the ambiguity that we found in the calculation of the $\rho\pi\pi$ coupling [16].

We first give here the general derivation which connects the strong vertex $A \to B + \pi^{(i)}$ to the weak transition matrix element $A \to B$. The matrix element for the two-particle decay $A \to B + \pi^{(i)}$ is connected to the vertex function $\langle B|J^{(i)}(0)|A\rangle$ by

$$M_{A\to B+\pi} = i(2\pi)^4\delta^4(p_A - p_B - k)\langle B|J^{(i)}_{(0)}|A\rangle\,, \tag{43a}$$

where $J^{(i)}_{(0)}$ is the source of the pion field

$$(-\Box + m_\pi^2)\varphi^{(i)}(x) = J^{(i)}(x) \tag{43b}$$

and p_A, p_B k are the momenta of the particles involved. We can write

$$M_{A\to B\pi} = i\int d^4x \exp[ikx]\langle B|J^{(i)}(x)|A\rangle\,.$$

Inserting (43b) we get

$$M_{A\to B\pi} = i\int d^4x \exp[-ikx](k^2 + m_\pi^2)\langle B|\varphi^{(i)}(x)|A\rangle\,. \tag{44}$$

We then take the off-mass-shell limit $k^2 \to 0$ and use PCAC:

$$\partial_\mu A_\mu^{(i)}(x) = C m_\pi^2 \varphi^{(i)}(x) \qquad (i = 1, 2, 3). \tag{45}$$

If we take (45) between two nucleon states we find

$$C = \frac{M_N g_A}{g_{\pi N}}\,,$$

where $g_{\pi N}^2/4\pi = 14.6$ and where g_A assumes the value which our quark model yields for it: $g_A = \frac{5}{3}$; M_N is the nucleon mass. We remark that we use here the nucleon mass and not the quark mass because we know nothing about the matrix element of (45) between two *free* quark states; we only know the effective value of *e.g.* $\varphi^{(i)}(x)$ between *bound* quarks moving with a nonrelativistic velocity. It was this last matrix element which determined $f_q = \frac{3}{5}f$ (see (32)). We have then

$$\begin{aligned} M_{A\to B\pi} &= \frac{1}{C}\, i\int d^4x \exp[ikx]\langle B|\partial_\mu A_\mu^{(i)}(x)|A\rangle\,,\\ &= \frac{1}{C}\, k_\mu^{(i)}\int d^4x \exp[ikx]\langle B|A_\mu^{(i)}(x)|A\rangle\,. \end{aligned} \tag{46}$$

---

[16] A similar approach has been developed by D. SUTHERLAND: *Oxford Thesis* (1966) (unpublished).

If we take now $k_\mu \to 0$, we obtain then for the lowest-order term in k_μ:

$$(47)\qquad M_{\mathrm{A}\to\mathrm{B}\pi} = \frac{1}{C} k_\mu^{(i)} \int \mathrm{d}^4x \langle B|A_\mu^{(i)}(x)|A\rangle = \frac{1}{C} k_\mu^{(i)} (2\pi)^4 \delta^4(p_\mathrm{A} - p_\mathrm{B}) \langle B|A_\mu^{(i)}(x)|A\rangle .$$

We apply now this last formula to the calculation of the $\rho\pi\pi$ coupling constant. Here the two outcoming pions are in a p-wave; because of Bose statistics, only the antisymmetric part of the wave function contributes. We have then clearly for this amplitude

$$(48)\qquad M = \frac{1}{C}\frac{1}{2}\left[k_\mu^{(i)} \langle \pi^{(j)}|A_\mu^{(i)}|\varrho\rangle - k_\mu^{(j)} \langle \pi^{(i)}|A_\mu^{(j)}|\varrho\rangle\right].$$

This result allows us to calculate the strong matrix element for $\rho \to \pi\pi$ in terms of the transition matrix element for a weak decay. In the nonrelativistic quark model the matrix element of, *e.g.*,

$$\langle \pi^+|A_\mu^{(0)}|\varrho^+(s_z = 0)\rangle$$

is given by

$$\langle \pi^+|\sum_i \sigma_{zi}\tau_i^{(0)}|\varrho^+(s_z = 0)\rangle ,$$

where the sum over i goes over the different quarks. For $\rho^+(s_z = 0) \to \pi^+ + \pi^0$ we find then

$$(49)\qquad M_z = \frac{1}{2C}(k_z^{(+)} - k_z^{(0)}) .$$

Using then

$$\frac{1}{2C} = \frac{g_{\pi\mathrm{n}}}{2M_N g_A} = \frac{3f}{5\mu} = \frac{fq}{\mu}$$

(see eqs. (45), (34), (32)), we get then

$$(50)\qquad M_z = \frac{fq}{\mu}(k_z^{(+)} - k_z^{(0)}).$$

This is the same result that we found before in Sect. **3**·3, eq. (36).

Appendix III

Tables for the quark structure ([17]) of the 36 mesons and 56 baryons. For the particles with spin different from zero, we represent only the state with the highest z-component.

([17]) V. F. Weisskopf: $SU_2 \to SU_3 \to SU_6$, Cern report 66-19 (1966).

TABLE I. – *The Mesons.*

The vector mesons	The pseudoscalar mesons
$\varrho^+ = \overset{\uparrow}{\bar n}\overset{\uparrow}{p}$	$\pi^+ = (1/\sqrt{2})(\overset{\uparrow}{\bar n}\overset{\downarrow}{p} - \overset{\downarrow}{\bar n}\overset{\uparrow}{p})$
$\varrho^0 = (1/\sqrt{2})(\overset{\uparrow}{\bar p}\overset{\uparrow}{p} - \overset{\uparrow}{\bar n}\overset{\uparrow}{n})$	$\pi^0 = \frac{1}{2}(\overset{\uparrow}{\bar p}\overset{\downarrow}{p} - \overset{\downarrow}{\bar p}\overset{\uparrow}{p} - \overset{\uparrow}{\bar n}\overset{\downarrow}{n} + \overset{\downarrow}{\bar n}\overset{\uparrow}{n})$
$\varrho^- = \overset{\uparrow}{\bar p}\overset{\uparrow}{n}$	$\pi^- = (1/\sqrt{2})(\overset{\uparrow}{\bar p}\overset{\downarrow}{n} - \overset{\downarrow}{\bar p}\overset{\uparrow}{n})$
$K^{*+} = \overset{\uparrow}{\bar\lambda}\overset{\uparrow}{p}$	$K^+ = (1/\sqrt{2})(\overset{\uparrow}{\bar\lambda}\overset{\downarrow}{p} - \overset{\downarrow}{\bar\lambda}\overset{\uparrow}{p})$
$K^{*0} = \overset{\uparrow}{\bar\lambda}\overset{\uparrow}{n}$	$K^0 = (1/\sqrt{2})(\overset{\uparrow}{\bar\lambda}\overset{\downarrow}{n} - \overset{\downarrow}{\bar\lambda}\overset{\uparrow}{n})$
$\bar K^{*0} = \overset{\uparrow}{\bar n}\overset{\uparrow}{\lambda}$	$\bar K^0 = (1/\sqrt{2})(\overset{\uparrow}{\bar n}\overset{\downarrow}{\lambda} - \overset{\downarrow}{\bar n}\overset{\uparrow}{\lambda})$
$K^{*-} = \overset{\uparrow}{\bar p}\overset{\uparrow}{\lambda}$	$K^- = (1/\sqrt{2})(\overset{\uparrow}{\bar p}\overset{\downarrow}{\lambda} - \overset{\downarrow}{\bar p}\overset{\uparrow}{\lambda})$
$\omega = (1/\sqrt{2})(\overset{\uparrow}{\bar p}\overset{\uparrow}{p} + \overset{\uparrow}{\bar n}\overset{\uparrow}{n})$	$\eta = (1/\sqrt{12})(2\overset{\uparrow}{\bar\lambda}\overset{\downarrow}{\lambda} - 2\overset{\downarrow}{\bar\lambda}\overset{\uparrow}{\lambda} - \overset{\uparrow}{\bar p}\overset{\downarrow}{p} + \overset{\downarrow}{\bar p}\overset{\uparrow}{p} - \overset{\uparrow}{\bar n}\overset{\downarrow}{n} + \overset{\downarrow}{\bar n}\overset{\uparrow}{n})$
$\varphi = \overset{\uparrow}{\bar\lambda}\overset{\uparrow}{\lambda}$	$X^0 = (1/\sqrt{6})(\overset{\uparrow}{\bar p}\overset{\downarrow}{p} - \overset{\downarrow}{\bar p}\overset{\uparrow}{p} + \overset{\uparrow}{\bar n}\overset{\downarrow}{n} - \overset{\downarrow}{\bar n}\overset{\uparrow}{n} + \overset{\uparrow}{\bar\lambda}\overset{\downarrow}{\lambda} - \overset{\downarrow}{\bar\lambda}\overset{\uparrow}{\lambda})$

TABLE II. – *The baryons.*

The Decuplet
$\mathcal{N}^{*++} = \overset{\uparrow}{p}\overset{\uparrow}{p}\overset{\uparrow}{p}$
$\mathcal{N}^{*+} = (1/\sqrt{3})(\overset{\uparrow}{p}\overset{\uparrow}{p}\overset{\uparrow}{n} + \overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{p} + \overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{p})$
$\mathcal{N}^{*0} = (1/\sqrt{3})(\overset{\uparrow}{n}\overset{\uparrow}{n}\overset{\uparrow}{p} + \overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{n} + \overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{n})$
$\mathcal{N}^{*-} = \overset{\uparrow}{n}\overset{\uparrow}{n}\overset{\uparrow}{n}$
$Y^{*+} = (1/\sqrt{3})(\overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{p} + \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} + \overset{\uparrow}{p}\overset{\uparrow}{p}\overset{\uparrow}{\lambda})$
$Y^{*0} = (1/\sqrt{6})(\overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{p} + \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} + \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} + \overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{\lambda} + \overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{\lambda})$
$Y^{*-} = (1/\sqrt{3})(\overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\uparrow}{n}\overset{\uparrow}{\lambda})$
$\Xi^{*0} = (1/\sqrt{3})(\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} + \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{\lambda} + \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda})$
$\Xi^{*-} = (1/\sqrt{3})(\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} + \overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{\lambda} + \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda})$
$\Omega^- = \overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}$

TABLE III. – *The baryon octet.*

$$P = (1/\sqrt{18})\,(2\overset{\uparrow}{p}\overset{\downarrow}{n}\overset{\uparrow}{p} + 2\overset{\uparrow}{p}\overset{\uparrow}{p}\overset{\downarrow}{n} + 2\overset{\downarrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{p} - \overset{\uparrow}{p}\overset{\downarrow}{p}\overset{\uparrow}{n} - \overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\downarrow}{p} - \overset{\downarrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{p} - \overset{\uparrow}{n}\overset{\downarrow}{p}\overset{\uparrow}{p} - \overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\downarrow}{p} - \overset{\downarrow}{p}\overset{\uparrow}{p}\overset{\uparrow}{n})$$

$$N = (1/\sqrt{18})\,(-2\overset{\downarrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{n} - 2\overset{\uparrow}{n}\overset{\uparrow}{n}\overset{\downarrow}{p} - 2\overset{\uparrow}{n}\overset{\downarrow}{p}\overset{\uparrow}{n} + \overset{\uparrow}{p}\overset{\downarrow}{n}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\downarrow}{n} + \overset{\downarrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\downarrow}{n}\overset{\uparrow}{p} + \overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\downarrow}{n} + \overset{\downarrow}{n}\overset{\uparrow}{n}\overset{\uparrow}{p})$$

$$\Sigma^+ = (1/\sqrt{18})\,(2\overset{\uparrow}{p}\overset{\downarrow}{\lambda}\overset{\uparrow}{p} + 2\overset{\uparrow}{p}\overset{\uparrow}{p}\overset{\downarrow}{\lambda} + 2\overset{\downarrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{p} - \overset{\uparrow}{p}\overset{\downarrow}{p}\overset{\uparrow}{\lambda} - \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\downarrow}{p} - \overset{\downarrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} - \overset{\uparrow}{\lambda}\overset{\downarrow}{p}\overset{\uparrow}{p} - \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\downarrow}{p} - \overset{\downarrow}{p}\overset{\uparrow}{p}\overset{\uparrow}{\lambda})$$

$$\Sigma^0 = (1/\sqrt{36})\,(2\overset{\uparrow}{n}\overset{\downarrow}{\lambda}\overset{\uparrow}{p} + 2\overset{\downarrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{p} + 2\overset{\uparrow}{p}\overset{\uparrow}{n}\overset{\downarrow}{\lambda} + 2\overset{\uparrow}{p}\overset{\downarrow}{\lambda}\overset{\uparrow}{n} + 2\overset{\downarrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{n} + 2\overset{\uparrow}{n}\overset{\uparrow}{p}\overset{\downarrow}{\lambda} - \overset{\downarrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} - \overset{\uparrow}{\lambda}\overset{\downarrow}{n}\overset{\uparrow}{p} - \overset{\uparrow}{p}\overset{\downarrow}{n}\overset{\uparrow}{\lambda} -$$
$$- \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\downarrow}{n} - \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\downarrow}{n} - \overset{\downarrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{\lambda} - \overset{\uparrow}{\lambda}\overset{\downarrow}{p}\overset{\uparrow}{n} - \overset{\downarrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} - \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\downarrow}{p} - \overset{?}{n}\overset{?}{p}\overset{\uparrow}{\lambda} - \overset{\downarrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{\lambda} - \overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\downarrow}{p})$$

$$\Sigma^- = (1/\sqrt{18})\,(-2\overset{\uparrow}{n}\overset{\downarrow}{\lambda}\overset{\uparrow}{n} - 2\overset{\uparrow}{n}\overset{\uparrow}{n}\overset{\downarrow}{\lambda} - 2\overset{\downarrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{n} + \overset{\uparrow}{\lambda}\overset{\downarrow}{n}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\downarrow}{n} + \overset{\downarrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} + \overset{\uparrow}{n}\overset{\downarrow}{n}\overset{\uparrow}{\lambda} + \overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\downarrow}{n} + \overset{\downarrow}{n}\overset{\uparrow}{n}\overset{\uparrow}{\lambda})$$

$$\Lambda = (1/\sqrt{12})\,(\overset{\uparrow}{p}\overset{\downarrow}{n}\overset{\uparrow}{\lambda} - \overset{\downarrow}{p}\overset{\uparrow}{n}\overset{\uparrow}{\lambda} - \overset{\uparrow}{n}\overset{\downarrow}{p}\overset{\uparrow}{\lambda} + \overset{\downarrow}{n}\overset{\uparrow}{p}\overset{\uparrow}{\lambda} + \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\downarrow}{n} - \overset{\uparrow}{\lambda}\overset{\downarrow}{p}\overset{\uparrow}{n} - \overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\downarrow}{p} + \overset{\uparrow}{\lambda}\overset{\downarrow}{n}\overset{\uparrow}{p} + \overset{\downarrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{p} -$$
$$- \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\downarrow}{p} - \overset{\downarrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{n} + \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\downarrow}{n})$$

$$\Xi^0 = (1/\sqrt{18})\,(-2\overset{\uparrow}{\lambda}\overset{\downarrow}{p}\overset{\uparrow}{\lambda} - 2\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\downarrow}{p} - 2\overset{\downarrow}{p}\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda} + \overset{\uparrow}{p}\overset{\downarrow}{\lambda}\overset{\uparrow}{\lambda} + \overset{\uparrow}{\lambda}\overset{\uparrow}{p}\overset{\downarrow}{\lambda} + \overset{\downarrow}{\lambda}\overset{\uparrow}{p}\overset{\uparrow}{\lambda} + \overset{\uparrow}{\lambda}\overset{\downarrow}{\lambda}\overset{\uparrow}{p} + \overset{\uparrow}{p}\overset{\uparrow}{\lambda}\overset{\downarrow}{\lambda} + \overset{\downarrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\uparrow}{p})$$

$$\Xi^- = (1/\sqrt{18})\,(2\overset{\uparrow}{\lambda}\overset{\downarrow}{n}\overset{\uparrow}{\lambda} + 2\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\downarrow}{n} + 2\overset{\downarrow}{n}\overset{\uparrow}{\lambda}\overset{\uparrow}{\lambda} - \overset{\uparrow}{n}\overset{\downarrow}{\lambda}\overset{\uparrow}{\lambda} - \overset{\uparrow}{\lambda}\overset{\uparrow}{n}\overset{\downarrow}{\lambda} - \overset{\downarrow}{\lambda}\overset{\uparrow}{n}\overset{\uparrow}{\lambda} - \overset{\uparrow}{\lambda}\overset{\downarrow}{\lambda}\overset{\uparrow}{n} - \overset{\uparrow}{n}\overset{\uparrow}{\lambda}\overset{\downarrow}{\lambda} - \overset{\downarrow}{\lambda}\overset{\uparrow}{\lambda}\overset{\uparrow}{n})$$

RIASSUNTO (*)

Si usa il modello a quark per il calcolo dei rapporti di decadimento dei barioni e dei mesoni. Secondo questo modello i 56 stati barionici contenuti nell'ottetto e nel decupletto fondamentali sono stati S di un sistema di 3 quark, e i 36 stati mesonici negli ottetti scalare e vettoriale sono stati S di un sistema quark-antiquark. Gli stati differiscono fra di loro nelle orientazioni relative dello spin e dello spin unitario dei quark e non molto nelle funzioni d'onda spaziali. Si calcolano i rapporti per le transizioni fra stati che sono: A) per interazione debole — emissione di coppie leptoniche; B) per interazione elettromagnetica — emissione di raggi γ o coppie di elettroni e C) per interazione forte — emissione di pioni. Si calcolano questi rapporti con lo stesso metodo per tutte le transizioni. In tutti i casi l'interazione è trasmessa solo da uno spin-flip (meccanico e/o isotopico). Inoltre gli stessi metodi si prestano al calcolo della annichilazione dei quark e antiquark nei mesoni. accompagnata o dall'emissione di una coppia di leptoni (interazione debole o elettromagnetica) o di una coppia di raggi γ. Questi metodi permettono il calcolo dei rapporti di tutti i processi di decadimento in cui il numero degli

(*) *Traduzione a cura della Redazione.*

adroni rimane invariato, senza fare alcuna ipotesi sull'interazione dei quark. Ci si serve solo delle note costanti di accoppiamento debole, elettromagnetico e forte e del momento magnetico del protone. I processi in cui un mesone scompare dipendono anche dal valore $|\psi(0)|^2$, che è il quadrato della funzione d'onda del sistema quark-antiquark a distanza nulla. Dal confronto con i rapporti osservati risulta che questa grandezza è dell'ordine del fm^{-3} e proporzionale alla massa del mesone. Sulla base del nostro modello si possono fare le seguenti predizioni per le ampiezze parziali dei decadimenti non ancora misurate: $\Sigma^0 \to \Lambda + \gamma$ 8.9 keV, $\omega \to e^+e^-$ 0.63 keV, $\rho \to \eta + \gamma$ 50 keV, $\varphi \to e^+e^-$ 1.0 keV, $\omega \to \eta + \gamma$ 6.3 keV, $\eta \to 2\gamma$ 1.3 keV, $\varphi \to \eta + \gamma$ 0.34 MeV, $\eta \to \pi^0 2\gamma$ 0.53 eV, $\eta \to 2\pi\gamma$ 0.16 keV. Si trova anche $\rho \to e^+e^-$ 5.8 keV, $\pi^0 \to 2\gamma$ 12 eV, $\rho \to 2\pi$ 185 MeV, $\omega \to 3\pi$ 14 MeV, valori che si accordano abbastanza bene coi dati sperimentali.

Процессы распада адронов и модель кварков.

Резюме (*). — Для вычисления интенсивностей распадов барионов и мезонов используется модель кварков. Согласно этой модели, 56 состояний барионов, входящих в фундаментальный октет и декуилет, представляют S-состояния системы трех кварков и 36 состояний мезонов в скалярном и векторном октетах представляют S-состояния системы кварк-антикварк. Состояния отличаются друг от друга относильной ориентацей спина и унитарного спина кварков, а также немного пространственными волновыми функциями. Вычисляются интенсивности для переходов между этими состояниями, которые представляют: A) слабое взаимодействие с излучением лептонной пары, B) электромагнитное взаимодействие с излучением γ-лучей или электронных пар, и C) сильное взаимодействие с излучением пионов. Эти интенсивности вычисляются с помощью единого метода для всех переходов. Во всех случаях взаимойствие осуществляется только за счет переворота спина (механического или изотопического). Кроме того, те же методы используются для вычисления аннигиляции кварка и антикварка в мезоны, которая сопровождается либо излучением лептонной пары (слабое или электромагнитное взаимодействие), либо пары γ-лучей. Эти методы позволяют вычислить интенсивности всех процессов распада, где число адронов остается неизменным, без какихлибо предположений относительно взаимодействий кварков. Все входящие величины являются известными: постоянные слабой, электромагнитной и сильной связи и магнитный момент протона. Процессы, в которых мезон исчезает, также зависят от величины $|\psi(0)|^2$, которая представляет квадрат волновой функции системы кварк-антикварк при нулевом расстоянии. Из сравнения с наблюдаемыми интенсивностями оказывается, что эта величина равна по порядку $(фм)^{-3}$ и пропорциональна массе мезона. На основе нашей модели можно сделать следующие предсказания для парциальной ширины распадов, которые еще не измерены: $\Sigma^0 \to \Lambda + \gamma$ 8.3 кэВ, $\omega \to e^+e^-$ 0.63 кэВ, $\rho \to \eta + \gamma$ 50 кэВ, $\varphi \to e^+e^-$ 1.0 кэВ, $\omega \to \eta + \gamma$ 6.3 кэВ, $\eta \to 2\gamma$ 0.45 кэВ, $\varphi \to \eta + \gamma$ 0.34 МэВ, $\eta \to \pi^0 2\gamma$ 0.53 кэВ, $\eta \to 2\pi\gamma$ 0.16 кэВ. Также оказывается, что: $\rho \to e^+e^-$ 5.8 кэВ, $\pi^0 \to 2\gamma$ 12 эВ, $\rho \to 2\pi$ 185 МэВ, $\omega \to 3\pi$ 14 МэВ, которые согласуетса очень хорошо с экспериментальыми данными.

(*) *Переведено редакцией.*

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