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Rediscovering the Newton-Wigner Operator from a Space-time Description of Quantum Field Theory.

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Summary. — The Newton-Wigner position operator is rediscovered directly from a space-time description of quantum field theory which is based on the actual physical situation where particles travel between emitters and detectors. It avoids methods used in nonrelativistic quantum physics and so-called wave equations.

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In view of the numerous experiments on the localization of particles by detectors [1], a space-time description of quantum field theory was developed [2] where actual probabilistic questions in *configuration* space may be asked in collision and decay processes [3]. The method is based on the physical situation where particles travel between emitters and detectors. The solution results from a *unitarity* expansion in configuration space of the so-called vacuum-to-vacuum transition amplitude [2, 4, 5]. The severe test of the associated completeness relation and the resulting consistent probabilistic interpretation is what has led us to this investigation. Although the so-called position operator is more in the spirit of nonrelativistic quantum physics, we show *directly* from our formulation that the derived position operator coincides, unambiguously, with the Newton-Wigner operator [6]. Unlike the present study, the latter ones [6] were in the spirit of nonrelativistic quantum physics, and also gave no hope in treating particle-particle interactions [3] in space-time.

Consider, for example, a scalar particle interacting with an external source $K(x)$. Then the vacuum-to-vacuum transition amplitude is given by the well-known expression [2, 4, 5]

$$\langle 0_+ | 0_- \rangle^K = \exp\left[\frac{i}{2} \int (\mathrm{d}x)(\mathrm{d}x')K(x)\Delta_+(x-x')K(x') \right],$$

$$\Delta_+(x-x') = i \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \frac{1}{2k^0} \exp[ik(x-x')], \qquad \text{for } x^0 > x'^0,$$

$k^0 = +\sqrt{\boldsymbol{k}^2 + m^2}$. Let $K(x) = K_1(x) + K_2(x)$, where the source $K_2(x)$ is switched on after the source $K_1(x)$ is switched off. Let y^0 denote any time in the intermediate range values between the times when K_1 is switched off and K_2 is switched on. In other words, at time y^0, both sources K_1, K_2 are not in operation. Then

$$\langle 0_+ | 0_- \rangle^K = \langle 0_+ | 0_- \rangle^{K_2} \exp\left[\int \mathrm{d}^3 \boldsymbol{y} (ia_2(y)^*)(ia_1(y)) \right] \langle 0_+ | 0_- \rangle^{K_1},$$

where

$$a(y) = \int (\mathrm{d}x)\Delta(y-x)K(x) = \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \frac{1}{\sqrt{2k^0}} \exp[iky]K(k),$$

$$\Delta(y-x) = \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \frac{\exp[ik(y-x)]}{\sqrt{2k^0}}$$

and the important equality

$$\int \mathrm{d}^3 \boldsymbol{y} |a(y)|^2 = \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \frac{1}{2k^0} |K(k)|^2.$$

Given that the source $K(x)$ has emitted one particle, then the (conditional) probability that it is found in a region $\Delta \subset R^3$, at time y^0, is [2, 3]

$$\int_\Delta \mathrm{d}^3 \boldsymbol{y} |\Phi(y)|^2,$$

where

$$\Phi(y) = Na(y), \qquad N\left(\int \mathrm{d}^3 \boldsymbol{y} |a(y)|^2 \right)^{-1/2},$$

defining the *spectral measure* of the position operator. In particular, the expectation value $E[Y^j]$ of a component Y^j of the position operator $\boldsymbol{Y}$ is then

$$E[Y^j] = \int_{R^3} \mathrm{d}^3 \boldsymbol{y}\, y^j |\Phi(y)|^2,$$

and from the definition of $a(y)$, the latter is given by

$$E[Y^j] = N\int \mathrm{d}^3\boldsymbol{y}\int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3}\exp\left[\frac{[-ik'y]}{\sqrt{2k'^0}}\right]\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{\sqrt{2k^0}}K^*(k')K(k)\left(-i\frac{\partial}{\partial k^j}\right)\exp[iky] =$$

$$= N\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{\sqrt{2k^0}}K^*(k)\left(i\frac{\partial}{\partial k^j}\frac{K(k)}{\sqrt{2k^0}}\right) =$$

$$= N\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2k^0}K^*(k)i\left[\frac{\partial}{\partial k^j} - \frac{k^j}{2k^{02}}\right]K(k),$$

and hence

$$Y^j = i\left[\frac{\partial}{\partial k^j} - \frac{k^j}{2k^{02}}\right],$$

coinciding with the Newton-Wigner operator.

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