A New OPEP.

C. VILLI

Istituto di Fisica dell'Università - Padova
Istituto Nazionale di Fisica Nucleare - Sezione di Padova

(ricevuto il 4 Agosto 1981)

Summary. — The one-pion exchange nucleon-nucleon potential (OPEP) is constructed in the framework of the static PS-PV theory of nuclear forces, taking into account Orear's data concerning the presumptive spatial dimension of the quark confinement region. The little-bag model, outlined by Brown and Rho, is developed on the basis of the conjecture that the spherically symmetric meson source should be nonuniform. The physical content of the chosen meson density is interpreted in terms of a Pais-Uhlenbeck nonlocal model of the nucleon. It is shown that the potential thus derived possesses realistic physical features (at least for even-parity triplet states) without resorting to the *ad hoc* introduction of the hard-core radius and to cutting-off procedures at short distances.

1. – Foreword.

It is often said that, as a man grows old, he turns back to those problems that worried him in his youth; this is perhaps the reason why over the past years my attention has again been attracted by the theoretical jungle which proliferated in the years '50 and '60 under the stimulus of innumerable attempts to construct the meson potential. None of them was conceived with the aim of giving a clear-cut answer to the following questions (1): *a*) How can a nucleon model be chosen with some hope of it being a correct model? *b*) How can one avoid putting tremendous efforts into calculations on a model which has no connection with reality? *c*) How can experimental data be used to discard wrong models and enable the research to converge towards the correct

(1) H. J. LIPKIN: *Phys. Rep.*, **3**, 173 (1973).

one? To avoid discussing false problems, I have printed in my mind, as a personal *vademecum*, the shocking opinion expressed by GOLDBERGER (2) twenty years ago: « There are few problems in modern theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between the nucleons. It is also true that scarcely ever has the world of physics owed so little to so many.... In general, in surveying the field, one is oppressed by the unbelievable confusion and conflict that exist. It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is ».

2. – Orear's data, the Brown and Rho little-bag model and the nucleon-nucleon potential.

Recent theoretical developments, stimulated by the construction of extended models of hadrons (3), will probably disclose new perspectives to a better understanding of the nucleon-nucleon interaction: so far, however, the utmost implications of such models are still unexplored. A quark bag model for nucleons (the so-called « little bag ») has been outlined by BROWN and RHO (4): their concluding remark is that « the resulting theory looks very much like the old Yukawa theory for a pion with a small distributed source, the source being now described in terms of quarks ». Let us recall that (by using standard notations) the nucleon-nucleon potential, deduced from the old Yukawa theory for one pion with a pointlike source, reads

$$U(r) = U_C(r) + S_{12}\,U_T(r) \equiv U_0(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,u_C(r) + S_{12}\,u_T(r)\}\,, \tag{2.1a}$$

$$u_C(r) = (1/\mu^3)\{\mu^2 F(\mu r) - 4\pi\delta(\boldsymbol{r})\}\,, \tag{2.1b'}$$

$$F(\mu r) = \exp[-\mu r]/r \equiv f(\mu r)/r\,, \tag{2.1b''}$$

$$u_T(r) = (1/\mu)^3\{3/r^3 + 3/r^2 + 1/r\}\,f(\mu r)\,, \tag{2.1c}$$

$$U_0 = (\mu c^2/3)(\mu/2M)^2\,g^2 = 0.249\,g^2\,\text{MeV}\,, \tag{2.1d}$$

the pion-nucleon coupling constant g^2 being an adimensional parameter: in the older literature it was customary to omit in $u_C(r)$ the delta-function, be-

---

(2) M. L. GOLDBERGER: *Proceedings of the Midwest Conference on Theoretical Physics*, (1960), p. 50.

(3) J. KOGUT and L. SUSSKIND: *Phys. Rep.* **8**, 75 (1973); A. CHODOS, R. L. JAFFE, K. JOHNSON and V. F. WEISSKOPF: *Phys. Rev. D*, **9**, 347 (1974); A. CHODOS, R. L. JAFFE, K. JOHNSON and G. B. THORN: *Phys. Rev. D*, **10**, 2499 (1974); G. T. FAIRLEY and E. J. SQUIRES: *Nucl. Phys. D*, **93**, 56 (1975); K. JOHNSON: *Acta Phys. Pol. B*, **6**, 865 (1975); P. HASENFRATZ and J. KUTI: *Phys. Rep. C*, **40**, 75 (1978).

(4) G. E. BROWN and M. RHO: *Phys. Lett. B*, **82**, 177 (1979).

cause it does not influence the wave function of the two interacting nucleons (*). The pathological aspects concealed in (2.1) have bothered many theorists for about forty years; the *ad hoc* introduction of cutting-off procedures at short distances ([5]) is theoretically clumsy. The purpose of this note is to set up a physically reasonable description of the one-pion exchange potential (OPEP) which does not have a tensor divergence like $1/r^3$ from the beginning, rather than to remove it by a suitable fudge (the hard core of radius r_c) after its appearance, and then extend such a fudge also to the even-triplet central part of the potential, in spite of the fact that its $1/r$ divergence at the origin is not in conflict with the existence of the deuteron bound state.

I shall attempt to construct a new OPEP taking into account that large-angle proton-proton scattering experiments ([6]) and the observed momentum distribution of muon and hadron pairs ([7]) have been explained by OREAR ([8]) assuming that the r.m.s. radius of the quark confinement region is, respectively,

$$(2.2) \qquad R_0 = (0.23 \pm 0.03)\cdot 10^{-13}\ \text{cm}\,, \qquad R_1 = (0.20 \pm 0.03)\cdot 10^{-13}\ \text{cm}\,.$$

BROWN and RHO have pointed out that the length R_0 can be considered as the r.m.s. radius of a uniform distribution of nucleon matter in a sphere of radius

$$(2.3) \qquad R \simeq \frac{3}{2}\,\hbar/Mc\,,$$

where M is the nucleon mass. We shall conjecture that such a distribution is spherically symmetric but nonuniform, and assume that its radial behaviour can be tentatively described by the 3-parameter function (normalized to 1)

$$(2.4) \qquad \varrho(r) = \lambda(m_1^2/4\pi)F(m_1 r) + (1-\lambda)(m_2^2/4\pi)F(m_2 r)\,,$$

(*) The numerical values given in this note have been calculated by assuming $Mc^2 = 938.9$ MeV and $\mu c^2 = 138.1$ MeV.

([5]) H. A. BETHE: *Phys. Rev.*, **57**, 260, 390 (1940).

([6]) J. HARTMANN. J. OREAR, J. VRIESLANDER, S. CONETTI, C. HOJVAT, D. G. RYAN, K. SHAHBAZIAN, D. G. STAIRS, J. TRISCHUK, W. FAISSLER, M. GETTNER, J. R. JOHNSON, T. KEPHART, E. POTHIER, D. POTTER, M. TAUTZ, P. BARANOV and S. RUSAKOV: *Phys. Rev. Lett.*, **39**, 975 (1977); H. DE KERRET, E. NAGY, R. S. ORR, M. REGLER, W. SCHMIDT-PARZEFAILL, K. WINTER, A. BRANDT, F. W. BÜSSER, H. H. DIBUM, G. FLÜGGE, F. NIEBERGALL, P. E. SCHUMACHER, K. R. SCHIFFERT, J. J. AUBERT, C. BROILL, G. COIGNET, J. FAVIER, L. MASSONNET, M. VIVARGENT, W. BARTL, H. EICHINGER, CH. GOTTFRIED and G. NEUHOFER: *Phys. Lett. B*, **68**, 374 (1977).

([7]) D. M. KAPLAN, R. J. FISK, A. S. ITO, H. JÖSTLEIN, J. A. APPEL, B. C. BROWN, C. N. BROWN, W. R. INNES, R. D. KEPHART, K. UENO, T. YAMANOUCHI, S. W. HERB, D. C. HOM, L. M. LEDERMAN, J. C. SENS, H. D. SNYDER and J. K. YOH: *Phys. Rev. Lett.*, **40**, 435 (1978); R. D. KEPHART: *Phys. Rev. Lett.*, **39**, 1440 (1977).

([8]) J. OREAR: *Phys. Rev. D*, **18**, 2484 (1978).

where, if we use the notation (2.1b''), $F(m_i r)$ are Yukawa functions, λ is an adimensional unknown parameter and m_i ($i = 1, 2$) are two unknown parameters homogeneous (in units $\hbar = c = 1$) to the inverse of a length. The Compton wave-length of the nucleon is $\hbar/Mc \simeq 0.21$ fm; since the average of the central values of (2.2) is 0.215 fm, we shall take into account Orear's data by putting

$$R_0 \simeq R_1 = \hbar/Mc\,. \tag{2.5}$$

As is well known, general quantum-mechanical arguments lead to the conclusion that the interaction between two nucleons is spread out at least over a distance of the order $\hbar/Mc$: it is remarkable that such a length is found to be very close to the r.m.s. radius of the conjectured quark confinement region. A first relation among the three parameters (m_1, m_2; λ) is obtained by equating the r.m.s. radius of (2.4) with the square of the Compton wave-length of the nucleon: it is found that

$$\frac{1}{6M^2} = \frac{\lambda}{m_1^2} + \frac{1-\lambda}{m_2^2}\,. \tag{2.6}$$

Two other independent relations are needed in order to specify the density (2.4): one of them will be derived from the prescription that the asymptotic behaviour of the nucleon-nucleon potential coincides with that given by the old Yukawa theory (sect. **4**), while the other will be deduced from the deuteron ground state (sect. **5**).

3. – A Pais-Uhlenbeck model of the nucleon.

It is interesting to investigate preliminarily what the physical content of the density (2.4), as concealed behind its analytical form, is (or might be). The little-bag model assumes that pions exist only outside the bag: the pion field is introduced so as to ensure the continuity of the axial vector current at the boundary of the bag. The opposite point of view consists in assuming that the pion field also exists in the interior of the bag. As a compromise between these two extreme points of view, I shall heuristically assume that *a*) a field Φ_1 is coupled (with strength g_1) to an unknown meson source $\mathscr{S}$, sunk in the nucleonic bag; *b*) the field Φ_1 interacts with an intermediate field Φ_2 (with coupling constant g_2) and *c*) the field Φ_2 interacts (with coupling constant g_3) with the pion field Φ_3: thus the pion field is not directly coupled to the source $\mathscr{S}$, but only through the linking field Φ_2.

Let us suppose that Φ_i ($i = 1, 2, 3$) are neutral and scalar fields. The Hamiltonian of the nucleon system is $\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_\text{I}$, where $\mathscr{H}_0$ and $\mathscr{H}_\text{I}$ are the unperturbed and, respectively, the interaction Hamiltonians. Let Π_i be the variables canonically conjugate to Φ_i; one has

$$(3.1a)\qquad \mathscr{H}_0 = \sum_{i=1}^{3} \mathscr{H}_{0i}\,, \qquad \mathscr{H}_{0i} = \tfrac{1}{2}\int \{\Pi_i^2 + (\nabla\Phi_i)^2 + m_i^2\Phi_i^2\}\,\mathrm{d}\boldsymbol{r}\,,$$

$$(3.1b)\qquad \mathscr{H}_{\mathrm{I}} = -\,g_1\mathscr{S}\Phi_1 - g_2\Phi_1\Phi_2 - g_3\Phi_2\Phi_3\,,$$

where $m_3 \equiv \mu$. Using the equations

$$(3.2)\qquad \delta\mathscr{H}/\delta\Phi_i = -\,\dot{\Pi}_i\,, \qquad \delta\mathscr{H}/\delta\Pi_i = \dot{\Phi}_i = \Pi_i\,,$$

where δ indicates variational differentiation, from (3.1) one obtains the system of coupled second-order differential equations

$$(3.3a)\qquad (\Box + m_1^2)\,\Phi_1 = g_1\mathscr{S} + g_2\Phi_2\,,$$

$$(3.3b)\qquad (\Box + m_2^2)\,\Phi_2 = g_2\Phi_1 + g_3\Phi_3\,,$$

$$(3.3c)\qquad (\Box + \mu^2)\,\Phi_3 = g_3\Phi_2\,,$$

$\Box$ being the d'Alembertian operator. From eqs. (3.3a), (3.3b) one deduces the fourth-order differential equation

$$(3.4)\qquad (\boldsymbol{\Omega} - g_2^2)\,\Phi_2 = g_3(\Box + m_1^2)\,\Phi_3 + g_1 g_2\mathscr{S}\,,$$

where $\boldsymbol{\Omega} = (\Box + m_1^2)(\Box + m_2^2)$ is the Pais-Uhlenbeck operator ([9]). The internal logic of the model is expressed by the inequalities

$$(3.5)\qquad m_1 > m_2 > m_3 \equiv \mu\,, \qquad g_1 \gg g_2 > g_3\,;$$

since $g_3 \ll g_1 g_2$, it is a plausible approximation to neglect the former term on the right-hand side of eq. (3.4). Let us suppose that the fields Φ_i are time independent and spherically symmetric and indicate with $\varphi_i(r)$ the functions describing their radial behaviour; furthermore, we make up for our complete ignorance of the spatial features of the source $\mathscr{S}$ (experimentally not accessible through the available accelerators) by resorting for its description to the delta-function $\delta(\boldsymbol{r})$. From eqs. (3.3c) and (3.4) one obtains

$$(3.6a)\qquad (\nabla^2 - \mu^2)\,\varphi_3(r) = -\,g_3\varphi_2(r)\,,$$

$$(3.6b)\qquad \{(\nabla^2 - m_1^2)(\nabla^2 - m_2^2) - g_2^2\}\,\varphi_2(r) = -\,g_1 g_2\,\delta(\boldsymbol{r})\,,$$

([9]) A. Pais and G. E. Uhlenbeck: *Phys. Rev.*, **79**, 145 (1950); P. Budini and L. Fonda: *Nuovo Cimento* **5**, 666 (1967).

∇^2 being the radial part of the Laplacian operator. The physical meaning of eqs. (3.6) is evident: the Klein-Gordon equation (3.6a) describes (in scalar and neutral approximation) the radial behaviour of the pion field in the presence of an effective meson source proportional to $\varphi_2(r)$, the radial dependence of which is governed by the Pais-Unlenbeck equation (3.6b). For $r \neq 0$ the latter equation becomes

$$(\mathbf{\Omega}_s + g_2^2)\varphi_2(r) = 0\,, \tag{3.7a}$$

where $\mathbf{\Omega}_s$ is the static part of the Pais-Uhlenbeck operator, *i.e.*

$$\mathbf{\Omega}_s = \boldsymbol{D}^4 + (1/r)\boldsymbol{D}^3 - (m_1^2 + m_2^2)\nabla^2 + m_1^2 m_2^2\,, \tag{3.7b}$$

with $\boldsymbol{D} = \mathrm{d}/\mathrm{d}r$.

Let us put $\varphi_2(r) = y(r)/r$; from (3.7) it is found that

$$\{\boldsymbol{D}^4 - (m_1^2 + m_2^2)\boldsymbol{D}^2 + m_1^2 m_2^2 + g_2^2\}\, y(r) = 0\,; \tag{3.8}$$

the general integral of (3.8), satisfying the condition $y(\infty) = 0$, is

$$\varphi_2(r) = -\, g_1 g_2 \{c_1 F(\alpha_1 r) + c_2 F(\alpha_2 r)\} \equiv -\, g_1 g_2 \sigma(r)\,, \tag{3.9}$$

where c_1 and c_2 are arbitrary constants of integration, $F(\alpha_i r)$ are Yukawa functions and

$$\alpha_1^2 = \tfrac{1}{2}\{m_1^2 + m_2^2 + \sqrt{(m_1^2 - m_2^2)^2 - 4g_2^2}\}\,, \tag{3.10a}$$

$$\alpha_2^2 = \tfrac{1}{2}\{m_1^2 + m_2^2 - \sqrt{(m_1^2 - m_2^2)^2 - 4g_2^2}\}\,. \tag{3.10b}$$

Assuming $4g_2^2 \ll (m_1^2 - m_2^2)^2$, from (3.10) one gets $\alpha_1 = m_1$ and $\alpha_2 = m_2$; the density $\sigma(r)$, defined in (3.9), is normalized to 1 independently of the values of m_1 and m_2, provided

$$c_1 = \lambda(m^2/4\pi)\,, \qquad c_2 = (1 - \lambda)(m_2^2/4\pi)\,. \tag{3.11}$$

Thus one finds that $\sigma(r)$ has the analytical form of $\varrho(r)$, given in (2.4); according to (3.6a), the effective pion-nucleon coupling constant turns out to be

$$g = g_1 g_2 g_3\,. \tag{3.12}$$

4. – A new OPEP.

Let us introduce the density (2.4) into the symmetrical pseudoscalar theory with pseudovector coupling. To overcome serious analytical difficulties (*a posteriori* proved to be physically irrelevant), the static second-order nucleon-

nucleon potential will be calculated as follows:

$$V(r) = (g/2M)^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\, v(r)\,, \tag{4.1a}$$

$$v(r) = \int \varrho(|\boldsymbol{r}'-\boldsymbol{r}_1|)\, G(\mu|\boldsymbol{r}'-\boldsymbol{r}''|)\, \varrho(|\boldsymbol{r}''-\boldsymbol{r}_2|)\,\mathrm{d}\boldsymbol{r}'\,\mathrm{d}\boldsymbol{r}''\,, \tag{4.1b}$$

where $r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ is the internucleon distance and $G \equiv F$ is the Green's function for the pion field. The PS-PV coupling constant appearing in the potential (4.1) has been deliberately indicated with the same symbol used in the scheme that led us to relation (3.12): the PS-PV coupling constant can be determined only by comparing theoretical predictions with experimental evidences, while the illustrative relation (3.12) shows, in a naïve way, that the remote origin of the phenomenologically determined coupling constant lies in the still unexplored meson clouds which are intertwined in the inner regions of the nucleon structure.

To evaluate the function $v(r)$ we put

$$\boldsymbol{r}'-\boldsymbol{r}_1 = \boldsymbol{X}\,,\qquad \boldsymbol{r}''-\boldsymbol{r}_2 = \boldsymbol{Y}\,,\qquad \boldsymbol{r}'-\boldsymbol{r}'' = \boldsymbol{Z}\,,\qquad \boldsymbol{r}'-\boldsymbol{r}_2 = \boldsymbol{U}\,; \tag{4.2}$$

the double volume element, suitable for a straightforward evaluation of the integral (4.1b), is

$$\mathrm{d}\boldsymbol{r}'\,\mathrm{d}\boldsymbol{r}'' = (1/r)\, XYZ\,\mathrm{d}X\,\mathrm{d}Y\,\mathrm{d}Z\,\mathrm{d}U\,\mathrm{d}\beta_1\,\mathrm{d}\beta_2\,. \tag{4.3}$$

The geometrical choices underlying the deduction of (4.3) are summarized in the following relations:

$$\left\{\begin{array}{llll} \mathrm{d}\boldsymbol{r}' & = X^2\,\mathrm{d}X\,\sin\alpha_1\,\mathrm{d}\alpha_1\,\mathrm{d}\beta_1\,, & \mathrm{d}\boldsymbol{r}'' & = Y^2\,\mathrm{d}Y\,\sin\alpha_2\,\mathrm{d}\alpha_2\,\mathrm{d}\beta_2\,,\\ U^2 & = X^2 + r^2 - 2Xr\cos\alpha_1\,, & Z^2 & = Y^2 + U^2 - 2UY\cos\alpha_2\,,\\ U\,\mathrm{d}U & = Xr\sin\alpha_1\,\mathrm{d}\alpha_1\,, & Z\,\mathrm{d}Z & = UY\sin\alpha_2\,\mathrm{d}\alpha_2\,,\\ \mathrm{d}\boldsymbol{r}' & = (1/r)\,UX\,\mathrm{d}U\,\mathrm{d}X\,\mathrm{d}\beta_1\,, & \mathrm{d}\boldsymbol{r}'' & = (1/U)\,YZ\,\mathrm{d}Y\,\mathrm{d}Z\,\mathrm{d}\beta_2\,; \end{array}\right. \tag{4.4}$$

the ranges of integration are

$$\left\{\begin{array}{ll} |X-r| \leqslant U \leqslant X+r\,, & 0 \leqslant Y \leqslant \infty,\\ |Y-U| \leqslant Z \leqslant Y+U\,, & 0 \leqslant Z \leqslant \infty. \end{array}\right. \tag{4.5}$$

The radial function (4.1b) explicitly reads ($i, j = 1, 2$)

$$v(r) = \lambda^2 v_{11}(r) + \lambda(1-\lambda)\{v_{12}(r) + v_{21}(r)\} + (1-\lambda)^2 v_{22}(r)\,, \tag{4.6}$$

$$v_{ij}(r) = (m_i^2 m_j^2/4\pi r)\, I_{ij}(r)\,, \tag{4.7a}$$

(4.7b) $$I_{ij}(r)=\int_0^r f(m_iX)\,\mathrm{d}X\int_{r-X}^{r+X}J_j(U)\,\mathrm{d}U+\int_r^\infty f(m_iX)\,\mathrm{d}X\int_{X-r}^{r+X}J_j(U)\,\mathrm{d}U\,,$$

(4.7c) $$J_j(U)=\int_0^U f(m_jY)\,\mathrm{d}Y\int_{U-Y}^{U+Y}f(\mu Z)\,\mathrm{d}Z+\int_U^\infty f(m_jY)\,\mathrm{d}Y\int_{Y-U}^{U+Y}f(\mu Z)\,\mathrm{d}Z\,,$$

where, according to the notations previously adopted, $f(m_ir)\equiv\exp[-m_ir]$. It is found that $\xi(m_i)=m_i^2/(m_i^2-\mu^2)$ and

(4.8a) $$v_{ii}(r)=\xi^2(m_i)[F(\mu r)-F(m_ir)-\{(m_i^2-\mu^2)/2m_i\}f(m_ir)]\,,$$

(4.8b) $$v_{12}(r)+v_{21}(r)=2\xi(m_1)\xi(m_2)\cdot\\ \cdot\left[F(\mu r)-\left(\frac{m_1^2-\mu^2}{m_1^2-m_2^2}\right)F(m_2r)+\left(\frac{m_2^2-\mu^2}{m_1^2-m_2^2}\right)F(m_1r)\right].$$

If we take into account that

(4.9a) $$(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})=\tfrac{1}{3}\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{\Omega}_{\mathrm{C}}+S_{12}\boldsymbol{\Omega}_{\mathrm{T}}\}\,,$$

(4.9b) $$\boldsymbol{\Omega}_{\mathrm{C}}=\boldsymbol{D}^2+(2/r)\boldsymbol{D}\,,\qquad \boldsymbol{\Omega}_{\mathrm{T}}=\boldsymbol{D}^2-(1/r)\boldsymbol{D}\,,$$

where $\boldsymbol{D}=\mathrm{d}/\mathrm{d}r$, the nucleon-nucleon potential (4.1) becomes

(4.10) $$V(r)=V_{\mathrm{C}}(r)+S_{12}V_{\mathrm{T}}(r)\equiv U_0(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)v_{\mathrm{C}}(r)+S_{12}v_{\mathrm{T}}(r)\}\,,$$

where U_0 is given by (2.1d), and

(4.11) $$\mu v_{\mathrm{C}}(r)=\boldsymbol{\Omega}_{\mathrm{C}}v(r)\,,\qquad \mu^3v_{\mathrm{T}}(r)=\boldsymbol{\Omega}_{\mathrm{T}}v(r)$$

are functions characterizing the radial behaviour of the central and, respectively, the tensor part of the potential.

Let us examine the first of eqs. (4.11). Since

(4.12) $$\boldsymbol{\Omega}_{\mathrm{C}}F(m_ir)=m_i^2F(m_ir)\,,\qquad \boldsymbol{\Omega}_{\mathrm{C}}f(m_ir)=m_i^2f(m_ir)-2m_iF(m_ir)\,,$$

one obtains $(m_3\equiv\mu)$

(4.13) $$v_{\mathrm{C}}(r)=\sum_{i=1}^{3}\{(A_i/\mu)F(m_ir)+B_if(m_ir)\}\,,$$

(4.14a) $$\begin{cases} A_1=-\dfrac{m_1^4\lambda^2}{(m_1^2-\mu^2)^2}+\dfrac{2m_1^4m_2^2\lambda(1-\lambda)}{\mu^2(m_1^2-\mu^2)(m_1^2-m_2^2)}\,,\\[2ex] A_2=-\dfrac{m_2^4(1-\lambda)^2}{(m_2^2-\mu^2)^2}-\dfrac{2m_1^2m_2^4\lambda(1-\lambda)}{\mu^2(m_2^2-\mu^2)(m_1^2-m_2^2)}\,,\\[2ex] A_3=\dfrac{m_1^4\lambda^2}{(m_1^2-\mu^2)^2}+\dfrac{2m_1^2m_2^2\lambda(1-\lambda)}{(m_1^2-\mu^2)(m_2^2-\mu^2)}+\dfrac{m_2^4(1-\lambda)^2}{(m_2^2-\mu^2)^2}\,,\end{cases}$$

$$(4.14b)\quad \begin{cases} B_1 = -\dfrac{m_1^5\lambda^2}{2\mu^3(m_1^2-\mu^2)}\,, \\ B_2 = -\dfrac{m_2^5(1-\lambda)^2}{2\mu^3(m_2^2-\mu^2)}\,, \\ B_3 = 0\,. \end{cases}$$

It is readily proved that the following relations hold independently of λ, namely

$$(4.15a)\qquad A_1 + A_2 + A_3 = 0\,,$$

$$(4.15b)\qquad \frac{1}{\mu}\left(\frac{A_1}{m_1^2}+\frac{A_2}{m_2^2}+\frac{A_3}{\mu^2}\right)+\frac{2B_1}{m_1^3}+\frac{2B_2}{m_2^3}=0\,.$$

From eq. (4.15b) it follows that the central potential possesses the property

$$(4.16)\qquad \int V_C(r)\,\mathrm{d}\boldsymbol{r} = 0\,;$$

the same property is also possessed by $U_C(r)$ given in (2.1).

To preserve for $\mu r > 1$ the asymptotic tail of $V_C(r)$, which is experimentally well established, we require that the condition

$$(4.17)\qquad A_3 = 1$$

should be satisfied. From the third of eqs. (4.14a) one obtains the second-degree equation in

$$(4.18)\qquad \lambda^2 - \frac{2m_2^2(m_1^2-\mu^2)\,\lambda}{\mu^2(m_1^2-m_2^2)} + \frac{(2m_2^2-\mu^2)(m_1^2-\mu^2)^2}{\mu^2(m_1^2-m_2^2)^2} = 0\,;$$

the only physically meaningful solution of eq. (4.18) is

$$(4.19)\qquad \lambda \equiv \lambda(m_1, m_2) = (m_1^2-\mu^2)/(m_1^2-m_2^2)\,.$$

As required by the logic underlying the model, one has

$$(4.20)\qquad \lambda(\infty, m_2) = 1\,;$$

it follows that in the limit $m_1 \to \infty$ the central part of the new OPEP coincides for $r > 0$ with the prediction of the old PS-PV Yukawa theory. The adimensional coefficients (4.14) of the function (4.13) take the form ($m_3 \equiv \mu$)

$$(4.21a)\qquad \begin{cases} A_1 = -\left(\dfrac{m_1^2}{m_1^2 - m_2^2}\right)^2 \left[1 + \dfrac{2m_2^2(m_2^2 - \mu^2)}{\mu^2(m_1^2 - m_2^2)}\right], \\ A_2 = \left(\dfrac{m_2^2}{m_1^2 - m_2^2}\right)^2 \left[\dfrac{2m_1^2(m_1^2 - \mu^2)}{\mu^2(m_1^2 - m_2^2)} - 1\right], \\ A_3 = 1\,, \end{cases}$$

$$(4.21b)\qquad A_1 + A_2 + 1 = 0\,,$$

$$(4.21c)\qquad \begin{cases} B_1 = -\dfrac{m_1^5(m_1^2 - \mu^2)}{2\mu^3(m_1^2 - m_2^2)^2}\,, \\ B_2 = -\dfrac{m_2^5(m_2^2 - \mu^2)}{2\mu^3(m_1^2 - m_2^2)^2}\,, \\ B_3 = 0\,. \end{cases}$$

The dependence of the central potential coefficients (4.21) on m_1 and m_2, correlated by Orear's constraints (2.6) in the intervals specified in (5.2), is shown in table I.

TABLE I. – *Numerical values of the central coefficients* (4.21) ($A_3 = 1$; $B_3 = 0$) *as functions of the parameters* m_1 *and* m_2, *correlated by Orear's constraint* (2.6) *in the intervals specified in* (5.2).

m_2	m_1	$-A_1$	A_2	$-B_1$	$-B_2$
1.50μ	11.10μ	1.086	0.086	703.733	$< 10^{-3}$
2.00μ	8.33μ	1.539	0.539	320.754	0.011
2.50μ	6.66μ	3.688	2.688	195.618	0.176
3.00μ	5.55μ	15.179	14.179	165.073	2.045
3.50μ	4.76μ	130.253	129.253	244.295	27.275
4.00μ	4.16μ	64 768.486	64 767.486	5958.722	4505.479

The physical content of $V_C(r)$ becomes more evident by writing the radial function (4.13) in the form

$$(4.22a)\qquad v_C(r) = (1/\mu^3)\{\mu^2 F(\mu r) - 4\pi d(r)\}\,,$$

$$(4.22b)\qquad d(r) = -(\mu^3/4\pi)\sum_{i=1}^{3}\{(A_i/\mu)F(m_i r) + B_i f(m_i r)\}\,;$$

the comparison of (4.21) with $u_C(r)$, given in (2.1b'), shows that the modification of the central OPEP, induced by the source (2.4), consists in replacing the contact interaction term with the function $d(r)$. Because of the spatial spreading of the delta-function, the Fourier transform (instead of being equal to 1 for all q) is

$$(4.23a)\qquad s(q^2;\, m_1,\, m_2) = \int d(r) \exp\,[i\boldsymbol{q}\cdot\boldsymbol{r}]\,\mathrm{d}\boldsymbol{r}\,,$$

$$(4.23b)\qquad s(q^2;\, m_1,\, m_2) = -\sum_{i=1}^{2}\left\{\frac{\mu^2 A_i}{q^2+m_i^2} + \frac{2\mu^3 m_i B_i}{(q^2+m_i^2)^2}\right\},$$

$$(4.23c)\qquad s(0;\, m_1,\, m_2) = s(q^2;\, \infty,\, m_2^2) = 1\,, \qquad s(\infty;\, m_1^2,\, m_2^2) = 0\,;$$

the result expressed by (4.23) has been eagerly, but unsuccessfully, sought for in the past. Since delta singularities are trivially brought about by the application of the Laplace operator to Yukawa functions

$$(4.24)\qquad \nabla^2 f(m_i r) = m_i^2 F(m_i r) - 4\pi\delta(\boldsymbol{r})\,,$$

their cancellation in (4.13) is due to the Hulthén functions appearing in $v(r)$: this is the reason why we have defined in (4.9*b*) the operator $\boldsymbol{\Omega}_{\mathrm{C}}$ simply as the radial part of the Laplacian. The emphasis put in the past upon the contact interaction [10] seems nowadays a naïve attempt to find a clue which might make up for the paradoxical drawback of the old PS-PV theory (the theory as was formulated conflicts with the existence of the deuteron bound state!), without resorting to a therapeutical cutting-off of lengths at short internucleon distances.

Let us now work out the analytical expression of the function $v_{\mathrm{T}}(r)$ characterizing the radial behaviour of the tensor potential $V_{\mathrm{T}}(r)$. As a consequence of the transformations

$$(4.25a)\qquad \boldsymbol{\Omega}_{\mathrm{T}} F(m_i r) = \{3/r^3 + 3m_i/r^2 + m_1^2/r\}\, f(m_i r)\,,$$

$$(4.25b)\qquad \boldsymbol{\Omega}_{\mathrm{T}} f(m_i r) = m_i^2 f(m_i r) + m_i F(m_i r)\,,$$

it is convenient to define the function

$$(4.26)\qquad w_{\mathrm{T}}(m_i r) = \frac{1}{\mu^3}\left\{\frac{3}{r^3} + \frac{3m_i}{r^2} + \frac{3m_i^2-\mu^2}{2r} + \frac{m_i(m_i^2-\mu^2)}{2}\right\} f(m_i r)\,.$$

It is found that

$$(4.27)\qquad v_{\mathrm{T}}(r) = \sum_{i=1}^{3} C_i w_{\mathrm{T}}(m_i r)\,,$$

([10]) M. M. LÉVY: *Phys. Rev.*, **88**, 72, 725 (1952); H. A. BETHE and F. DE HOFFMANN: *Mesons and Fields*, Vol. II (New York, N. Y., 1955).

where the adimensional coefficients C_i are given by

$$(4.28)\qquad \begin{cases} C_1 = -\dfrac{m_1^4\lambda^2}{(m_1^2-\mu^2)^2} + \dfrac{2m_1^2m_2^2\lambda(1-\lambda)}{(m_1^2-\mu^2)(m_1^2-m_2^2)}\,, \\[2ex] C_2 = -\dfrac{m_2^4(1-\lambda)^2}{(m_2^2-\mu^2)^2} - \dfrac{2m_1^2m_2^2\lambda(1-\lambda)}{(m_2^2-\mu^2)(m_1^2-m_2^2)}\,, \\[2ex] C_3 = \quad \dfrac{m_1^4\lambda^2}{(m_1^2-\mu^2)^2} + \dfrac{2m_1^2m_2^2\lambda(1-\lambda)}{(m_1^2-\mu^2)\,(m_2^2-\mu^2)} + \dfrac{m_2^4(1-\lambda)^2}{(m_2^2-\mu^2)^2}\,. \end{cases}$$

It is readily proved that the coefficients (4.28) possess the property, independent of λ,

$$(4.29)\qquad C_1 + C_2 + C_3 = 0\,.$$

The analytical form of C_3 identifies with that of A_3 given by the third of eqs. (4.14*a*). Thus the condition $C_3=1$, which for $\mu r>1$ preserves the asymptotic behaviour of $V_{\mathrm{T}}(r)$ in agreement with the old Yukawa theory, leads to the result (4.18), already obtained through the mathematically identical condition (4.17). Substituting (4.19) into (4.28), one obtains

$$(4.30)\qquad \begin{cases} C_1 = -\left(\dfrac{m_1^2}{m_1^2-m_2^2}\right)^2\left[1+\dfrac{2m_2^2(m_2^2-\mu^2)}{m_1^2(m_1^2-m_2^2)}\right], \\[2ex] C_2 = \quad \left(\dfrac{m_2^2}{m_1^2-m_2^2}\right)^2\left[\dfrac{2m_1^2(m_1^2-\mu^2)}{m_2^2(m_1^2-m_2^2)}-1\right], \\[2ex] C_3 = 1\,. \end{cases}$$

The dependence of the tensor potential coefficients (4.30) on m_1 and m_2, correlated by Orear's constraint (2.6) in the intervals specified in (5.2), is shown in table II.

TABLE II. – *Numerical values of the tensor coefficients* (4.30) ($C_3=1$) *as functions of the parameters* m_1 *and* m_2, *correlated by Orear's constraint* (2.6), *in the intervals specified in* (5.3).

m_2	m_1	$-C_1$	$-C_2$
1.50μ	11.10	1.038	0.038
2.00μ	8.33	1.126	0.126
2.50μ	6.66	1.407	0.407
3.00μ	5.55	2.424	1.424
3.50μ	4.76	10.279	9.279
4.00μ	4.16	3908.172	3907.172

The behaviour of (4.26) for $r \to 0$ is

$$\lim_{r\to 0} w(m_i r) \to \frac{1}{\mu^3}\left[\frac{3}{r^3} - \frac{\mu^2}{2r} - \frac{m_i^2(2m_i^2-\mu^2)}{4}\, r\right]; \tag{4.31}$$

it follows that, at short distances ($m_i r \ll 1$), the divergence of $V_T(r)$ like $1/r^3$ and $1/r$ are suppressed because the coefficients (4.30) satisfy the relation

$$C_1 + C_2 + 1 = 0\,; \tag{4.32}$$

the suppression of the divergence like $1/r^2$ at the origin occurs also in the old Yukawa theory. The unexpected result is that the radial tensor function of the OPEP, modified by the meson density (2.4), vanishes at the origin linearly with r, *i.e.*

$$\lim_{r\to 0} v_T(r) \to \alpha\mu r\,, \tag{4.33a}$$

$$\alpha = -\sum_{i=1}^{3}\left\{m_i^2(2m_i^2-\mu^2)/4\pi^4\right\} C_i\,, \tag{4.33b}$$

$$v_T(0) = 0\,. \tag{4.33c}$$

This property of $V_T(r)$ compels one to a critical rethinking of many current opinions based on the results obtained by fitting procedures used to explain the deuteron ground state and to extract information from the nucleon-nucleon elastic-scattering data. Two generations of physicists have been worried by the unphysical implications of the $1/r^3$ divergence of the old PS-PV tensor potential. The theory of nuclear forces had gained nothing from the attempts to suppress such a divergence or to reduce it to $1/r$ (for instance, mixed theories of the Moeller-Rosenfeld type, controversial relativistic calculations of second- and fourth-order potentials, etc.; in this connexion it is worthwhile mentioning a paper by NOYES and PANDYA [11]): only Bethe's paleonuclear therapy has survived, but the problem is still unsolved. This is probably due to the fact that, as shown by the model outlined in this note, the problem simply does not exist!

5. – The Fredholm equation for the deuteron S bound state.

In order to compare the predictions of the new OPEP with the experimental data, we need to know the parameters (m_1, m_2; λ) characterizing the density (2.4). It has already been pointed out that a first relation among them

---

(11) H. P. NOYES and S. P. PANDYA: *Phys. Rev.*, **102**, 269 (1956).

is given by (2.6). A second relation is obtained by substituting (4.14) into (2.6); it is found that

$$m_1 m_2 = \sqrt{6}\, M\mu\,. \tag{5.1}$$

Since $m_2 > \mu$ and $m_1 > m_2$, the intervals of variability of m_1 and m_2 are

$$\mu < m_2 < (\sqrt{6}\, M\mu)^{\frac{1}{2}} = 4.08\mu\,, \qquad 4.08\mu < m_1 < \sqrt{6}\, M = 16.64\mu\,; \tag{5.2}$$

m_1 is evaluated as a function of m_2 in tables I and II. A third relation is required to determine the numerical values of m_1 and m_2: we shall deduce it from the S-state of the deuteron.

The Schrödinger equation for the deuteron S-state in momentum space is

$$(p^2 + p_0^2)\Psi(p) = \Lambda^2 \int H(|\boldsymbol{p} - \boldsymbol{q}|)\, \Psi(q)\, \mathrm{d}\boldsymbol{q}\,, \tag{5.3}$$

where $p_0^2 = -MB_d/\hbar^2 = 0.110\mu^2$ ($B_d = -2.228$ MeV being the deuteron binding energy) and

$$\Lambda^2 = 3MU_0/\hbar^2 = \mu c^2 (M/\hbar^2)(\mu/2M)^2 g^2 = 0.036\mu^2 g^2\,; \tag{5.4}$$

the kernel of eq. (5.3) is the three-dimensional Fourier transform of $v_C(r)$,

$$H(|\boldsymbol{p} - \boldsymbol{q}|) = (2\pi)^{-3} \int v_C(r) \exp\left[i(\boldsymbol{p} - \boldsymbol{q})\cdot \boldsymbol{r}\right] \mathrm{d}\boldsymbol{r}\,, \tag{5.5a}$$

$$H(|\boldsymbol{p} - \boldsymbol{q}|) = \frac{1}{2\pi^2\mu} \sum_{i=1}^{3} \left\{ \frac{A_i}{m_i^2 + |\boldsymbol{p} - \boldsymbol{q}|^2} + \frac{2\mu m_i B_i}{(m_i^2 + |\boldsymbol{p} - \boldsymbol{q}|^2)^2} \right\}. \tag{5.5b}$$

Substituting (5.5b) into (5.3) and integrating over the angle between $\boldsymbol{p}$ and $\boldsymbol{q}$, one obtains the one-dimensional, homogeneous, singular Fredholm equation

$$\Psi(p) = \Lambda^2 \int_0^\infty K(p, q)\, \Psi(q)\, \mathrm{d}q\,, \tag{5.6}$$

where the new kernel is unsymmetric:

$$K(p, q) = \frac{q}{2\pi\mu p (p^2 + p_0^2)} \sum_{i=1}^{3} A_i \ln \left\{ \frac{m_i^2 + (p - q)^2}{m_i^2 + (p + q)} \right\} + \\ + \frac{4q^2}{\pi(p^2 + p_0^2)} \sum_{i=1}^{3} \frac{m_i B_i}{\{m_i^2 + (p - q)^2\}\{m_i^2 + (p + q)^2\}}\,. \tag{5.7}$$

As is well known, eq. (5.6) possesses one, and only one, continuous solution given by $\Psi(p) \equiv 0$, unless the parameter Λ^2 assumes certain special values.

The problem in that of finding the characteristic constants ([12]) of the kernel (5.7), which are the roots of Fredholm's determinantal equation

$$\Delta(\Lambda^2) = 1 + \sum_{n=1}^{\infty} (-1)^n (S_n/n!)\Lambda^{2n} = 0 ; \tag{5.8}$$

this is equivalent to saying that for given values of m_1 and m_2 the PS-PV pion-nucleon coupling constant g^2 must be a characteristic constant of (5.6) or, *vice versa*, that for a given value of g^2 the parameters m_1 and m_2, correlated by the independent relation (5.1), must be determined so that eq. (5.8) is satisfied. This mathematical aspect of the problem is fundamental: if it is ignored, the S-state of the deuteron can also be described with physically meaningless potentials, as is abundantly shown in the literature. The coefficients S_n are determined by the recurrence relations

$$S_n = \int_0^{\infty} T_{n-1}(q, q)\,\mathrm{d}q\,, \tag{5.9a}$$

$$T_n(p, q) = \alpha_n K(p, q) - n\int_0^{\infty} K(p, s)\,T_{n-1}(s, q)\,\mathrm{d}s\,, \tag{5.9b}$$

where $S_0 = 1$ and, therefore, $T_0(p, q) \equiv K(p, q)$. From (5.7) one has

$$K(q, q) = \frac{1}{2\pi\mu(q^2 + p_0^2)} \sum_{i=1}^{3} \left[A_i \ln\left\{1 + \left(\frac{2q}{m_i}\right)^2\right\} + \frac{8\mu q^2 B_i}{m_i(m_i^2 + 4q^2)}\right]. \tag{5.10}$$

Only $S_1 \equiv S_1(m_2)$ can be determined analytically; it is found ([13]) that

$$S_1(m_2) = \frac{1}{2\mu^2} \sum_{i=1}^{3} \left[\frac{\mu}{p_0} A_i \ln\left\{1 + \frac{2p_0}{m_i}\right\} + \frac{2\mu^2 B_i}{m_i(m_i + 2p_0)}\right]. \tag{5.11}$$

The evaluation of S_n for $n > 1$ is complicated and time consuming; it has been carried out up to $n = 3$ (the coefficients S_n corresponding to $n > 3$ have been found to be negligible). Consequently, the determinantal equation (5.8) has been reduced to a cubic equation in Λ^2, *i.e.*

$$S_3(m_2)\Lambda^6 - 3S_2(m_2)\Lambda^4 + 6S_1(m_2)\Lambda^2 - 6 = 0\,. \tag{5.12}$$

The analyses of the data concerning the nucleon-nucleon interaction localize

([12]) W. V. Lovitt: *Linear Integral Equations* (New York, N. Y., 1950).

([13]) I. S. Gradshteyn and I. M. Ryzhik: *Tables of Integrals, Series and Products* (New York, N. Y., 1965).

the value of the pion-nucleon coupling constant approximately within the interval

$$g^2 = 15.0 \pm 3.0 \; ; \tag{5.13}$$

it follows that the uncertitude on Λ^2, defined in (5.4), is

$$\Lambda^2 = (0.540 \pm 0.108)\mu^2 \, . \tag{5.14}$$

The parameter m_2 has been determined on the basis of what one might call « the deuteron S bound-state approximation », *i.e.* by equating (5.14) to the single positive real root of eq. (5.12); it has been found that

$$m_2 = (2.45 \pm 0.39)\mu \, . \tag{5.15}$$

Attempts have been made to solve eq. (5.8) by adopting the more realistic $S + D$ bound-state description of the deuteron. Puzzling mathematical difficulties have so far prevented the achievement of this goal; in spite of this failure, it has been possible to collect arguments for concluding that the $S + D$ description modifies the result (5.15) by less than $\sim 5\,\%$.

6. – The g^2-dependence of the radial form of the new even-triplet OPEP.

From (5.15) one obtains, using eqs. (5.1) and (4.14), the value of m_1 and, respectively, the value of λ. The potential coefficients (4.17) and (4.27), calculated with the parameters m_1 and m_2 thus determined, are strictly related to the OPEP behaviour in even-parity triplet states. It is by no means obvious that the values of such coefficients are physically meaningful also for the description of the nucleon-nucleon interaction in other states, which do not come into play in the description of the deuteron ground state (even-parity singlet and odd-parity singlet and triplet states). For instance, a variation interval for m_2 different from (5.15) could be determined for singlet-even states by fitting the zero energy limit of the effective range theory:

$$\lim_{k\to 0} \{\text{tg}\, \delta_0(k)/k\} = -\, a_0 \, , \tag{6.1}$$

where $\delta_0(k)$ is the asymptotic S singlet phase shift, k is the momentum in the centre-of-mass system and a_0 is the experimental Fermi length ([14]). Similarly, from the available information on the elastic nucleon-nucleon scattering data, it is possible to determine m_2 both for odd-singlet and odd-triplet states. In

([14]) J. M. Blatt and V. F. Weisskopf: *Theoretical Nuclear Physics* (New York, N. Y., 1952).

conclusion, from (4.10) one can deduce four one-pion exchange potentials, each one describing the nucleon-nucleon interaction in even- and odd-singlet and triplet states: this possibility discloses theoretical perspectives which are more significant than those originally offered by the phenomenological nucleon-nucleon potentials of Gammel, Christian and Thaler ([15]). This problem will not be discussed in this note in which we shall confine ourselves to the even-triplet OPEP only.

The coefficients (4.21) and (4.30), as functions of g^2, are

$$g^2 = 12.0:\ m_1 = 8.08\mu\,,\qquad m_2 = 2.06\mu\,,\qquad \lambda = 1.053\,, \tag{6.2a}$$

$$\left\{\begin{array}{lll} A_1 = -\,1.659\,, & B_1 = -\,297.082\,, & C_1 = -\,1.151\,,\\ A_2 = 0.659\,, & B_2 = -\,0.016\,, & C_2 = 0.151\,,\\ A_3 = 1\,, & B_3 = 0\,, & C_3 = 1; \end{array}\right. \tag{6.2b}$$

$$g^2 = 15.0:\ m_1 = 6.79\mu\,,\qquad m_2 = 2.45\mu\,,\qquad \lambda = 1.125\,, \tag{6.3a}$$

$$\left\{\begin{array}{lll} A_1 = -\,3.301\,, & B_1 = -\,202.400\,, & C_1 = -\,1.364\,,\\ A_2 = 2.301\,, & B_2 = -\,0.137\,, & C_2 = 0.364\,,\\ A_3 = 1\,, & B_3 = 0\,, & C_3 = 1; \end{array}\right. \tag{6.3b}$$

$$g^2 = 18.0:\ m_1 = 5.86\mu\,,\qquad m_2 = 2.84\mu\,,\qquad \lambda = 1.269\,, \tag{6.4a}$$

$$\left\{\begin{array}{lll} A_1 = -\,9.118\,, & B_1 = -\,166.865\,, & C_1 = -\,1.924\,,\\ A_2 = 8.118\,, & B_2 = -\,0.945\,, & C_2 = 0.924\,,\\ A_3 = 1\,, & B_3 = 0\,, & C_3 = 1\,. \end{array}\right. \tag{6.4b}$$

The g^2-dependence of the radial form of $V(r)$ substantially modifies the fitting procedures based on the search for a phenomenological correlation between g^2 and the radius r_c of the hard core. It is worthwhile noting that for $g^2 = 12.0$ the value of the parameter m_2 is approximately equal to the inverse of the range of the two-pion exchange potential, while for $g^2 = 15$ the value of m_1 practically identifies with that of the inverse Compton wave-length of the nucleon. To visualize quantitatively the radial behaviour of the potential we give in table III the roots of the equation $v_C(r_0) = 0$, the internucleon distances r_1^C and r_1^T corresponding to the maximum of $v_C(r)$ and, respectively, $v_T(r)$, and the values of the maxima. It is remarkable *a*) that the maximum of $v_C(r)$ lies in the pion cloud, at an internucleon distance approximately equal to the

([15]) J. L. Gammel, R. S. Christian and R. M. Thaler: *Phys. Rev.*, **105**, 311 (1957).

TABLE III. – *Roots of the equation* $v_C(r_0) = 0$, *internucleon distances* r_1^C *and* r_1^T *corresponding to the maximum of* $v_C(r)$ *and, respectively,* $v_T(r)$ *parametrized as functions of* g^2; *the values of the maxima are also tabulated.*

g^2	μr_0	μr_1^C	$v_C(r_1^C)$	μr_1^T	$v_T(r_1^T)$
12.0	0.726	0.949	0.362	0.191	33.902
15.0	0.789	1.076	0.322	0.191	29.513
18.0	0.836	1.098	0.307	0.191	14.928

Compton wave-length of the pion, and *b*) that the maximum of $v_T(r)$ is sunk into the nucleonic bag, at an internucleon distance (practically independent of g^2) larger by a factor ~ 1.27 than the Compton wave-length of the nucleon. The radial functions $v_C(r)$ and $v_T(r)$ are plotted in fig. 1*a*, 1*b*; it is seen that

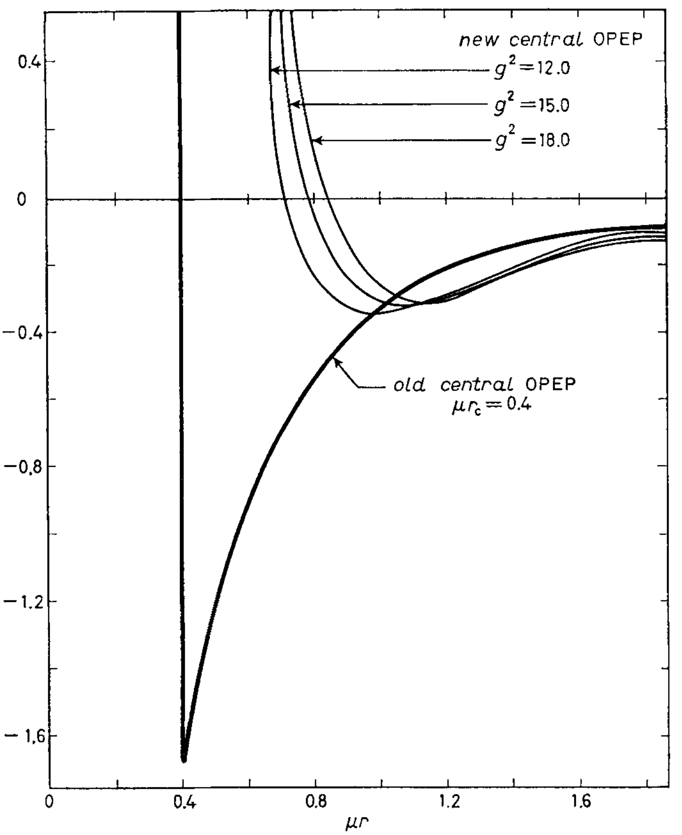

Fig. 1*a*. – Comparison of the radial functions $u_C(r)$ and $v_C(r)$ corresponding to the old (——) and to the new (——) even-triplet central OPEP, respectively: both of them are plotted with negative sign. The radius of the hard core of the former has been indicatively assumed to be $\mu r_c = 0.4$. The g^2-dependence of the form of $v_C(r)$ is clearly exhibited. The finite values of $v_C(r)$ at $r = 0$ are 1) $g^2 = 12.0$: $v_C(0) = -308.1$; 2) $g^2 = 15.0$: $v_C(0) = -218.3$; 3) $g^2 = 18.0$: $v_C(0) = -197.2$.

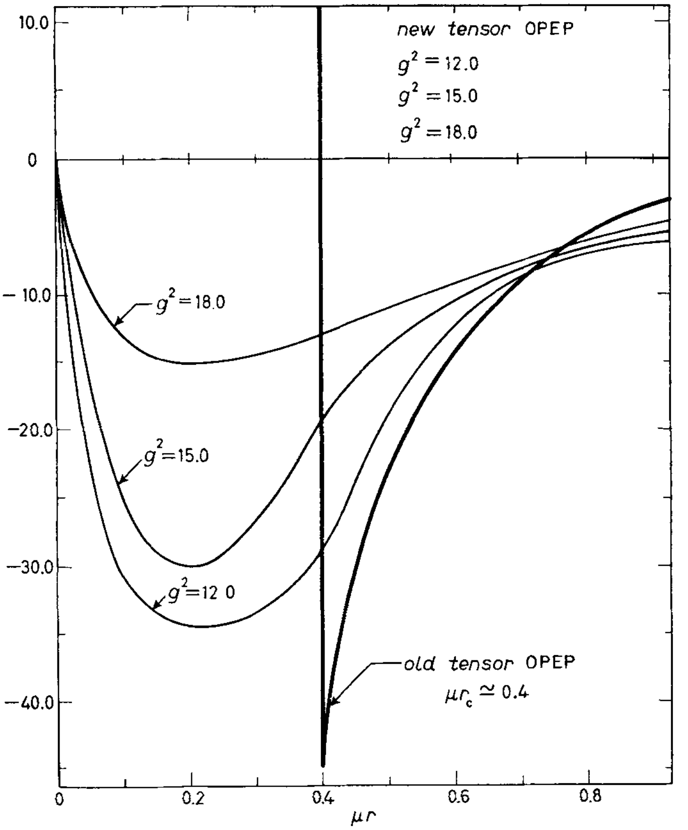

Fig. 1*b*. – Comparison of the radial functions $u_T(r)$ and $v_T(r)$ corresponding to the old (———) and to the new (———) even-triplet tensor OPEP, respectively: both of them are plotted with negative sign. The radius of the hard core of the former has been indicatively assumed to be $\mu r_c = 0.4$. The g^2-dependence of the form of $v_C(r)$ is remarkable. The values of the parameter α, defined in (4.30*b*), are 1) $g^2 = 12.0$: $\alpha = -2432.7$; 2) $g^2 = 15.0$: $\alpha = -1427.7$; 3) $g^2 = 18.0$: $\alpha = -1089.4$.

both the central and tensor parts of the potential exhibit a longer tail than the Yukawa one: this is primarily due to the non-Yukawian (exponential) term appearing in (4.8*a*) and to the mathematical implications brought about by the transformation (4.1*a*) which was used to obtain a realistic PS-PV potential from an unrealistic neutral and scalar scheme.

The central potential $V_C(r)$ is repulsive for $r < r_0$ and attractive for $r > r_0$ for the nucleon-nucleon states characterized by a negative eigenvalue of the operator

$$O_C = (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \,. \tag{6.5}$$

For example, by assuming $g^2 = 12.0$, the finite value of $V_C(r)$ at the origin, for an even-parity triplet state, is

$$V_C(0) = -3U_0 \sum_{i=1}^{3} \{(m_i/\mu) A_i + B_i\} \simeq 2.94 Mc^2 . \tag{6.6}$$

Thus in an even-triplet state the potential $V_C(r)$, however free from artificial core constraints, simulates at distances $r < r_0$ a radial behaviour, which it would clearly be misleading to ascribe to a presumptive existence of a soft core. Because of the influence of the source (2.4) the potential $V_C(r)$ deviates from the conventional even-triplet central OPEP in the region in which the two-pion exchange potential and recoil effects are expected to become effective. The dominating repulsion at short distances ($r < r_0$) practically absorbs the contributions of higher-order central potentials and conceals or minimizes more complicated effects which, however, could not be correctly accounted for in a static pseudovector coupling approximation. It is amusing to note that the simulation of core effects occurs in a rather straightforward way in the central potential, but not in the tensor one, although the hard core was originally introduced only in the latter in order to ensure the existence of the deuteron bound state.

The phenomenological correlation between g^2 and the hard-core radius r_c searched for in fitting procedures based on the old OPEP is replaced in the new OPEP scheme by the g^2-dependence of the form of the radial behaviour of the central and tensor parts of $V(r)$. Clearly, the coupling constants determined in the two cases are not equal: let g_0^2 be the coupling constant determined (together with r_c) in the former case. Let us define in three-dimensional space the n-th order moments of the functions $u_C(r)$ and $u_T(r)$ truncated at the phenomenological hard-core radius r_c (for the sake of simplicity, we make no distinction between hard cores related to either even-triplet central or tensor interaction). Apart from unessential constant factors, they are ($x = \mu r$)

$$\mathscr{M}_n^C = \int_{x_C}^{\infty} x^{n+2} u_C(r)\, dx , \qquad \mathscr{M}_n^T = \int_{x_C}^{\infty} x^{n+2} u_T(x)\, dx , \tag{6.7a}$$

where $x_c \equiv \mu r_c$. The corresponding n-th–order moments of the functions $v_C(r)$ and $v_T(r)$ are

$$\mathscr{N}_n^C = \int_0^{\infty} x^{n+2} v_C(x)\, dx , \qquad \mathscr{N}_n^T = \int_0^{\infty} x^{n+2} v_T(x)\, dx . \tag{6.7b}$$

A clue for understanding the link between the predictions of the old even-triplet OPEP and the new one is given by the fact that the following relations hold:

$$g_0^2 \mathscr{M}^C = g^2 \mathscr{N}_n^C , \qquad g_0^2 \mathscr{M}_n^T = g^2 \mathscr{N}_n^T ; \tag{6.8}$$

for $n = 0$ the relation $g_0^2 \mathscr{M}_0 = g^2 \mathscr{N}_0$ is not true, because the cut-off on $u_C(r)$ prevents the property (4.16) from being valid also for $U_C(r)$, as happens in the old Yukawa theory with the contact interaction term. Let $\gamma = g^2/g_0^2$; for g_0^2 and g^2 included in the interval (5.13) the ratio γ possesses a rather large interval of variability

$$0.667 \leqslant \gamma \leqslant 1.500 \,, \tag{6.9}$$

which ensures the validity of equalities (6.8). The peculiar role played in the tensor interaction by the hard-core radius r_c appears clearly by investigating the equality

$$\mathscr{M}_0^{\mathrm{T}} = \gamma \mathscr{N}_0^{\mathrm{T}} \,, \tag{6.10a}$$

$$\mathscr{M}_0^{\mathrm{T}} = 3\int_{x_c}^{\infty} F(x)\,\mathrm{d}x + (4 + x_c) f(x_c) \,, \tag{6.10b}$$

$$\mathscr{N}_0^{\mathrm{T}} = 3\{\ln(m_1/\mu) + C_2 \ln(m_1/m_2)\} + \\ + (3\mu^2/2m_1^2) - (3\mu^2/2)\{(1/m_2)^2 - (1/m_1)^2\}\, C_2 - \tfrac{3}{2} \,. \tag{6.10c}$$

Let us assume $\mu r_c = 0.35 \pm 0.05$; it is found that

$$\mathscr{M}_0^{\mathrm{T}} = 5.501 \pm 0.424 \,, \tag{6.11}$$

$$g^2 = 12.0\colon\ \mathscr{N}_0^{\mathrm{T}} = 5.360 \,, \qquad \gamma = 1.260 \pm 0.079 \,; \tag{6.12a}$$

$$g^2 = 15.0\colon\ \mathscr{N}_0^{\mathrm{T}} = 5.312 \,, \qquad \gamma = 1.035 \pm 0.080 \,; \tag{6.12b}$$

$$g^2 = 18.0\colon\ \mathscr{N}_0^{\mathrm{T}} = 5.720 \,, \qquad \gamma = 0.962 \pm 0.074 \,. \tag{6.12c}$$

To test the validity of properties (4.33), I have undertaken the calculation of the quadrupole moment of the deuteron. Let $\Psi_S(r)$ and $\Psi_D(r)$ be the radial wave functions for the S and D triplet states; the coupled second-order differential equations to be solved are

$$\Psi_S''(\zeta) - \{v_C(\zeta) - \eta_1\}\Psi_S(\zeta) = \eta_2 v_T(\zeta)\Psi_D(\zeta) \,, \tag{6.13a}$$

$$\Psi_D(\zeta) - \{v_C(\zeta) - 2v_T(\zeta) + 6\zeta^{-2} - \eta_1\}\Psi_D(\zeta) = \eta_2 v_T(\zeta)\Psi_S(\zeta) \,, \tag{6.13b}$$

where $\zeta = \Lambda r$, Λ being defined in (5.4), and

$$\eta_1 = B_d/3U_0 \,, \qquad \eta_2 = -\sqrt{8} \,. \tag{6.13c}$$

Preliminary results indicate that the new even-triplet OPEP is beautifully fit to reproduce the experimental value of the quadrupole moment of the deuteron consistently with the expected D-state probability. This matter will be discussed in a forthcoming paper. Calculations of the fourth-order contributions to the new OPEP are in progress: the form factor of the pion-nucleon vertex is described as the three-dimensional Fourier transform of the density (2.4) with the parameter specified by (4.19) and (5.1), *i.e.*

$$G(q^2, m^2) = \frac{a(m^2)}{q^2 + m^2} - \frac{b(m^2)}{m^2 q^2 + c}, \tag{6.14}$$

$$a(m^2) = m^4(m^2 - \mu^2)/(m^4 - 6\mu^2 M^2), \tag{6.15a}$$

$$b(m^2) = 6\mu^4 M^2(6M^2 - m^2)/(m^4 - 6\mu^2 M^2), \tag{6.15b}$$

$$c = 6\mu^2 M^2, \tag{6.15c}$$

$$G(0, m^2) = G(q^2, \infty) = 1, \tag{6.16}$$

where the parameter $m \equiv m_1$, varying in the interval specified in (5.2), has to be determined, together with g^2, from the experimental data.

The OPEP scheme developed in this note shows that the phantomatic hard core of radius r_c is a conceptually wicked trick invented (and, unfortunately, taken too seriously) in order to explain the deuteron ground state and the low-energy nucleon-nucleon elastic-scattering data, using the unrealistic potential predicted by the old PS-PV theory. According to the new OPEP, the pion-nucleon coupling constant g^2 and only one of the parameters m_1 or m_2 have to be determined from the data: all the coefficients of the potential (4.10) are g^2-dependent consistently with three experimental prescriptions: *a*) Orear's data (sect. **2**), *b*) the asymptotic coincidence of the new OPEP with the old one (sect. **4**) and *c*) the deuteron binding energy (sect. **5**). Clearly, the surreptitious parameter r_c has nothing to do with the structure of the nucleon. The identification of r_c with an intrinsic parameter of the nucleon structure, initially prophesied by BETHE, was fallaciously considered trustworthy also as a consequence of a serious misinterpretation of Jastrow's analyses ([16]): this has strongly contributed to disbanding the theory of nuclear forces and to creating (with the help of a critically unrestricted use of Tamm-Dankoff and other, more sophisticated, methods) the « unbelievable confusion and conflict » drastically pointed out by GOLDBERGER (sect. **1**).

* * *

I am very much indebted to Dr. L. SALVADORI for taking care of the publication of this note.

([16]) R. JASTROW: *Phys. Rev.*, **81**, 165 (1951).

Note added in proofs.

I thank Dr. P. F. Bortignon for calling my attention, after the completion of this work, to the paper by D. Vento, M. Rho and G. E. Brown: *Phys. Lett. B*, **103**, 285 (1981).

● RIASSUNTO

Si costruisce il potenziale di scambio internucleonico di un pione (PSIUP) nell'ambito della teoria statica PS-PV delle forze nucleari, tenendo conto dei dati di Orear relativi alle presunte dimensioni spaziali della regione di confinamento dei quarks. Il «little-bag model», delineato da Brown e Rho, è sviluppato sulla base della congettura che la sorgente mesonica, a simmetria sferica, non sia uniforme. Il contenuto fisico della densità scelta è interpretato in termini di un modello non locale del nucleone inteso alla Pais-Uhlenbeck. Si mostra che il potenziale che ne deriva possiede aspetti fisici realistici (per lo meno negli stati pari di tripletto di spin) senza ricorrere all'introduzione *ad hoc* del raggio del nocciolo impenetrabile e a procedimenti di taglio alle piccole distanze.

Новый потенциал с одно-пионным обменом.

Резюме (*). — Конструируется нуклон-нуклонный потенциал с одно-пионным обменом в рамках статической PS-PV теории ядерных сил, учитывая данные Орира, касающиеся предполагаемых пространственных размеров области удержания кварков. Развивается модель мешков, описанная Брауном и Ро, на основе предположения, что сферически симметричный мезонный источник должен быть неоднородным. Физическое содержание выбранной мезонной плотности интерпретируется в терминах нелокальной модели Пейса-Уленбека нуклона. Показывается, что предложенный потенциал обладает реалистичными физическими свойствами (по крайней мере, для триплетных состояний с положительной четностью), без введения радиуса остова и процедур обрезания на малых расстояниях.

(*) *Переведено редакцией.*

Direttore responsabile: RENATO ANGELO RICCI

Stampato in Bologna dalla Tipografia Compositori coi tipi della Tipografia Monograf
Questo fascicolo è stato licenziato dai torchi l'1-II-1982