LETTERE AL NUOVO CIMENTO VOL. 5, N. 10 4 Novembre 1972

Does the Axial Vector Current Influence Space-Time?

K. HAYASHI

Max-Plank-Institut für Physik und Astrophysik - München
Institute of Physics, University of Tokyo - Tokyo

(ricevuto il 19 Settembre 1972)

In particle physics the axial vector current, together with the vector current, has played an important role in current algebras (1) and chiral dynamics (2). Does it play any role in other branch of physics? In this letter we argue that the axial vector current, along with the energy-momentum tensor, of the spin-$\frac{1}{2}$ baryons and leptons may influence the space-time structure through the spin angular-momentum tensor contributing to gravitational sources. In high-energy physics both the mass and spin of the hadrons are dynamically incorporated into the concept of Regge poles which has been successful in analysing the scattering data. On a macroscopic scale, however, the spin will play no significant role as well as the electric charges. Therefore it appears quite reasonable that Einstein's macroscopic theory of gravitation lets the mass, or the stress tensor, of macroscopic bodies alone be a unique source of gravitation and at the same time lets it be a sole agency in generating a curved space-time. If we take such a microscopic viewpoint as that by which gravitation arises through individual nucleons and electrons, or more generally spin-$\frac{1}{2}$ baryons and leptons, straightforward extrapolation of Einstein's idea to the realm of particle physics would not be justified. In this circumstance we would rather think that the mass *and* spin, or the energy-momentum tensor *and* spin angular-momentum tensor, are both contributing to gravitational sources and hence have influence on the space-time structure.

In order to avoid unnecessary confusion we emphasize the *macroscopic* nature of Einstein's gravitational field equations. The stress tensor of macroscopic bodies appearing therein as a source tensor of gravitation is represented by *macroscopic observables*, for example, the energy density, the fluid velocity and the pressure in the perfect-fluid model of a star. This stress tensor has nothing to do with the energy-momentum tensor of individual hadrons and leptons, except for that of the electromagnetic field which has a macroscopic and classical limit. *Hence it would be incorrect to judge our theory to be developed below as an alternative theory to Einstein's.* It must be remembered that two theories stand on a different scale, one microscopic and the other macroscopic, and that a relation between two could be established by taking the macroscopic limiting

(1) M. GELL-MANN: *Phys. Rev.*, **125**, 1067 (1962); *Physics*, **1**, 63 (1964).
(2) J. SCHWINGER: *Phys. Lett.*, **24** B, 473 (1967); *Phys. Rev. Lett.*, **18**, 923 (1967).

procedure of our theory. Now we shall see how our hypothesis proposed previously nay be formulated and how a resulting theory may be reduced to Einstein's theory by such a limit.

Let $\boldsymbol{L} = \boldsymbol{L}_M + \boldsymbol{L}_G$ be a total Lagrangian for a system consisting of a spin-$\frac{1}{2}$ particle and the gravitational field, where $\boldsymbol{L}_M$ denotes a material part involving arbitrary gravitational interactions and $\boldsymbol{L}_G$ an arbitrary free gravitational part. It is well known that the gravitational variables are not the metric tensor in this case but the tetrads, $b_k{}^\mu(x)$, or their inverse $b_{k\mu}(x)$, where the Greek index refers to a Riemann space and the Latin index to the local Lorentz space. See our previous papers for further details ([3,4]). First we are going to construct $\boldsymbol{L}_G$ by demanding its action integral be invariant under A) a general co-ordinate transformation, B) a Lorentz transformation (with respect to the Latin index), and C) $\boldsymbol{L}_G$ of quadratic form in the first derivatives of the tetrads. The most general form of $\boldsymbol{L}_G$ to meet these demands was already given partly in ref. ([3]) and here perfectly given with 4 arbitrary real coefficients:

$$\boldsymbol{L}_G = b(\alpha T^2_{klm} + \beta V^2_k + \gamma A^2_k + \eta V_k A_k)\,, \tag{1}$$

where the Lorentz tensors are invariant under a general co-ordinate transformation,

$$\left\{\begin{aligned} T_{klm} &= (\tfrac{1}{2})(C_{klm} + C_{lkm}) + (\tfrac{1}{6})(\delta_{mk} V_l + \delta_{ml} V_k) - (\tfrac{1}{3})\,\delta_{kl} V_m\,,\\ V_k &= C_{mmk}\,,\\ \text{and}\qquad & \\ A_k &= (i/6)\,\varepsilon_{klmn} C_{lmn}\,, \end{aligned}\right. \tag{2}$$

are the irreducible components of $C_{klm} = b_l{}^\mu b_m{}^\nu (b_{k\mu,\nu} - b_{k\nu,\mu})$. We use the notation, $b = \det(b_{k\mu})$. The above Lagrangian can be rewritten into a more convenient form as

$$\boldsymbol{L}_G = \boldsymbol{L}_E/2\varkappa^2 + b\big\{(\alpha + \beta)\,V^2_k - (9/4\lambda^2)\,A^2_k + \eta V_k A_k\big\}\,, \tag{3}$$

where $\boldsymbol{L}_E = \sqrt{-g}\,R$ ($R =$ the Riemann scalar) is the Einstein's free gravitational Lagrangian, and we adjust an overall coefficient with the Einstein's gravitational constant as $3\alpha = \varkappa^{-2}$ and introduce the notation, $\alpha - 4\gamma/9 = \lambda^{-2}$. Note that in (3) we suppressed a four-divergence arising from the Riemann scalar because it does not affect the field equations. From (3), though $\boldsymbol{L}_M$ is yet unspecified, we can derive the field equations of gravitation:

$$\boldsymbol{G}_{kl} - \varkappa^2 \boldsymbol{B}_{kl} = -\varkappa^2 \boldsymbol{T}_{kl}\,, \tag{4}$$

where the first member on the left is the well-known Einstein tensor, now rewritten in terms of the tetrads, $\boldsymbol{G}_{kl} = b(R_{kl} - (\frac{1}{2})\,\delta_{kl} R)$,

$$\boldsymbol{B}_{kl} = -(b_m{}^\mu \boldsymbol{F}_{klm})_{,\mu} + \big\{(\tfrac{1}{2})\,C_{lmn}\boldsymbol{F}_{kmn} - C_{mnk}\boldsymbol{F}_{mnl}\big\} + \delta_{kl}\{\text{the second terms of (3)}\}\,, \tag{5}$$

$$\boldsymbol{F}_{klm} = 4b\big\{(\alpha + \beta)\,\delta_{k[l} V_{m]} + (3i/8\lambda^2)\,\varepsilon_{klmn} A_n + (\eta/2)\big(\delta_{k[l} A_{m]} - (i/6)\,\varepsilon_{klmn} V_n\big)\big\}\,, \tag{6}$$

([3]) K. Hayashi and T. Nakano: *Progr. Theor. Phys.*, **38**, 491 (1967).
([4]) K. Hayashi: *Lett. Nuovo Cimento*, **5**, 529 (1972).

and the energy-momentum tensor is defined as a source tensor of gravitation with respect to $\boldsymbol{L}_M$ yet unspecified:

$$T_{kl} = -(\delta \boldsymbol{L}_M/\delta b_{k\mu})\, b_{l\mu} \,. \tag{7}$$

Here symbol $\delta/\delta b_{k\mu}$ means taking the Euler derivative with respect to the inverse tetrads Observing that the second terms in (5), enclosed by curly brackets, are symmetric in k and l (this can be proved by using the irreducible tensors (2)), we decompose the field equations into the symmetric and skewsymmetric parts:

$$\boldsymbol{G}_{kl} - \varkappa^2 \boldsymbol{B}_{(kl)} = -\varkappa^2 \boldsymbol{T}_{(kl)} \,, \tag{8}$$

$$-(b_m{}^\mu \boldsymbol{F}_{[kl]m})_{,\mu} = \boldsymbol{T}_{[kl]} \,, \tag{9}$$

where parentheses enclosing indices mean symmetrization and square brackets mean antisymmetrization. Making use of the invariance property B) mentioned previously, we can derive the *generalized tetrode relation* between the antisymmetric part of the energy-momentum tensor and divergence of the spin tensor:

$$T_{kl} - T_{lk} = \boldsymbol{S}^\mu{}_{kl,\mu} \,, \tag{10}$$

where

$$\boldsymbol{S}^\mu{}_{kl} = (\partial \boldsymbol{L}_M/\partial \psi_{,\mu})(iS_{kl})\,\psi + (\partial \boldsymbol{L}_M/\partial b_{m\nu,\mu})(iS_{kl})_{mn} b_{n\nu} \,. \tag{11}$$

Here S_{kl} is the spin matrix; for a spin-$\frac{1}{2}$ particle $S_{kl} = (\gamma_k\gamma_l - \gamma_l\gamma_k)/4i$ and for the tetrads $(S_{kl})_{mn} = -i(\delta_{km}\delta_{ln} - \delta_{kn}\delta_{lm})$. Formula (10) can be easily obtained from eq. (2.22) of ref. [3]. Assuming the boundary condition that there is no gravitation in the absence of the nucleon's spin, we can integrate (9) with (10) only to find

$$\boldsymbol{F}_{[kl]m} = -(\tfrac{1}{2})\boldsymbol{S}_{mkl} \,, \tag{12}$$

where $\boldsymbol{S}_{mkl} = b_{m\mu}\boldsymbol{S}^\mu{}_{kl}$. Next we turn our attention to the remaining free parameters, β, γ, and η. Can they take arbitrary values? To answer this question, we make use of the weak-field approximation specified by

$$b_{k\mu} \to \delta_{km} + a_{km} \,. \tag{13}$$

It is not necessary in this approximation to distinguish Greek index from Latin index. It follows from (8) and (9) that

$$\Box S_{kl} + \varkappa^2(\alpha+\beta)(\partial_k\partial_l - \delta_{kl}\Box)\, S_{mm} + (i/3)\,\eta\varkappa\lambda\varepsilon_{mnj(k}\partial_{l)}\partial_j A_{mn} = -\varkappa T_{(kl)} \,, \tag{14}$$

$$\Box A_{kl} - (i/3)\,\eta\lambda^2\varepsilon_{mnj[k}\partial_{l]}\partial_j A_{mn} = -\lambda T_{[kl]} \,, \tag{15}$$

where we have used the new linearized variables

$$\varkappa S_{kl} = a_{(kl)} - (\tfrac{1}{2})\,\delta_{kl} a_{mm} \,,$$

$$\lambda A_{kl} = a_{[kl]} \,,$$

together with the divergence-free conditions $\partial_l S_{kl} = 0 = \partial_l A_{kl}$. Thus the parity-violating term $\eta V_k A_k$ in $\boldsymbol{L}_G$ makes both sets of field equations coupled each other. We shall set $\eta = 0$ so as to let each set of field equations closed in itself. Thus one may find the retarded solution to (14) with the absence of the last term on the left-hand side [5]:

$$(16)\qquad S_{kl}(cr, \boldsymbol{x}) = \frac{\varkappa}{4\pi}\int \mathrm{d}^3 y\, \frac{T_{(kl)}(ct - |\boldsymbol{x}-\boldsymbol{y}|, \boldsymbol{y}) + (\xi/1-3\xi)\,\delta_{kl} T_{mm}}{|\boldsymbol{x}-\boldsymbol{y}|} + \\ + \frac{\varkappa}{(4\pi)^2}\,\frac{\xi}{1-3\xi}\iint \mathrm{d}^3 y\, \mathrm{d}^3 z\, \frac{\partial_k \partial_l T_{mm}(ct - |\boldsymbol{x}-\boldsymbol{y}| - |\boldsymbol{y}-\boldsymbol{z}|, \boldsymbol{z})}{|\boldsymbol{x}-\boldsymbol{y}||\boldsymbol{y}-\boldsymbol{z}|},$$

where the argument of T_{mm} appearing in the first integral is $T_{mm}(ct - |\boldsymbol{x}-\boldsymbol{y}|, \boldsymbol{y})$ and $\xi = \varkappa^2(\alpha+\beta)$. The first term in S_{kl} represents the retarded solution to the usual wave equations; gravitation propagates on the backward light-cone. On the other hand the second term implies that gravitation propagates *inside* the backward light-cone to its vertex, thus violating Huyghens' principle, however small ξ might be. We shall set $\alpha + \beta = 0$. From (6) we obtain the completely antisymmetric tensor

$$(17)\qquad \boldsymbol{F}_{klm} = (3/2\lambda^2)\, bi\varepsilon_{klmn} A_n\,,$$

and hence from (12) we get upon multipling it the Levi-Civita tensor

$$(18)\qquad A_k = (\lambda^2/18i)\,\varepsilon_{klmn} \boldsymbol{S}_{lmn} = -(\lambda^2/3)\, S_k\,,$$

where $\boldsymbol{S}_{klm} = bS_{klm}$ and S_k denotes the totally antisymmetric part of the spin tensor. Inserting (17) with (18) into (8), we finally reach the desired form of the field equation of gravitation:

$$(19)\qquad R_{kl} - (\tfrac{1}{2})\,\delta_{kl} R = -\varkappa^2\{T_{(kl)} - (\tfrac{1}{4})(C_{lmn} S_{kmn} - 2C_{mnk} S_{lmn})\} - (\lambda^2/4)\,\delta_{kl} S_m^2\,.$$

The left-hand sides of the resulting field equations are same as that of Einstein's; it is the Einstein tensor, now rewritten in terms of the tetrads. The right-hand side, however, differs from that of Einstein's. Our equations involve the spin tensor, besides the energy tensor, in the source tensor of gravitation. It should be noticed here that the physical meaning of a gravitational source tensor in Einstein's theory is *different* from ours; in the former it is the stress tensor of macroscopic bodies as a phenomenological representation of matter, completely independently of how constituent particles interact with gravitation. In our theory, however, a gravitational source is the energy-momentum tensor of individual spin-$\frac{1}{2}$ baryons and leptons, and its properties may be completely determined when a particular gravitational interaction of these particles is known. For example, whether the energy-momentum tensor is symmetric or not depends crucially on how a spin-$\frac{1}{2}$ particle couples to the gravitational field. It it *happens* to be symmetric, then the spin tensor must be vanishing owing to the generalized Tetrode relation (10), thus having no influence on spacetime. *To choose such a specific gravitational interaction as symmetrizes the energy-momentum tensor is equivalent to the dynamical assumption that the spin tensor is forced to play no role in curving space-time.* There is, however, no reason why the microscopic energy-momentum tensor must be sym-

(5) I am indebted to Dr. N. SETO for informing me his results on (14) by private communication.

metric, because we do not know which type of gravitational interactions of a spin-$\frac{1}{2}$ particle is allowed or forbidden. Under these circumstances there might be no other way than proceeding with trials and errors.

First let us consider the *simplest* gravitational coupling [3] in that it does not involve the derivatives of the gravitational field and it reduces, without gravitation, to the Dirac Lagrangian;

$$\boldsymbol{L}_M = b\{(\tfrac{1}{2})\, b_k{}^\mu(\bar{\psi}\gamma_k\psi_{,\mu} - \bar{\psi}_{,\mu}\gamma_k\psi) + m\bar{\psi}\psi\}\,. \tag{20}$$

From (7) we get

$$\boldsymbol{T}_{kl} = b\{(\tfrac{1}{2})\, b_k{}^\mu(\bar{\psi}\gamma_l\psi_{,\mu} - \bar{\psi}_{,\mu}\gamma_l\psi)\}\,. \tag{21}$$

The spin tensor is defined by (11) as

$$\boldsymbol{S}_{klm} = -\,b(i/2)\,\varepsilon_{klmn} J^5_n\,, \tag{22}$$

where we denote the axial vector current as $J^5_k = i\bar{\psi}\gamma_5\gamma_k\psi$. Hence it follows from (18) that $A_k = (\lambda^2/6)\, J^5_k$. *If spin-$\frac{1}{2}$ baryons and leptons interact with the gravitational field according to the above Lagrangian* (20), *then the axial vector current will affect the space-time continuum through the form of the spin tensor.* On a macroscopic scale, however, such an effect due to the axial vector current will be negligible with the possible exception of the neutron stars with strong magnetic field which tends to align spin orientation.

Secondly, can we find such a gravitational interaction as symmetrizes the energy-momentum tensor? To do so, we just add a more complicated axial vector interaction to $\boldsymbol{L}_M$ (20):

$$\boldsymbol{L}_M - (\tfrac{3}{4})\, A_k J^5_k b\,. \tag{23}$$

The spin tensor arising from the second term just counteracts the spin tensor (22) from the first term, thus rendering the total spin tensor null: $T_{kl} = \boldsymbol{T}_{lk}$.

It would be worth noting that the gravitational interaction (20) may be obtained by replacing the ordinary by covariant derivatives with respect to an *asymmetric affine connexion*

$$\Gamma^\lambda{}_{\mu\nu} = b_k{}^\lambda b_{k\mu,\nu}\,,$$

while the latter interaction (23) may be derived by using the covariant derivative with respect to the Christoffel symbol. We can envisage more complex interactions of a spin-$\frac{1}{2}$ particle, but it is difficult to find an affine connexion corresponding to each of these interactions. *This fact implies that it is not always clever to attack the present problem by the conventional geometrical approach to gravitation.* Presumably the gravitational intaraction of spin-$\frac{1}{2}$ baryons and leptons would be very complicated, thus making the energy-momentum tensor *generally asymmetric.* For example, some authors investigated if it conserves parity and time-reversal [6]. The conclusion reached up

---

[6] J. LEITNER and S. OKUBO: *Phys. Rev.*, **136**, B 1542 (1964); K. HIIDA and Y. YAMAGUCHI: *Progr. Theor. Phys. Suppl. (Kyoto), Commemoration Issue for the XXX Anniversary of the Meson Theory by Dr. Yukawa* (1965); K. HAYASHI: *Parity, charge conjugation and time reversal in the gravitational interaction; the possible existence of a massless scalar particle and Schwinger's equal-time commutation relation for energy density,* preprint, MPI.

to now is rather unsatisfactory and not yet conclusive; one cannot rule out the possible violation of these discrete symmetries. We cannot help leaving the answer to this question to further researches in the future.

Finally, we close this letter by remarking that the macroscopic limit taken in our field equations brings them to the Einstein's field equations of gravitation (presumably with the exception of the neutron stars with strong magnetic fields) whatever gravitational interactions of spin-$\frac{1}{2}$ baryons and leptons may exist, because all the spin polarization effects will be averaged out in this limiting process and hence the stress tensor of macroscopic bodies will have *no memory* of microscopic gravitational interactions whatsoever.

Direttore responsabile: GIULIANO TORALDO DI FRANCIA

Stampato in Bologna dalla Tipografia Compositori coi tipi della Tipografia Monograf
Questo fascicolo è stato licenziato dai torchi il 30-X-1972