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Nonleptonic Weak Decays of Charmed Baryons (*).

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Summary. — The nonleptonic decay rates $\Lambda_c^+ \to p\bar{K}^0$ and $\Lambda_c^+ \to \Lambda\pi^+$ are calculated by using current algebra and an evaluation of the matrix element by $\langle B_f|H_W^{p.c.}|B_i\rangle$ using nonrelativistic SU_6 wave functions. The results are found to be in good agreement with experiment. The results are also compared with earlier quark model and MIT bag model calculations.

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1. – Introduction.

In 1978, RIAZUDDIN and FAYYAZUDDIN (1) deduced the $\Delta I = \frac{1}{2}$ rule and the $(d/f)_W$ ratio in nonleptonic weak decays of noncharmed baryons by considering weak-meson boson exchange graphs as the analogue of gluon exchange. In this paper we apply the same standard current algebra techniques, extended to SU_4, and obtain a good fit to the known nonleptonic-decay rates $\Lambda_c^+ \to p\bar{K}^0$, $\Lambda_c^+ \to \Lambda\pi^+$ using the same scale parameter of Riazuddin and Fayyazuddin (1,2). We compare our analysis with quark model results and with a recent calculation using the MIT bag.

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(1) RIAZUDDIN and FAYYAZUDDIN: *Phys. Rev. D*, **18**, 1578 (1978); **19**, 1630 (1978).

(2) M. D. SCADRON: *Rep. Prog. Phys.*, **44**, 213 (1981).

2. – Combining current algebra and quark amplitudes.

The standard point of our analysis is the weak Hamiltonian density

$$H_{\mathrm{W}} = \frac{G}{2\sqrt{2}}(J^{\mu}J^{+}_{\mu} + J^{\mu+}J_{\mu}) \tag{1}$$

with the hadronic part of the weak $V-A$ left-handed SU_4 quark current [3,4] given by

$$J_{\mu} = \bar{u}\gamma_{\mu}(1-i\gamma_5)(d\cos\theta_{\mathrm{C}} + s\sin\theta_{\mathrm{C}}) + \bar{c}\gamma_{\mu}(1-i\gamma_5)(s\cos\theta_{\mathrm{C}} - d\sin\theta_{\mathrm{C}}) \tag{2}$$

with $G = 1.026\cdot 10^{-5}\, m_{\mathrm{p}}^{-2}$ and where u, d, s and c stand for the respective quark fields and θ_{C} is the Cabibbo angle. As a first approximation we ignore the short-distance QCD effects [5] so that the Cabibbo-enhanced charm-changing effective Hamiltonian is

$$H_{\mathrm{W}}^{\mathrm{eff}} = \frac{G}{2\sqrt{2}}\cos^2\theta_{\mathrm{C}}[\bar{u}\gamma_{\mu}(1-i\gamma_5)d][\bar{s}\gamma^{\mu}(1-i\gamma_5)c]\,. \tag{3}$$

This obeys the selection rule $\Delta C = \Delta S = \Delta I = 1$. Later we shall comment upon short-distance modifications of (3).

The matrix element for baryon decay processes can be written as [2]

$$M = -\langle B^{f}(p_f)P^{j}(q)|H_{\mathrm{W}}|B^{i}(p_i)\rangle = \bar{u}_{\mathrm{B}^f}[iA + B\gamma_5]u_{\mathrm{B}^i}\,, \tag{4}$$

where A and B are the (parity violating) s-wave and (parity conserving) p-wave amplitudes, respectively. The three-hadron matrix elements of the weak Hamiltonian may be reduced to baryon-to-baryon transition elements of H_{W} by applying standard soft-meson techniques [6], to give

$$M = \frac{i}{f_{\mathrm{P}}}\langle B^{f}|[Q_5^{j}, H_{\mathrm{W}}]|B^{i}\rangle + M_{\mathrm{p}}(q) + M_{\mathrm{fac}}(q)\,. \tag{5}$$

Here Q_5^j is the axial generator associated with the meson P^j and f_{P} is the corresponding pseudoscalar-meson decay constant ($f_{\pi} = 93$ MeV, $f_{\mathrm{K}} = 112$ MeV).

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(3) N. CABIBBO: *Phys. Rev. Lett.*, **10**, 531 (1963).
(4) S. GLASHOW, J. ILIOPOULOS and L. MAIANI: *Phys. Rev. D*, **2**, 1285 (1970).
(5) B. W. LEE and M. K. GAILLARD: *Phys. Rev. Lett.*, **33**, 108 (1974); G. ALTARELLI and L. MAIANI: *Phys. Lett. B*, **52**, 351 (1974).
(6) R. E. MARSHAK, RIAZUDDIN and C. P. RYAN: *Theory of Weak Interactions in Particle Physics* (New York, N. Y., 1969).

$M_p(q)$ are possible pole term contributions and $M_{fac}(q)$ represent the factorization (quark decay diagram) contributions.

The $V-A$ structure of H_W leads to $[Q_5^j, H_W] = -[Q^j, H_W]$, so that

$$M = -\frac{i}{f_P}\langle B^f|[Q^j, H_W]|B^i\rangle + M_p(q) + M_{fac}(q)\,, \tag{6}$$

where Q^j is the SU_4 charge with the quantum numbers of the meson P^j and operates on the baryon states to its left and right as a SU_4 generator.

The s-wave contributions of the $\frac{1}{2}^+$ baryon pole terms are suppressed, whereas the commutator term contributes only to the s-wave amplitudes A, so that

$$A = A_{fac} - \frac{1}{f_P}\langle B^f|[Q^j, H_W^{p.c.}]|B^i\rangle\,, \tag{7a}$$

$$B = B_{fac} + B_{pole}\,, \tag{7b}$$

where B_{pole} is obtained from fig. 1 as (²)

$$B_{pole} = -(m_f + m_i)\sum_n\left(\frac{g_{fnj}H_{ni}^{p.c.}}{(m_i - m_n)(m_f + m_n)} - \frac{H_{fn}^{p.c.}g_{nij}}{(m_n - m_f)(m_i + m_n)}\right) \tag{8}$$

with $\langle n|H_W^{p.c.}|i\rangle = H_{ni}^{p.c.}$. Here g_{fnj} is the strong-coupling constant.

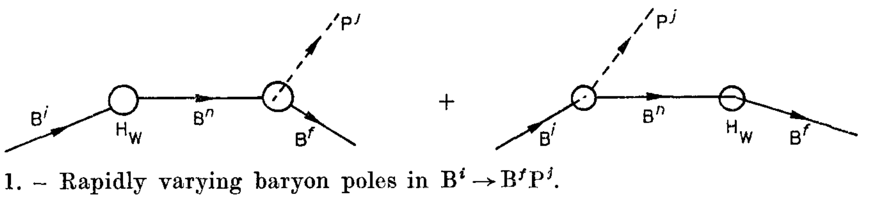

Fig. 1. – Rapidly varying baryon poles in $B^i \to B^f P^j$.

Note that, because Q^j is a generator of SU_4 and the $\frac{1}{2}^+$ baryon states we are considering are members of the 20 multiplet of SU_4, both amplitudes are described as a sum of terms involving transitions of the form $\langle B_2|H_W^{p.c.}|B_1\rangle$.

Finally, using SU_4, the factorization contributions can be specified in terms of the measured form factors of current transitions between known baryons, as has been discussed by BURAS (⁷).

3. – Charmed $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to p\bar{K}^0$ decays.

Since there are experimental data on only two charmed-hyperon decays $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to p\bar{K}^0$, we shall give the details of the calculation and the

(⁷) A. J. BURAS: *Nucl. Phys. B*, **109**, 373 (1976).

results for just these decays. The relevant formulae for these two processes are:

1) $\Lambda_c^+ \to \Lambda\pi^+$. In this case the current commutator term vanishes, so that

$$(9a)\qquad A = A_{\text{fac}}\,,$$

$$(9b)\qquad B = B_{\text{fac}} - \frac{2}{\sqrt{3}}\, dg(\Lambda_c + \Lambda)\left(\frac{\langle\Sigma^+|H_W^{\text{p.c.}}|\Lambda_c^+\rangle}{(\Lambda_c - \Sigma^+)(\Lambda + \Sigma^+)} - \frac{\langle\Lambda|H_W^{\text{p.c.}}|\Sigma_c^0\rangle}{(\Sigma_c^0 - \Lambda)(\Lambda_c^+ + \Sigma_c^0)}\right).$$

2) $\Lambda_c^+ \to p\bar{K}^0$:

$$(10a)\qquad A = A_{\text{fac}} + \frac{1}{\sqrt{2} f_K}\langle\Sigma^+|H_W^{\text{p.c.}}|\Lambda_c^+\rangle\,,$$

$$(10b)\qquad B = B_{\text{fac}} + \frac{(\Lambda_c^+ + P)\sqrt{2}(f-d)g\langle\Sigma^+|H_W^{\text{p.c.}}|\Lambda_c^+\rangle}{(\Lambda_c^+ - \Sigma^+)(\Sigma^+ + P)}\,.$$

Here the masses of the baryons are denoted by the corresponding particle symbols. f and d denote the usual SU_3 antisymmetric and symmetric meson-baryon coupling with $f + d = 1$ and $f/d = \frac{1}{2}$. $g = 13.45$ is the pion-nucleon coupling constant.

The task now is to compute $\langle B_2|H_W^{\text{p.c.}}|B_1\rangle$. Riazuddin and Fayyazuddin calculated these matrix elements for noncharmed-hyperon decays using non-relativistic SU_2 quark wave functions for the baryons. We repeat the same calculation noting that the wave functions for Λ_c^+, Σ_c^0 are obtained from the corresponding strange hyperons by making the replacement $s \to c$ [8]. The quark scattering diagram is shown in fig. 2.

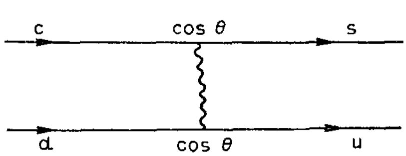

Fig. 2. – W scattering of quarks in the charm-changing nonleptonic weak Hamiltonian density.

The effective H_W is the Fourier transform of the nonrelativistic limit of fig. 2. This gives in leading order

$$(11)\qquad H_W^{\text{p.c.}} = \frac{1}{\sqrt{2}}\, G\cos^2\theta_C \sum_{i>j}(\alpha_i^- \beta_j^+ + \beta_i^+ \alpha_j^-)(1 - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\,\delta^3(\boldsymbol{r})$$

[8] J. Finjord and F. Ravndal: *Phys. Lett. B*, **58**, 61 (1975).

and $H_W^{p.v.} = 0$. Here α_i^- and β_j^+ are operators which, respectively, transform a c-quark into a s-quark and a d-quark into a u-quark. Working out the spin and unitary-spin components of the various constituent quark baryon transitions, using the nonrelativistic SU_6 wave functions [9], we find

$$\langle\Sigma^+|H_W^{p.c.}|\Lambda_c^+\rangle = \langle\Lambda|H_W^{p.c.}|\Sigma_c^0\rangle = \frac{1}{\sqrt{6}}\,\mathrm{ctg}\,\theta_C\langle p|H_W^{p.c.}|\Sigma^+\rangle\,, \tag{12}$$

where $\langle p|H_W^{p.c.}|\Sigma^+\rangle$ is the transition matrix element evaluated by RIAZUDDIN and FAYYAZUDDIN [1,2]. Thus

$$\langle\Sigma^+|H_W^{p.c.}|\Lambda_c^+\rangle = -\frac{27G\cos^2\theta_C}{8\sqrt{12}\,\pi\alpha_s}(\Sigma-\Lambda)\left(\frac{\hat{m}^2}{1-\hat{m}/m_s}\right)_{\text{cons}}, \tag{13}$$

where α_s ($q \approx 1$ GeV) $\simeq 0.5$, Σ, Λ denote the masses of the respective baryons, $\hat{m}$ and m_s are the constituent masses of the nonstrange and strange quarks, respectively, with $\hat{m} = 340$ MeV and $m_s = 510$ MeV.

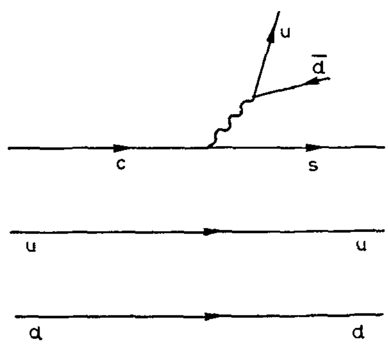

Fig. 3. – Quark diagram for $\Lambda_c^+ \to \Lambda\pi^+$.

The factorization or quark diagram contributions to $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to p\bar{K}^0$ are given in fig. 3 and 4, respectively. The decay amplitude for fig. 3 is given

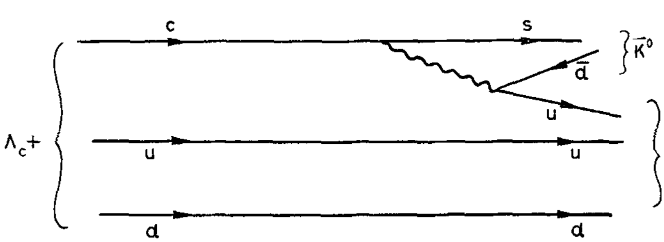

Fig. 4. – Quark diagram for $\Lambda_c^+ \to p\bar{K}^0$.

(9) W. THIRRING: *Acta Phys. Austriaca Suppl.*, **2**, 205 (1965).

by

$$M_{\text{fac}} = -\frac{iG\cos^2\theta_C}{2} f_\pi q^\mu \langle \Lambda | J_\mu^{\Delta c=1}(\bar{s}c) | \Lambda_c^+ \rangle . \tag{14}$$

To obtain the amplitude for fig. 4 it is convenient to perform the Fierz transformation

$$\sum_{i,j=1}^{3} (\bar{u}^i d^i)(\bar{s}^j c^j) = \sum_{i,j=1}^{3} (\bar{s}^j d^i)(\bar{u}^i c^j) . \tag{15}$$

This equation is valid for $V-A$ currents and Fermi statistics of the quarks. It transforms the Hamiltonian to a form containing neutral $V-A$ currents. Because the particles are colour singlets, the amplitude corresponding to fig. 4 is obtainable from the effective Hamiltonian

$$\frac{1}{2\sqrt{2}} G \cos^2\theta_C \frac{1}{3} \left((\bar{s}d)(\bar{u}c) + \text{h.c.}\right), \tag{16}$$

where we have included the colour suppression factor $\frac{1}{3}$. Thus the amplitude for fig. 4 is

$$M_{\text{fac}} = -\frac{iG\cos^2\theta_C}{6} f_K q^\mu \langle p | J_\mu^{\Delta c=1}(\bar{u}c) | \Lambda_c^+ \rangle . \tag{17}$$

The current matrix elements in (14) and (17) have been related by BURAS (7) to the measured form factors of current transitions involving known baryons.

4. – Results.

Finally we get

1) $\Lambda_c^+ \to \Lambda\pi^+$:

$$A = -\frac{Gf_\pi}{2}\cos^2\theta_C H_1^{3*}(q^2)(\Lambda_c - \Lambda) , \tag{18a}$$

$$B = \frac{Gf_\pi}{2}\cos^2\theta_C H_3^{3*}(q^2)(\Lambda_c + \Lambda) + \frac{3}{4}\frac{G\cos^2\theta_C}{\pi\alpha_s} g(\Lambda_c + \Lambda)(\Sigma - \Lambda)\left(\frac{\hat{m}^2}{1-\hat{m}/m_s}\right)_{\text{cons}} \cdot \left(\frac{1}{(\Lambda_c - \Sigma^+)(\Lambda + \Sigma^+)} - \frac{1}{(\Sigma_c^0 - \Lambda)(\Lambda_c^+ + \Sigma_c^0)}\right); \tag{18b}$$

2) $\Lambda_c^+ \to p\bar{K}^0$:

$$(19a)\qquad A = -\frac{Gf_K}{2\sqrt{6}}\cos^2\theta_C H_1^{3*}(q^2)(\Lambda_c - P) - \frac{27}{16\sqrt{6}}\frac{G\cos^2\theta_C}{\pi\alpha_s f_K}(\Sigma-\Lambda)\left(\frac{\hat{m}^2}{1-\hat{m}/m_s}\right)_{\text{cons}},$$

$$(19b)\qquad B = \frac{Gf_K}{2\sqrt{6}}\cos^2\theta_C H_3^{3*}(q^2)(\Lambda_c + P) + \frac{9}{8\sqrt{6}}\frac{G\cos^2\theta_C}{\pi\alpha_s}g(\Sigma-\Lambda)\left(\frac{\hat{m}^2}{1-\hat{m}/m_s}\right)_{\text{cons}}\frac{\Lambda_c^+ + P}{(\Lambda_c^+ - \Sigma^+)(\Sigma^+ + P)}.$$

As described in ref. (7) the $q^2 = 0$ values of the form factors H_1^{3*} and H_3^{3*} are fixed from the vector and axial vector form factors of known baryons

$$(20)\qquad H_1^{3*}(0) = 1\,,\qquad H_3^{3*}(0) = g_A(f_A + \tfrac{1}{3}d_A)\,.$$

We take $g_A = 1.254$ and $f_A/d_A = \frac{2}{3}$. Following KORNER *et al.* (10) we use the invariant form factors $H_1^{3*}(q^2)$ and $H_3^{3*}(q^2)$ to continue from $q^2 = 0$ to $q^2 = m_p^2$, where m_p is the mass of the relevant pseudoscalar meson, *i.e.* either m_π or m_K. We use the standard dipole form factor of the form

$$\left(1-\frac{q^2}{m^2_{F^*,D^*}}\right)^{-2}$$

with $m_{F^*} = 2.14$ GeV and $m_{D^*} = 2.006$ GeV. Since nothing is known about the mass values of the axial vector mesons F_A and D_A that appear in the form factors H_3^{3*}, we use the same mass values for these as for D* and F*.

The theoretical values of the partial widths Γ for $\Lambda\pi^+$ and $p\bar{K}^0$ are compared with the experimental values and with other calculations in table I (11).

TABLE I. – *Partial widths for* $\Lambda_c^+ \to \Lambda\pi^+$ *and* $p\bar{K}^0$ *in units of* $10^{11}\,s^{-1}$.

Γ	Experiment	Quark model (10)	MIT bag (11)	Current algebra, present calculation
$\Lambda_c^+ \to \Lambda\pi^+$	0.54 ± 0.5	0.8	too large by a factor of $3\div5$	0.77
$\Lambda_c^+ \to p\bar{K}^0$	$1.00^{+0.86}_{-0.78}$	8.9	1.40	1.64

(10) J. G. KORNER, G. KRAMER and J. WILLRODT: *Z. Phys. C*, **2**, 117 (1979).
(11) D. EBERT and W. KALLIES: CERN preprint TH. 3598.

We see that our model fits the known experimental results better than either the quark model calculations of Korner *et al.*, which seriously over-estimates the width for the $p\bar{K}^0$ mode, or the MIT bag model calculation, which seriously over-estimates the $\Lambda\pi^+$ mode by a factor of $3\div 5$. Our results are well within the experimental errors for both modes.

However, there is a caveat to these claims. We have not included possible short-distance effects of strong-interaction QCD. The appearance of a new term $(\bar{s}d)(\bar{u}c)$ (neutral current) interaction is expected from the short-distance expansion of the W-boson exchange amplitude in an asymptotically free gauge theory of coloured quarks. One obtains the effective Hamiltonian ([12])

$$H^{W}_{\text{eff}} = \frac{1}{2\sqrt{2}} G \cos^2\theta_C \{C_1(\bar{u}d)(\bar{s}c) + C_2(\bar{s}d)(\bar{u}c) + \text{h.c.}\}. \tag{21}$$

The new effective Hamiltonian will change the quark diagram amplitude for $\Lambda^+_c \to \Lambda\pi^+$ (fig. 3) by the factor $C_1 + \frac{1}{3} C_2$ and the amplitude corresponding to fig. 4 by the factor $C_1 + 3C_2$. Though these effects are appreciable, they are not as significant as the effect of employing H^{W}_{eff} of eq. (21) in evaluating the matrix element $\langle\Sigma^+|H^{\text{p.c.}}_{W}|\Lambda^+_c\rangle$. Using the values of C_1 and C_2 preferred by Korner *et al.*, $C_1 = 1.315$, $C_2 = -0.585$, we find that the matrix element $\langle\Sigma^+|H^{\text{p.c.}}_{W}|\Lambda^+_c\rangle$ is enhanced by a factor of 1.9. The total effect of including these short-distance factors is to significantly increase the widths for both decay modes to

$$\Gamma_{\Lambda\pi^+} = 1.61\cdot 10^{11}\,\text{s}^{-1} \quad \text{and} \quad \Gamma_{p\bar{K}^0} = 2.87\cdot 10^{11}\,\text{s}^{-1}\,,$$

which are too large as compared to the experimental values. However, we notice that the ratio of the decay widths still very fits well (table II).

TABLE II. – *Ratio of partial widths* $R = \Gamma(\Lambda\pi^+)/\Gamma(p\bar{K}^0)$.

	Experiment	Without short-distance factors	With short-distance factors
$R = \Gamma(\Lambda\pi^+)/\Gamma(p\bar{K}^0)$	0.54	0.47	0.56

The conclusion we reach is that the unmodified Hamiltonian gives very good results, whereas the QCD-corrected effective Hamiltonian does not fit the experimental rates. The modified Hamiltonian also does not give as good results of charmed-meson decays as obtained by ignoring short-distance correc-

([12]) M. K. Gaillard, B. W. Lee and J. L. Rosner: *Rev. Mod. Phys.*, **47**, 227 (1975); J. Ellis, M. K. Gaillard and D. V. Nanopoulos: *Nucl. Phys. B*, **100**, 313 (1975); G. Altarelli, N. Cabibbo and L. Maiani: *Phys. Rev. Lett.*, **35**, 635 (1975).

tions ([13]). As suggested by GUBERINA *et al.* ([14]) one possible solution would be to include the effects of soft gluons. However, our results are a significant improvement on earlier calculations.

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([13]) M. D. SCADRON: University of Arizona preprint (1983).
([14]) B. GUBERINA, D. TADIĆ and J. TRAMPETIĆ: *Z. Phys. C*, **13**, 251 (1982).

RIASSUNTO (*)

Si calcolano i tassi di decadimento non leptonico $\Lambda_c^+ \to p\bar{K}^0$ e $\Lambda_c^+ \to \Lambda\pi^+$ per mezzo dell'algebra delle correnti ed anche una valutazione dell'elemento di matrice $\langle B_f|H_W^{p.c.}|B_i\rangle$ usando funzioni d'onda SU_6 non relativistiche. Si trova che i risultati sono in buon accordo con l'esperimento. I risultati sono anche confrontati con precedenti calcoli del modello dei quark e del modello a sacca del MIT.

(*) *Traduzione a cura della Redazione.*

Нелептонные слабые распады очарованных барионов.

Резюме (*). — Вычисляются интенсивности нелептонных распадов $\Lambda_c^+ \to p\bar{K}^0$ и $\Lambda_c^+ \to \Lambda\pi^+$, используя алгебру току и оценку матричного элемента $\langle B_f|H_W^{p.c.}|B_i\rangle$ с помощью нерелятивистских волновых функций. Полученные результаты согласуются с экспериментом. Результаты также сравниваются с вычислениями в рамках модели кварков и MIT модели «мешка».

(*) *Переведено редакцией.*