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ON DISCRETE SUBGROUPS CONTAINING A LATTICE IN A HOROSPHERICAL SUBGROUP

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ABSTRACT

We showed in [Oh] that for a simple real Lie group G with real rank at least 2, if a discrete subgroup Γ of G contains lattices in two opposite horospherical subgroups, then Γ must be a non-uniform arithmetic lattice in G, under some additional assumptions on the horospherical subgroups. Somewhat surprisingly, a similar result is true even if we only assume that Γ contains a lattice in one horospherical subgroup, provided Γ is Zariski dense in G.

1. Introduction

Let G be a connected semisimple algebraic $\mathbb{R}$-group. A unipotent subgroup U of G is called a **horospherical** $\mathbb{R}$-subgroup if U is the unipotent radical of some proper parabolic $\mathbb{R}$-subgroup of G.

The main theorem in this paper is as follows:

1.1. THEOREM: *Let G be a connected absolutely simple $\mathbb{R}$-split algebraic group with rank at least 2. Suppose that G is not of type A_2. Let Γ be a discrete Zariski dense subgroup of $G(\mathbb{R})$. Then Γ is a non-uniform arithmetic lattice in $G(\mathbb{R})$ if and only if there exists a horospherical $\mathbb{R}$-subgroup U of G such that $\Gamma \cap U$ is Zariski dense in U.*

1.2. COROLLARY: *Let G be as in Theorem 1.1 and U a horospherical $\mathbb{R}$-subgroup of G.*

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(1) *Let X be any subset of $U(\mathbb{R})$ such that the subgroup generated by the elements of X is Zariski dense in U.*

(2) *Let Y be any subset of $G(\mathbb{R})$ such that the subgroup $\Gamma_{X\cup Y}$ generated by the elements of $X \cup Y$ is Zariski dense.*

Then either $\Gamma_{X\cup Y}$ is a non-uniform arithmetic lattice in $G(\mathbb{R})$ or $\Gamma_{X\cup Y}$ is not discrete.

The proof of Theorem 1.1 is based on some results obtained in [Oh], which we will recall in the following.

Two parabolic subgroups are called **opposite** if their intersection is a common Levi subgroup in both of them. Two horospherical subgroups are called **opposite** if they are the unipotent radicals of opposite parabolic subgroups.

1.3. THEOREM (cf. [Oh, Theorem 0.2]): *Let G be a connected absolutely simple $\mathbb{R}$-algebraic group with real rank at least 2 and U_1, U_2 a pair of opposite horospherical $\mathbb{R}$-subgroups of G. Let F_1 and F_2 be lattices in $U_1(\mathbb{R})$ and $U_2(\mathbb{R})$, respectively. Suppose that G is split over $\mathbb{R}$ and that if G is of type A_2, then U_1 is not the unipotent radical of a Borel subgroup. Then F_1 and F_2 generate a discrete subgroup if and only if there exists a $\mathbb{Q}$-form of G with respect to which U_1 and U_2 are defined over $\mathbb{Q}$ and $F_i \subset U_i(\mathbb{Z})$ for each $i = 1, 2$. Furthermore, the discrete subgroup generated by F_1 and F_2 is a subgroup of finite index in $G(\mathbb{Z})$.*

In [Oh, Theorem 0.2], we proved that if the subgroups F_1 and F_2 as in the statement of Theorem 1.3 generate a discrete subgroup, then there exists a $\mathbb{Q}$-form of G such that the subgroup Γ_{F_1,F_2} generated by F_1 and F_2 is commensurable to the subgroup $G(\mathbb{Z})$. We deduce Theorem 1.3 from this theorem; since G is absolutely simple and hence the center of G is trivial, we can assume that $G \subset \mathrm{SL}_N(\mathbb{C})$ by considering the adjoint representation of G. Since Γ_{F_1,F_2} is an arithmetic subgroup, it is not difficult to see that there exists a Γ_{F_1,F_2}-invariant lattice L in $\mathbb{Q}^N$ (cf. [P-R, Proposition 4.2]); hence $\Gamma_{F_1,F_2} \subset G^L = \{g \in G \mid g(L) \subset L\}$. Now by applying the automorphism of $\mathrm{SL}_N(\mathbb{C})$ that changes the standard basis to a basis of L, we can change the $\mathbb{Z}$-form of G so that $G(\mathbb{Z}) = G^L$, proving Theorem 1.3.

The following corollary follows from the above Theorem 1.3 and Theorem 7.1.1 in [Ma].

1.4. COROLLARY: *Let G be a connected absolutely simple $\mathbb{R}$-split algebraic group with rank at least 2 and Γ a discrete subgroup of $G(\mathbb{R})$. Suppose that G is not of type A_2. Then Γ is a non-uniform lattice in $G(\mathbb{R})$ if and only if there* *exists a pair U_1, U_2 of opposite horospherical $\mathbb{R}$-subgroups of G such that $\Gamma \cap U_i$ is Zariski dense in U_i for each $i = 1, 2$.*

Using Corollary 1.4, Theorem 1.1 is then a consequence of the following proposition.

1.5. PROPOSITION: *Let G be a connected semisimple $\mathbb{R}$-algebraic group and Γ a discrete Zariski dense subgroup of $G(\mathbb{R})$. Suppose that there exists a horospherical $\mathbb{R}$-subgroup U of G such that $\Gamma \cap U$ is Zariski dense in U. Then there exists a pair U_1, U_2 of opposite horospherical $\mathbb{R}$-subgroups of G such that $U \subset U_1$ and $\Gamma \cap U_i$ is Zariski dense in U_i for each $i = 1, 2$.*

For simplicity, Theorem 1.3 is stated here only for $\mathbb{R}$-split groups but we proved it in much greater generality (see [Oh, Theorem 0.3]). All the above theorems are valid for all situations in which Theorem 0.3 in [Oh] holds.

Remark: Theorem 1.3 was conjectured by G. Margulis in greater generality (see [Oh1] for the full statement of the conjecture).

2. Unipotent subgroups of Γ

2.1. Definition: Let H be a locally compact topological group, and D and M closed subgroups of H. We say that D is M-**proper** if the natural map $D/(D \cap M) \to H/M$ is proper, and M-**compact** if the factor space $D/(D \cap M)$ is compact.

2.2. LEMMA [Ma, Lemma 2.1.4]: *If M is a discrete subgroup of H, D_1 is M-proper and D_2 is M-compact, then $D_1 \cap D_2$ is M-compact.*

2.3. Let G be a connected semisimple algebraic $\mathbb{R}$-group. For a subgroup F of $G(\mathbb{R})$, we denote by $N(F)$ the normalizer of F in G, by $[F, F]$ the commutator subgroup of F, and by $S(F)$ the subgroup $[N(F), N(F)] \cap G(\mathbb{R})$. The notation $\overline{F}$ denotes the Zariski closure of F.

The following theorem in [Ma], which was originally stated only for the case where Γ is a lattice, holds for arbitrary discrete subgroups, as we can see from the proof given in [Ma].

THEOREM [Ma, Lemma 5.2.2]: *Let Γ be a discrete subgroup of $G(\mathbb{R})$ and F a unipotent subgroup of Γ such that $\overline{F} \cap \Gamma = F$. Then $S(\overline{F})$ is Γ-proper.*

2.4. COROLLARY: *Let G be a connected semisimple algebraic $\mathbb{R}$-group and Γ a discrete subgroup of $G(\mathbb{R})$. Let F_1 and F_2 be unipotent subgroups of Γ such* *that $\overline{F}_i \cap \Gamma = F_i$ for each $i = 1, 2$. Then $S(\overline{F}_1) \cap \overline{F}_2$ is Γ-compact. In particular, $S(\overline{F}_1) \cap \overline{F}_2 \cap \Gamma$ is a co-compact lattice in $S(\overline{F}_1) \cap \overline{F}_2$.*

Proof: By Theorem 2.3, $S(\overline{F_1})$ is Γ-proper. Since a Zariski dense discrete subgroup of a unipotent algebraic group is a co-compact lattice, $\overline{F_2} \cap G(\mathbb{R})$ is Γ-compact. Therefore, by Lemma 2.2, $S(F_1) \cap F_2$ is Γ-compact. ∎

Note that the above corollary gives a way of obtaining a subgroup of $\overline{F}_2$ that intersects Γ as a lattice when $S(\overline{F_1}) \cap \overline{F_2}$ is non-trivial.

3. Horospherical subgroups and their intersections with Γ

3.1. Let G be a connected semisimple algebraic $\mathbb{R}$-group, S a maximal $\mathbb{R}$-split torus of G and T a maximal $\mathbb{R}$-torus containing S. Denote by ${}_{\mathbb{R}}\Phi = \Phi(S, G)$ (resp. $\Phi = \Phi(T, G)$) the set of roots of G with respect to S (resp. T). We choose compatible orderings on Φ and ${}_{\mathbb{R}}\Phi$, and let ${}_{\mathbb{R}}\Delta$ be the simple roots of ${}_{\mathbb{R}}\Phi$ with respect to this ordering. Let $j\colon \Phi \to {}_{\mathbb{R}}\Phi \cup \{0\}$ be the map induced by the restriction to S.

For each $b \in \Phi(T, G)$, we denote by U_b the unique one parameter root subgroup associated with b. For $\theta \subset {}_{\mathbb{R}}\Delta$, $[\theta]$ denotes the $\mathbb{Z}$-linear combinations of elements of θ which are roots in ${}_{\mathbb{R}}\Phi$. A subset $\Psi \subset {}_{\mathbb{R}}\Phi$ is called **closed** if $a, b \in \Psi$ and $a + b \in {}_{\mathbb{R}}\Phi$ imply $a + b \in \Psi$. If Ψ is closed, then we denote by G_Ψ the subgroup generated by T and the subgroups U_a, $a \in j^{-1}(\Psi \cup \{0\})$, and by G_Ψ^* the subgroup generated by the subgroups U_a, $a \in j^{-1}(\Psi)$. If G_Ψ^* is unipotent, then it will also be denoted by U_Ψ and the set Ψ in this case will be called **unipotent**.

For $\theta \subset {}_{\mathbb{R}}\Delta$, we define the following closed subsets of ${}_{\mathbb{R}}\Phi$:

$$\pi_\theta = [\theta] \cup {}_{\mathbb{R}}\Phi^+, \quad {\pi_\theta}^- = [\theta] \cup {}_{\mathbb{R}}\Phi^-, \quad \beta_\theta = {}_{\mathbb{R}}\Phi^+ - [\theta], \quad {\beta_\theta}^- = {}_{\mathbb{R}}\Phi^- - [\theta].$$

For the sake of simplicity, we shall denote by P_θ, ${P_\theta}^-$, V_θ and ${V_\theta}^-$ the subgroups G_{π_θ}, $G_{{\pi_\theta}^-}$, U_{β_θ} and $U_{{\beta_\theta}^-}$, respectively.

We recall some well-known facts about parabolic subgroups and horospherical subgroups (cf. [B-T]). Any parabolic (resp. horospherical) $\mathbb{R}$-subgroup of G is conjugate to P_θ (resp. V_θ) for some $\theta \subset {}_{\mathbb{R}}\Delta$. Any pair of opposite parabolic (resp. horospherical) $\mathbb{R}$-subgroups is conjugate to the pair P_θ, ${P_\theta}^-$ (resp. V_θ, ${V_\theta}^-$) for some $\theta \subset {}_{\mathbb{R}}\Delta$. Note that $P_\emptyset$ is a minimal parabolic $\mathbb{R}$-subgroup and $V_\emptyset$ is a maximal horospherical $\mathbb{R}$-subgroup. Any parabolic $\mathbb{R}$-subgroup containing $P_\emptyset$ is of the form P_θ for some $\theta \subset {}_{\mathbb{R}}\Delta$ and any horospherical $\mathbb{R}$-subgroup contained in $V_\emptyset$ is of the form V_θ for some $\theta \subset {}_{\mathbb{R}}\Delta$.

Let ${}_{\mathbb{R}}W = N(S)/C(S)$ be the relative Weyl group where $C(S)$ denotes the centralizer of S in G. For any $w \in {}_{\mathbb{R}}W$ and any subgroup H of G normalized by $C(S)$, we denote by wHw^{-1} the subgroup $n_w H n_w^{-1}$ where n_w is a representative of w in $N(S)$. We fix $w_0 \in {}_{\mathbb{R}}W$ such that $w_0 V_\emptyset w_0^{-1} = V_\emptyset^-$. Then w_0 takes Φ^+ into Φ^- and there exists an involution i of Φ such that $w_0(i(a)) = -a$ for all $a \in \Phi$. We call i the **opposition involution** of Φ (cf. [B-T, 2.1]). We have that $w_0 P_\theta w_0^{-1} = P_{i(\theta)}^-$ and $w_0 V_\theta w_0^{-1} = V_{i(\theta)}^-$. If $i(\theta) = \theta$, then P_θ (resp. V_θ) is called **reflexive** and so are all the elements in the conjugacy class of P_θ (resp. V_θ). Note that a parabolic subgroup P is reflexive if and only if its conjugacy class contains a parabolic subgroup opposite to P.

It is not difficult to see that the following lemma holds.

3.2. Lemma: *The followings are all equivalent.*

(1) *V_θ is reflexive;*
(2) *$i(\theta) = \theta$;*
(3) *V_θ and $w_0 V_\theta w_0^{-1}$ are opposite;*
(4) *$N(V_\theta) \cap w_0 V_\theta w_0^{-1}$ is trivial.*

3.3. Lemma [B-T, Proposition 3.22]: *If ψ and ϕ are two closed subsets of ${}_{\mathbb{R}}\Phi$ and ψ is unipotent, then $U_\psi \cap G_\phi = U_\psi \cap G_\phi^* = U_{\psi\cap\phi}$.*

3.4. Lemma [B-T, Lemma 4.12]: *Let G be a connected semisimple algebraic $\mathbb{R}$-group and P_1, P_2 two parabolic subgroups of G. Then the set M of all elements $g \in G$ such that P_1 and gP_2g^{-1} contain opposite minimal parabolic subgroups is Zariski dense and open.*

3.5. Corollary: *Let G be a connected semisimple $\mathbb{R}$-algebraic group and P a reflexive parabolic $\mathbb{R}$-subgroup of G. Then the set M of the elements $g \in G$ such that P and gPg^{-1} are opposite is Zariski dense and open.*

Proof: This follows from Lemma 3.4 and Proposition 4.10 in [B-T], which says that two conjugate parabolic subgroups are opposite if they contain opposite minimal parabolic subgroups. ∎

3.6. Proposition: *Let G be a connected semisimple algebraic $\mathbb{R}$-group, Γ a Zariski dense subgroup of $G(\mathbb{R})$ and U a horospherical $\mathbb{R}$-subgroup of G such that $\Gamma \cap U$ is Zariski dense in U. Then there exists an element $h \in G$ such that*

(1) *$hUh^{-1} = V_\theta$ for some $\theta \subset {}_{\mathbb{R}}\Delta$,*
(2) *$w_0 hUh^{-1} w_0^{-1} \cap h\Gamma h^{-1}$ is Zariski dense in $w_0 hUh^{-1} w_0^{-1}$.*

Proof: By Lemma 3.4, there exists an element $\gamma \in \Gamma$ such that $B_1 \subset N(U)$ and $B_2 \subset \gamma N(U)\gamma^{-1}$ for some pair B_1, B_2 of opposite minimal parabolic subgroups.

Therefore $U \subset R_u(B_1)$ and $\gamma U\gamma^{-1} \subset R_u(B_2)$. Since $R_u(B_1)$ and $R_u(B_2)$ are opposite maximal horospherical subgroups, there exists $h \in G(\mathbb{R})$ such that

$$hR_u(B_1)h^{-1} = V_\emptyset \quad \text{and} \quad hR_u(B_2)h^{-1} = V_\emptyset^-.$$

Therefore $hUh^{-1} \subset V_\emptyset$; hence $hUh^{-1} \subset V_\theta$ for some $\theta \subset {}_{\mathbb{R}}\Delta$, and $h\gamma U\gamma^{-1}h^{-1} \subset V_\emptyset^-$. Since $h\gamma U\gamma^{-1}h^{-1} \cap h\Gamma h^{-1} = h\gamma(U \cap \Gamma)\gamma^{-1}h^{-1}$ and $U \cap \Gamma$ is Zariski dense in U by the assumption, we have that $h\gamma U\gamma^{-1}h^{-1} \cap h\Gamma h^{-1}$ is Zariski dense in $h\gamma U\gamma^{-1}h^{-1}$. It only remains to show that $h\gamma U\gamma^{-1}h^{-1} = w_0 hUh^{-1}w_0^{-1}$. Since $w_0 V_\emptyset w_0^{-1} = V_\emptyset^-$, we have $w_0^{-1}h\gamma U\gamma^{-1}h^{-1}w_0 \subset V_\emptyset$. Observe that $hN(U)h^{-1}$ and $w_0^{-1}h\gamma N(U)\gamma^{-1}h^{-1}w_0$ are two conjugate parabolic subgroups which contain the same minimal parabolic subgroup $P_\emptyset$. So $hN(U)h^{-1} = w_0^{-1}h\gamma N(U)\gamma^{-1}h^{-1}w_0$ (cf. [B-T, 4.3]). Since the normalizer of a parabolic subgroup is the parabolic subgroup itself, there exists an element $n \in N(U)$ such that $hn = w_0^{-1}h\gamma$. Therefore $h\gamma U\gamma^{-1}h^{-1} = w_0 hUh^{-1}w_0^{-1}$, proving the proposition. ∎

3.7. COROLLARY: *Let G, Γ, U be as above and U be conjugate to V_θ for some $\theta \subset {}_{\mathbb{R}}\Delta$. Then for some $h \in G(\mathbb{R})$, $h\Gamma h^{-1}$ intersects both V_θ and $V_{i(\theta)}^-$ Zariski densely.*

4. Proof of Proposition 1.5

4.1. PROPOSITION: *Let G be a connected semisimple algebraic $\mathbb{R}$-group and Γ a discrete Zariski dense subgroup of $G(\mathbb{R})$. Suppose that there exists a horospherical $\mathbb{R}$-subgroup U of G such that $\Gamma \cap U$ is Zariski dense in U. Then there exists a reflexive horospherical $\mathbb{R}$-subgroup V of G such that $U \subset V$ and $\Gamma \cap V$ is Zariski dense in V.*

Proof: Note that a maximal horospherical $\mathbb{R}$-subgroup of G is always reflexive and the dimensions of the maximal horospherical $\mathbb{R}$-subgroups of G are all equal. Therefore by induction on the dimension of U, it is enough to prove that if U is not reflexive, then there exists a horospherical $\mathbb{R}$-subgroup V such that $V \cap \Gamma$ is Zariski dense in V and $U \subsetneq V$.

We use the notation ${}_{\mathbb{R}}\Phi$, ${}_{\mathbb{R}}\Delta$ etc. from section 3.1. By Corollary 3.7, there is no loss of generality in assuming that $U = V_\theta$ for some $\theta \subset {}_{\mathbb{R}}\Delta$ and $V_{i(\theta)}^- \cap \Gamma$ is Zariski dense in $V_{i(\theta)}^-$. Let $U' = V_{i(\theta)}^-$. Note that $w_0 U w_0^{-1} = U'$. Suppose that U is not reflexive. Then by Lemma 3.2, $N(U) \cap U'$ is not trivial. By Lemma 3.3, we have that $[N(U), N(U)] \cap U' = N(U) \cap U'$. Therefore it follows from Corollary 2.4 that $N(U)(\mathbb{R}) \cap U'(\mathbb{R})$ is Γ-compact; hence $N(U) \cap U' \cap \Gamma$ is Zariski dense in

$N(U) \cap U'$. Hence $V \cap \Gamma$ is Zariski dense in V where $V = (N(U) \cap U') \ltimes U$. To finish the proof of the proposition, it remains to show that V is a horospherical $\mathbb{R}$-subgroup of G, which we will do in Proposition 4.2. ∎

Note that for $U = V_\theta$ for $\theta \subset {}_{\mathbb{R}}\Delta$, we have

$$(N(U) \cap w_0 U w_0^{-1}) \ltimes U = (P_\theta \cap V_{i(\theta)}^-) \ltimes V_\theta.$$

4.2. Proposition: *Let $U = V_\theta$. Then $(N(U) \cap w_0 U w_0^{-1}) \ltimes U$ is a horospherical $\mathbb{R}$-subgroup of G.*

Proof: Note that $N(V_\theta) = P_\theta$. Let $L = P_\theta \cap P_\theta^-$ and $M = [L, L]$. Then M is a connected semisimple algebraic $\mathbb{R}$-group and $M = G_{[\theta]}^*$, i.e., the subgroup generated by U_a, $a \in j^{-1}[\theta]$. It is well known that $S \cap M$ is a maximal $\mathbb{R}$-split torus of M and the restriction of $[\theta]$ to $S \cap M$ gives all of $\Phi(S \cap M, M)$, which will be denoted by Φ_M. The root system Φ_M has the induced ordering from Φ.

Since $w_0 U w_0^{-1} = V_{i(\theta)}^- = U_{{}_{\mathbb{R}}\Phi^- - [i(\theta)]}$ and the set ${}_{\mathbb{R}}\Phi^- - [i(\theta)]$ is unipotent, we have that $N(U) \cap w_0 U w_0^{-1} = M \cap w_0 U w_0^{-1} = U_{\Phi_M^- - [\theta \cap i(\theta)]}$ by Lemma 3.3. It follows that $M \cap w_0 U w_0^{-1}$ is a horospherical $\mathbb{R}$-subgroup of M. Note that $V_\emptyset \cap M$ is a maximal horospherical $\mathbb{R}$-subgroup of M. There exists an element $w \in N(S \cap M) \cap M$ such that $w(V_\emptyset \cap M) w^{-1} = V_\emptyset^- \cap M$ (cf. section 3.1). Let j be the opposition involution of Φ_M such that $w(j(a)) = -a$ for all $a \in \Phi_M$.

Then

$$wU_{\Phi_M^- - [\theta \cap i(\theta)]} \ltimes U_{{}_{\mathbb{R}}\Phi^+ - [\theta]} w^{-1} = wU_{\Phi_M^- - [\theta \cap i(\theta)]} w^{-1} \ltimes U_{{}_{\mathbb{R}}\Phi^+ - [\theta]}$$

since w normalizes U.

Since w sends $\Phi_M^- - [\theta \cap i(\theta)]$ to $\Phi_M^+ - [j(\theta \cap i(\theta))]$,

$$wU_{\Phi_M^- - [\theta \cap i(\theta)]} w^{-1} \ltimes U_{\Phi^+ - [\theta]} = U_{\Phi_M^+ - [j(\theta \cap i(\theta))]} \ltimes U_{{}_{\mathbb{R}}\Phi^+ - [\theta]} = U_{{}_{\mathbb{R}}\Phi^+ - [j(\theta \cap i(\theta))]}.$$

So $wU_{\Phi_M^- - [\theta \cap i(\theta)]} \ltimes U_{{}_{\mathbb{R}}\Phi^+ - [\theta]} w^{-1}$ is the horospherical $\mathbb{R}$-subgroup $V_{j(\theta \cap i(\theta))}$. This proves that $(M \cap w_0 U w_0^{-1}) \ltimes U$ is a horospherical $\mathbb{R}$-subgroup of G. ∎

4.3. Proof of Proposition 1.5: By Proposition 4.1, we may assume that U is reflexive. By Corollary 3.5, there exists $\gamma \in \Gamma$ such that U and $\gamma U \gamma^{-1}$ are opposite. Since $\gamma U \gamma^{-1} \cap \Gamma = \gamma(U \cap \Gamma)\gamma^{-1}$, it suffices to set $U_1 = U$ and $U_2 = \gamma U \gamma^{-1}$ to prove the proposition. ∎

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