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Nucleon Polarizabilities in the Constituent Quark Model.

M. DE SANCTIS and D. PROSPERI
Dipartimento di Fisica, I Università di Roma «La Sapienza»
P.le A. Moro 2, 00185 Roma
INFN - Sezione di Roma

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Summary. — The spectrum of the nonstrange baryons in the constituent quark model is critically analysed. The electromagnetic properties of the nucleon are calculated with particular emphasis on the electric and magnetic polarizabilities. Relativistic corrections up to order m^{-2} are consistently considered.

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1. – Introduction.

In recent years there have been many attempts to study some electromagnetic properties of nonstrange baryons as the form factors and the dipole polarizabilities in the framework of QCD-inspired quark potential models [1-4].

In such models explicit gluonic degrees of freedom are eliminated and gluon-exchange effects are incorporated by a phenomenological potential between quarks. Only the one-gluon exchange is explicitly taken into account, while the strong coupling constant and the quark mass are considered as «renormalized» parameters simulating higher-order QCD effects. The confinement is supplemented by a two-body potential with a simple radial dependence [4, 5].

These models lead to the determination of explicit baryon wave-functions and energies consistent with the experimental mass spectrum. Moreover these wave-functions can be used for the calculations of the electromagnetic properties of the baryons, that are considered to be particularly suitable to test the validity of the underlying baryon model [3].

In the present work after having selected a nonrelativistic effective Hamiltonian we firstly study the nonstrange baryon mass spectrum, putting into evidence some problems due to the specific form of the model, then we calculate some relevant electromagnetic quantities of the nucleon and of the $\Delta(1232)$ resonance. Particular interest is devoted to the magnetic and electric polarizabilities. For the latter we give an original estimate of the relativistic corrections required by the general consistency of the model. These relativistic corrections are calculated up to order $1/m^2$, according to the general expressions obtained in a previous work [6]. Moreover, we critically analyse the relation between the numerical results for the polarizabilities and the form of the mass spectrum.

2. – The Hamiltonian and its matrix elements.

The main aim in studying the baryon mass spectrum is to reproduce the energies of the low-lying resonances by using the *same* Hamiltonian for *all* the spin-isospin multiplets.

We assume that the Hamiltonian for the baryons is of the following form [3, 7]:

$$H = \sum_i m_i + H_0 + H_{\text{hyp}} + H_{\text{rad}} + E_0 . \tag{1}$$

In eq. (1) $m_i \equiv m$ is the mass of the constituent quark. The second term H_0 represents the kinetic energy and the harmonic-oscillator confining potential written as ($\hbar = c = 1$)

$$H_0 = \sum_i \frac{p_i^2}{2m} + \frac{1}{6} m\omega^2 \sum_{i<j} r_{ij}^2 , \tag{2}$$

The third term H_{hyp} stands for the hyperfine interaction, given by

$$H_{\text{hyp}} = \frac{2\alpha_s}{3m^2} \sum_{i<j} \left\{ \frac{8}{3} \frac{b^3}{\pi^{1/2}} \exp[-b^2 r_{ij}^2]\, \boldsymbol{s}_i \boldsymbol{s}_j + \frac{1}{r_{ij}^3} \left[\frac{3(\boldsymbol{s}_i \boldsymbol{r}_{ij})(\boldsymbol{s}_j \boldsymbol{r}_{ij})}{r_{ij}^2} - \boldsymbol{s}_i \boldsymbol{s}_j \right] \right\} . \tag{3}$$

The first contribution of the equation above represents the spin-spin interaction in which the usual $\delta(\boldsymbol{r}_{ij})$ space dependence has been replaced by a Gaussian function in order to eliminate any inessential singularity and to obtain some improvement in the prediction for the charge radius of the proton. The second term represents the tensor interaction, as it is usually given in the literature [7, 8].

Further, H_{rad} given by [3, 7]

$$H_{\text{rad}} = \sum_{i>j} U(\boldsymbol{r}_{ij}) \tag{4}$$

represents the nonharmonic central contribution to the one-gluon and confining potentials. We do not specify the form of $U(\boldsymbol{r}_{ij})$ because as it will be shown in the following, the matrix elements of this interaction, in the adopted harmonic-oscillator base, can be expressed in terms of only three parameters that are directly fitted to the experimental data.

Finally, E_0 is a splitting constant that is employed to adjust the ground-state energies of the negative-parity multiplets.

Let us remark that in other works [7-9] the various sectors of the baryon spectrum were described by means of different Hamiltonians: in particular H_{rad} was employed *only* for the positive-parity *excited* states.

The separation of the center of mass and internal notion is easily accomplished by using the standard definitions of the Jacobi coordinates $\boldsymbol{\rho}$, $\boldsymbol{\lambda}$ and $\boldsymbol{R}$ [7]. We choose the harmonic-oscillator spatial wave functions (up to $N = 2$ quanta) as a convenient set of base states of our study.

The construction of the space, spin and isospin symmetric zeroth-order wavefunctions is also performed according to the phase conventions given in ref. [7].

In this base the harmonic-oscillator Hamiltonian is obviously diagonal, while H_{hyp} and H_{rad} can be treated as perturbations. The matrix elements of H_{hyp} are obtained by means of standard calculations.

Our results becomes coincident with those given in ref. [7, 8] when the width of the Gaussian function in the spin-spin interaction is reduced to 0, that is $b \to \infty$.

The nonvanishing space integrals involved in the matrix elements of H_{rad}, that are defined as

$$R^{L_\rho}_{N'_\rho N_\rho} = R^{L_\rho}_{N_\rho N_{\rho'}} = \int \mathrm{d}^3\rho\, \psi^*_{N'_\rho L_\rho 0}(\boldsymbol{\rho})\, U(\sqrt{2}\,\boldsymbol{\rho})\, \psi_{N_\rho L_\rho 0}(\boldsymbol{\rho})\,, \tag{5}$$

take the form

$$R^0_{00} = \frac{\alpha^3}{\pi^{1/2}}\, 4a_0\,, \tag{6a}$$

$$R^1_{11} = \frac{\alpha^3}{\pi^{1/2}}\, \frac{8}{3}\, a_2\,, \tag{6b}$$

$$R^0_{02} = \frac{\alpha^3}{\pi^{1/2}}\, 4\left[\sqrt{\frac{3}{2}}\, a_0 - \sqrt{\frac{2}{3}}\, a_2\right], \tag{6c}$$

$$R^2_{22} = \frac{\alpha^3}{\pi^{1/2}}\, \frac{16}{15}\, a_4\,, \tag{6d}$$

$$R^0_{22} = \frac{\alpha^3}{\pi^{1/2}}\, \frac{8}{3}\left[\frac{9}{4}\, a_0 - 3a_2 + a_4\right]. \tag{6e}$$

The quantities $a_m\,(m = 0, 2, 4)$ are given by

$$a_m = \int_0^\infty \rho^2\, \mathrm{d}\rho\, \exp\left[-\alpha^2\rho^2\right] (\alpha\rho)^m\, U(\sqrt{2}\,\boldsymbol{\rho}) \tag{7}$$

with

$$\alpha = (m\omega)^{1/2}\,. \tag{8}$$

As anticipated before, the quantities a_0, a_2 and a_4 will be directly fitted to the experimental data. As a side remark let us note that in studying the *excited* baryons in ref. [7] the 0-quanta state was ignored. Moreover in ref. [3] the mixing between this state and the state with two quanta of radial excitation, that is given by R^0_{02}, was omitted.

3. – The mass spectrum.

The energies of the resonances in the baryon spectrum and their amplitudes in terms of the base states are obtained by diagonalizing the entire Hamiltonian H.

A reasonable reproduction of the experimental baryon spectrum (see fig. 1 and 2) [10], is given by the values of the parameters shown hereunder:

$$(9)\qquad \begin{cases} m = 0.32\,\text{GeV}\,, \\ \omega = 0.25\,\text{GeV}\,, \\ \delta = 0.32\,\text{GeV}\,, \\ b = 12.0\,\text{fm}^{-1}\,, \\ a_0 = -2.91\cdot 10^{-2}\,\text{fm}^3\,\text{GeV}\,, \\ a_2 = -5.48\cdot 10^{-2}\,\text{fm}^3\,\text{GeV}\,, \\ a_4 = -6.06\cdot 10^{-2}\,\text{fm}^3\,\text{GeV}\,, \end{cases}$$

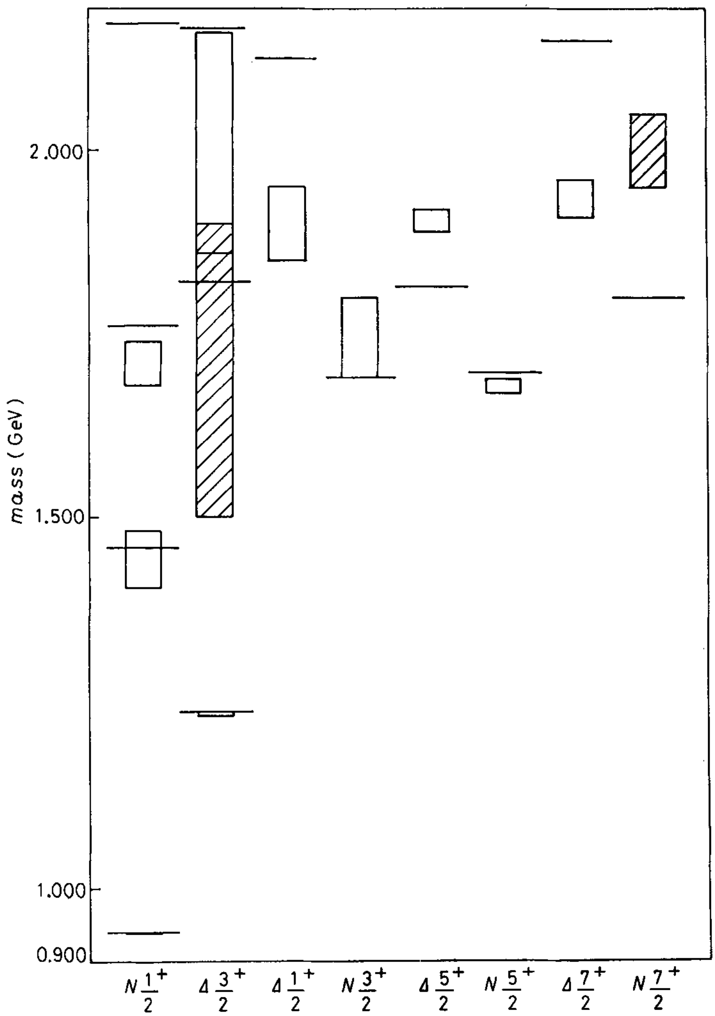

Fig. 1. – Spectrum of nonstrange positive-parity baryons. Shadowed areas are referred to doubtful resonances. The black lines are the predictions of the model.

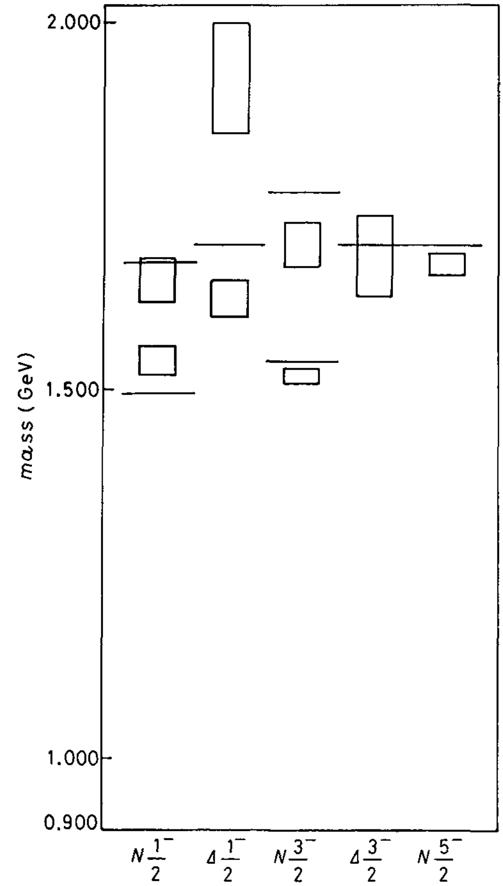

Fig. 2. – Spectrum of nonstrange negative-parity baryons. The black lines are the predictions of the model.

where δ, that is related to the Δ-$\mathcal{N}$ mass difference, is defined as

$$\delta = \frac{4\alpha_s \alpha^3}{3\sqrt{2\pi}\, m^2}. \tag{9a}$$

Furthermore, for all the positive-parity baryons we set $E_0^+ = 0$, while in order to fit the energies of the negative-parity states we set $E_0^- = +0.322\,\text{GeV}$.

Some words of comments are required about this point: in the truncated harmonic-oscillator base that has been adopted, it is not possible to fit, with the same parameters, both the positive- and the negative-parity states. In fact the positive-parity excited states are essentially given by harmonic-oscillator wave-functions with $N = 2$ quanta, while the negative-parity states are constructed with $N = 1$ wave-functions. In order to fit the experimental data, the energy difference between the $N = 2$ and $N = 1$ levels should be significantly reduced. In our opinion this objective could be achieved in a natural way by means of a modification of the space wave functions of the base.

Finally, we notice that in the $N(3/2)^+$ multiplet we have neglected the contribution of the space antisymmetric state in order to obtain a better fit of the resonance. For the reader's convenience the explicit composition of the nucleon and of the Δ(1232) resonance wave-functions are shown hereunder in the self-explaining notation of ref. [3].

$$(10a)\qquad |N(940)\rangle = a_S\left|N^2S_S\frac{1}{2}^+\right\rangle + a_S'\left|N^2S_S'\frac{1}{2}^+\right\rangle + a_M\left|N^2S_M\frac{1}{2}^+\right\rangle + a_D\left|N^4D_M\frac{1}{2}^+\right\rangle + a_A\left|N^2P_A\frac{1}{2}^+\right\rangle$$

and

$$(10b)\qquad |\Delta(1232)\rangle = b_S\left|\Delta^4S_S\frac{3}{2}^+\right\rangle + b_S'\left|\Delta^4S_S'\frac{3}{2}^+\right\rangle + b_D\left|\Delta^4D_S\frac{3}{2}^+\right\rangle + b_D'\left|\Delta^2D_M\frac{3}{2}^+\right\rangle,$$

with the following amplitudes:

$$(11a)\qquad \begin{cases} a_S = 0.986\,, \\ a_S' = -0.715\cdot 10^{-2}\,, \\ a_M = -0.153\,, \\ a_D = -0.645\cdot 10^{-1}\,, \\ a_A = -0.909\cdot 10^{-2} \end{cases}$$

and

$$(11b)\qquad \begin{cases} b_S = -0.982\,, \\ b_S' = -0.131\,, \\ b_D = -0.920\cdot 10^{-1}\,, \\ b_D' = -0.104\,. \end{cases}$$

We also notice that the amplitudes of the negative-parity states are very similar to those given in ref. [8].

4. – The electromagnetic properties of the nucleon.

We now go to study some relevant electromagnetic properties of the nucleon and of the Δ(1232) resonance as they result from our model. Firstly we discuss the static quantities, *i.e.* the charge radii and the magnetic moments, then the magnetic and electric polarizabilities.

The results for the static quantities obtained by using the amplitudes of eq. (11*a*) are shown in the first column of table I. In the second column we report the results that would be obtained by ignoring the mixing produced by H_{hyp} and H_{rad}.

Let us note that the state mixing is essential in order to reproduce the non-vanishing neutron charge radius [9]. Our value for the proton charge radius is in better agreement with the experimental determination in comparison with the commonly quoted result for the quark model ($\langle r_p^2\rangle^{1/2} \simeq 0.6$ fm [4, 11, 12]). However more efforts should be devoted to understand the reason of the remaining discrepancy. This problem could be probably solved by taking into account the smearing of the quark charge due to the coupling with virtual mesons [3, 11].

TABLE I. – *Charge radii and magnetic moments for nucleons and* $\Delta(1232)$ *resonance.*

	Our model	No mixing	Experimental [9, 10, 13]
$\langle r_p^2 \rangle^{1/2}$ (fm)	0.74	0.70	0.81
$\langle r_n^2 \rangle / \langle r_p^2 \rangle$	−0.11	0.00	−0.15
μ_p (n.m.u.)	2.87	3.00	2.793
μ_n (n.m.u.)	−1.89	−2.00	−1.913
$\langle r_\Delta^2 \rangle^{1/2}$ (fm)	1.07	1.00	
μ_Δ (n.m.u.)	5.82	5.86	5.7 ± 1.0

Finally we note that the satisfactory agreement of the magnetic moments is a consequence of the fact that these quantities are essentially determined by the symmetry properties of the wave-function. However some care should be exercized in interpreting these results because they could be substantially modified by the insertion of an anomalous magnetic moment of the quarks [3], exchange and relativistic corrections, etc.

In passing we notice that by means of the amplitudes of eq. (10*b*) some properties of the $\Delta(1232)$ resonance can be calculated. Specifically, for the magnetic moment of the charge state with $T_3 = 3/2$ we obtain the value reported in table I, that is in fair agreement with the experimental determination [13]. We also display the value of the charge radius, for which, up to now, no experimental determination is available.

For the photon scattering polarizabilities—in the center-of-mass reference frame—we use the results of our previous work [6]. In the case of elastic scattering on a spin-1/2 particle the only nonvanishing terms are:

$$P_0(E1, E1) = -\frac{(Ze)^2}{M}\sqrt{6} + \omega_\gamma^2 \sqrt{6}\,[\alpha_E + \alpha_{RC} + \alpha_R]\,, \tag{12}$$

$$P_1(E1, E1) = -\sqrt{\frac{2}{3}}\,\omega_\gamma \frac{Ze}{M}\langle 0\|\mu\|0\rangle + \omega_\gamma \frac{(Ze)^2}{M^2}\,, \tag{13}$$

$$P_0(M1, M1) = -\sqrt{\frac{3}{2}}\,\omega_\gamma \left(\frac{Ze}{M}\right)^2 + \omega_\gamma^2 \sqrt{6}\,[\chi_P + \chi_R + \chi_D]\,, \tag{14}$$

$$P_1(M1, M1) = +\frac{2}{3}\,\omega_\gamma |\langle 0\|\mu\|0\rangle|^2\,, \tag{15}$$

$$P_1(M2, E1) = -\frac{\sqrt{10}}{6}\,\omega_\gamma \frac{Ze}{M}\langle 0\|\mu\|0\rangle\,. \tag{16}$$

For the reader's convenience the most widely used definition of the polarizabilities has been adopted in the equations above [6, 14]. The quantities α_E, α_{RC} and α_R occurring in eq. (12) for the electric polarizability, respectively stand for

$$\alpha_E = \frac{2}{3}\sum_{n\neq 0}\frac{|\langle 0|\boldsymbol{D}|n\rangle|^2}{E_n - E_0} \text{ (dispersive term)}, \tag{17}$$

$$\alpha_{RC} = \frac{2}{3} Ze\left[\sum_{n\neq 0}\frac{\langle 0|\boldsymbol{O}^{RC}|n\rangle\langle n|\boldsymbol{D}|0\rangle}{E_n - E_0} + \sum_{n\neq 0}\frac{\langle 0|\boldsymbol{D}|n\rangle\langle n|\boldsymbol{O}^{RC}|0\rangle}{E_n - E_0}\right] \text{ (relativistic corrections)}, \tag{18}$$

$$\alpha_R = \frac{1}{3}\frac{Ze}{M}\langle 0|r^2|0\rangle \text{ (retardation term)}. \tag{19}$$

We also recall that the relativistic correction operator $\boldsymbol{O}^{RC}$ has the form

$$\boldsymbol{O}^{RC} = -\frac{1}{2M}\sum_{i=1}^{3}(\boldsymbol{r}'_i h_i + h_i \boldsymbol{r}'_i), \tag{20}$$

where M is the nucleon mass, $\boldsymbol{r}'_i$ is the coordinate of the i-th quark relative to the center of mass and h_i is the Hamiltonian of the i-th quark as defined in ref. [6].

In eq. (14) for the magnetic polarizabilities we have defined

$$\chi_P = \frac{2}{3}\sum_{n\neq 0}\frac{|\langle 0|\boldsymbol{\mu}|n\rangle|^2}{E_n - E_0} \text{ (dispersive paramagnetic term)}, \tag{21}$$

$$\chi_R = -\frac{1}{2M}\langle 0|\boldsymbol{D}^2|0\rangle \text{ (retardation term)}, \tag{22}$$

$$\chi_D = -\frac{1}{6m}\langle 0|\sum_{i=1}^{3} e_i^2 \boldsymbol{r}_i'^2|0\rangle \text{ (diamagnetic term)}. \tag{23}$$

In the following we focus our attention on the dynamical contributions given by the terms of eqs. (17)-(19) and (21)-(23).

In the electric polarizabilities α_E and α_{RC} the intermediate states belonging to the following multiplets have been considered: $N(1/2)^-$, $\Delta(1/2)^-$ and $N(3/2)^-$. In the adopted model, these states are represented by $N=1$ quantum harmonic-oscillator wave-functions. In order to give the *whole* strength of the electric dipole transitions, also $N=3$ quanta states should be considered. However, given the preliminary character of the present work, these contributions have been omitted. In fact, by considering eq. (17) for α_E one easily realizes that these contributions must be much *smaller* than those given by the $N=1$ states: firstly because of the presence of higher-energy denominators, secondly because only the $N=2$ small terms in the nucleon wave-function can give a nonvanishing contribution.

In the adopted approximation the explicit calculation of α_E is straightforward. The calculations of the matrix elements of $\boldsymbol{O}^{RC}$ require some words of explanation. By exploiting the full antisymmetry of the states [7,8] the matrix elements of $\boldsymbol{O}^{RC}$ can be put in the form

$$\langle n|\boldsymbol{O}^{RC}|0\rangle = \frac{\sqrt{6}}{2M}\langle n|(\boldsymbol{\lambda} h_3 + h_3\boldsymbol{\lambda})|0\rangle . \tag{24}$$

Furthermore, by writing the explicit expression of h_3 in terms of the total potential V

$$h_3 = \frac{\boldsymbol{p}_3'^2}{2m} + \frac{1}{2}(V - V_{12}) \tag{25}$$

and by recalling that the $V\cdot\boldsymbol{\lambda}$ terms give vanishing matrix elements due to the antisymmetry of the states, one obtains

$$\langle n|\boldsymbol{O}^{\mathrm{RC}}|0\rangle = \frac{\sqrt{6}}{2M}\langle n|\left\{\boldsymbol{\lambda},\left(\frac{\boldsymbol{p}_3'^2}{2m}-\frac{1}{2}V_{12}\right)\right\}_+|0\rangle\,. \tag{26}$$

Also, for the same reason, the splitting constant E_0^-, is uninfluential in the matrix elements evaluation. The remainder of the calculation is tedious but straightforward.

Let us finally note that in our $1/m^2$ nonrelativistic approximation discussed in ref. [6], the contributions of the hyperfine interaction to V_{12} are neglected.

Our results for α_E, α_{RC}, α_R and for the sum of the three terms $\bar{\alpha}=\alpha_E+\alpha_{RC}+\alpha_R$, are shown in table II. Our estimate of α_E looks significantly smaller than current results obtained from simplified quark models with no state mixing [2, 4]. We note that α_E strictly depends on the energies of the negative-parity states and on the proton charge radius which are, up to now, unsolved problems of the constituent quark model.

TABLE II. – *Electric and magnetic polarizabilities of the nucleon. All the entries are in units of* $10^{-4}\,\mathrm{fm}^3$.

	Neutron (our model)	Proton (our model)	Proton (experimental) [2]
α_E	6.0	6.0	
α_{RC}	0.0	− 0.4	
α_R	0.0	2.8	
$\bar{\alpha}$	6.0	8.4	11.6 ± 2.4
χ_P	7.9	7.9	
χ_R	− 3.3	− 3.3	
χ_D	− 2.3	− 3.8	
$\bar{\chi}$	2.3	0.8	3.0 ± 2.4
$\bar{\alpha}+\bar{\chi}$	8.3	9.2	14.2 ± 0.6

In our opinion, the estimate of $\bar{\alpha}$ given by other authors [2, 4] seems to be in almost fortuitous agreement with the experimental determination: in fact the retardation term, that is proportional to $\langle r_p^2\rangle$, is clearly underestimated, while the dispersive term is probably overestimated because, in that model, it has been taken $\omega \simeq 0.315$ GeV that is smaller than the energy difference between the electric-dipole levels and the nucleon.

The relativistic corrections for the proton give a contribution that is 5 per cent of the dispersive plus retardation terms. We think that this ratio should be substantially model independent. We also note that the relativistic corrections tend to *reduce* the whole electric polarizability.

Finally we discuss our estimates of the magnetic polarizabilities also shown in table II. For the dispersive paramagnetic term χ_P, the $N(1/2)^+$ (except the ground state), $\Delta(3/2)^+$, $\Delta(1/2)^+$ and $N(3/2)^+$ multiplets have been considered, but the multiplets different from $\Delta(3/2)^+$ give only 1 per cent of the total χ_P. Substantial and critical cancellations are due to the diamagnetic (χ_D) and retardation (χ_R) terms. As before, in table II $\bar{\chi}$ represents the sum of the three contributions.

In conclusion we recall that the sum of the electric and magnetic polarizabilities is related to the σ_{-2} sum rule by forward dispersion relations

$$\bar{\alpha} + \bar{\chi} = \frac{\sigma_{-2}}{2\pi^2}. \tag{27}$$

The comparison of our estimates with the results of an analysis of Compton scattering experiments [2] is shown in table II.

5. – Conclusions.

In summary, we have analyzed the predictions of the constituent quark model both for the mass spectrum of the baryons and for some selected electromagnetic properties of the nucleon. We have seen that these two aspects are intimately connected. New theoretical efforts should be devoted to develop a model that *simultaneously* fits the baryon spectrum and the related electromagnetic properties: a base modification seems to be necessary, the role of the mesonic degrees of freedom should be carefully investigated.

Also, in the present work, the relativistic corrections have been considered, obtaining a small but nonnegligible contribution to the electric polarizabilities.

Finally, a better determination of the nucleon polarizabilities, in particular of the magnetic one, should also help to understand the influence of internal nucleon structure on photon scattering by complex nuclei [4].

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