Black Holes and Elementary Particles.

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The aim of the present report is to suggest a plausible conceptual connection between black holes and the world of strong interaction along defining an internal quantum thermodynamics of elementary (including hadrons) particles. DE BROGLIE postulated [1] that the energy of an elementary particle is proportional to a temperature associated to that particle through the equation $h\nu = mc^2 = kT$, where ν is the frequency and T the temperature (h and K stand, respectively, for the Planck and the Boltzmann constants). Besides other theoretical difficulties [2], this equation can be shown to be unphysical as it predicts internal temperatures extremely high for ordinary particles and, consequently, for vacuum. It is for this reason that we cannot be able to consider de Broglie's equation in order to establish a link between the thermodynamics of black holes and elementary particles. For, we proceed as follows.

In quantum mechanics the linear momentum of a particle is given by the familiar equation $\hat{p}\Psi = p\Psi$, where $\hat{p}$ is the linear-momentum operator. Let Ψ be the quasi-classical wave function $\Psi = \mathrm{R} \exp[iS/\hbar]$, R and S being real. One finds

$$\hat{p}\Psi = \nabla S \Psi - i\hbar \nabla R \exp[iS/\hbar] ,$$

where ∇S is the classical linear momentum of the given particle. Consider an elementary free particle, classically at rest ($\nabla S = 0$). It will possess a *quantum* linear momentum and, thereby, a *quantum* velocity given by

$$v_{\mathrm{q}} = -i\hbar \nabla R / mR .$$

Associated to v_{q} is a *quantum* current

$$j_{\mathrm{q}} = -i\hbar \nabla P / 2m , \qquad (1)$$

(1) L. DE BROGLIE: *La thermodynamique de la particule isolée* (Paris, 1964), p. 93.
(2) Note, for example, that the temperature of the subquantum medium (D. BOHM and J. P. VIRGIER: *Phys. Rev.*, **96**, 208 (1956)), associated by DE BROGLIE (1) to vacuum, would depend on the frequency of the particles, so that in spite of that, it seems natural to attribute a unique characteristic temperature to the subquantum medium.

where $P \equiv P(x) = R(x)^2$ is the probability density for the particle to be found at a given point x of space. The hypothesis is now that the existence of the *quantum* current (1) is associated to an intrinsic diffusion coefficient d of the particle which should be defined in terms of the particle current induced by the *quantum* probability density gradient:

$$j = -id\nabla P . \tag{2}$$

From (1) and (2), we get

$$dm = \hbar/2 . \tag{3}$$

Note that the Lorentz invariance required by $\hbar$ implies $d = d_0\gamma$, where $\gamma = (1 - v^2/c^2)^{\frac{1}{2}}$ and d_0 is the rest diffusion coefficient for the given particle, and that, if d is responsible for the quantum indetermination of the particle position, eq. (3) ensures that indetermination is inversely related to the mass of the particle.

Let us now use the Einstein celebrated relation $d = kT/\varrho$ (ϱ and T being here an intrinsic friction coefficient and an internal temperature of the particle, respectively). It follows that

$$kTm = \varrho\hbar/2 . \tag{4}$$

Because of the indefiniteness in the value of ϱ, the temperature may be, in principle, so small as one wants. We must try the two following possibilities: i) T is Lorentz invariant and ϱ transforms: $\varrho = \varrho_0/\gamma$, and ii) ϱ is Lorentz invariant, while T transforms: $T = T_0\gamma$. Although relativistic invariance for the temperature is claimed by some authors [3-5] (mainly under the argument that, since time is not a variable in reversible thermodynamics, the classical concept of temperature remains unaffected by the change of the time concept introduced by relativity), most writers are in favour of a relativistic variation of temperature [6]. Therefore, we tentatively assume that [7] $T = T_0\gamma$ and ϱ is a Lorentz invariant.

Consider now a Schwarzschild black hole of mass M. According to the Hawking calculation [8] of the quantum particle creation occurring during the gravitational collapse of a body to form a black hole, this emits thermal radiation at a temperature given by

$$kTM = (c^3/8\pi G)\hbar/2 , \tag{5}$$

where G is the gravitational Newton constant. It is quite gratifying indeed to note the deep analogy holding between expressions (4) and (5); actually, they become formally identical if we put

$$\varrho = c^3/8\pi G ,$$

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[3] L. D. LANDSBERG: *Nature (London)*, **212**, 571 (1966).
[4] N. G. VAN KAMPEN: *Phys. Rev.*, **173**, 295 (1968).
[5] G. CAVALLERI and G. SALGARELLI: *Nuovo Cimento A*, **62**, 792 (1969).
[6] See, for example, R. G. NEWBURGH: *Nuovo Cimento B*, **52**, 219 (1979).
[7] There are actually two different approaches to deal with the Lorentz transformation of temperature. M. PLANCK (*Ann. Phys. (Leipzig)*, **26**, 1 (1908)) considered $T = T_0\gamma$, while H. OTT (*Z. Phys.*, **175**, 70 (1963)) took $T = T_0/\gamma$. We use here Planck transformation because $T = T_0/\gamma$ would imply that ϱ varies as $\varrho = \varrho_0/\gamma^2$, which is in contradiction with an argument given later in the text.
[8] S. W. HAWKING: *Commun. Math. Phys.*, **43**, 199 (1975).

which is consistent with the assumption of taking ϱ as a Lorentz invariant. Moreover, since the value of ϱ derived from this equality would yield temperatures extremely high for elementary particles, I suggest that the Hawking's temperature of the black hole (eq. (5)) is actually the limiting temperature for gravitational interaction of the more general temperature given by eq. (4), which is defined for any particle. We could consider, therefore, black holes as the elementary particles *associated* to gravitational interaction, much as we usually consider hadrons to be the elementary particles associated to strong interaction. In this way, the difference between a black hole and a hadron should be that the binding force responsible for the particle collapse is the gravitational force for the former, while for the latter it should be the strong one. Leptons and even quarks should be then considered as objects collapsed by forces considerably more intense than the strong ones. Accordingly, the general expression for ϱ, in the case of a spherical uncharged particle, should be

$$\varrho = c^3/8\pi\tau\,, \tag{6}$$

where τ would stand for the universal constant (expressed in dimension $L^3 M^{-1} T^{-2}$) characterizing the interaction responsible for the particle collapse. If $\tau = G$, we will deal with black holes, while if τ equalized the universal constant (in vacuum) of strong interaction, N, it would give rise to hadrons. N is here defined from the dimensionless quantity $Ng^2/\hbar c$, where g is a *strong charge* having the dimension of mass ([9-12]).

For a generic electrically charged nonspherical elementary particle (which includes black holes), one would have

$$\varrho = c^3/4\pi\tau\chi\,, \tag{7}$$

where χ is given by ([13])

$$\chi = [2 - e^2/m^2\tau + 2(1 - e^2/m^2\tau - c^2J^2/m^4\tau^2)^{\frac{1}{2}}]^{\frac{1}{2}}\,, \tag{8}$$

e and J standing, respectively, for the electric charge and the intrinsic angular momentum (including the spin) of the given particle (black hole, hadron or lepton). Accordingly, there must be a self-consistent theoretical model in which gravitational, strong and the suggested superstrong interactions were actually unified. Although restricted to gravitational and strong forces, there are indeed some previous arguments that seem in favour of such a model. We know that the forces derived from these two types of interactions are always attractive and that, since their quanta are themselves field sources, both are given by nonlinear equations, so that, in terms of gauge theories, we would eventually make recourse to non-Abelian gauge theories like quantum chromodynamics ([14]). In particular, CALDIROLA *et al.* ([15]) have already suggested a unified theory of strong and gravitational interactions under postulating a hierarchy of interaction universes.

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(9) E. RECAMI and P. CASTORINA: *Lett. Nuovo Cimento*, **15**, 347 (1976).
(10) R. MIGNANI: *Lett. Nuovo Cimento*, **16**, 6 (1976).
(11) I. S. HUGHES: *Elementary Particles* (Harmondwarth, 1972).
(12) P. CALDIROLA, M. PAVŠIČ and E. RECAMI: *Phys. Lett. A*, **66**, 9 (1978).
(13) L. SMARR: *Phys. Rev. Lett.*, **30**, 71 (1973).
(14) An excellent survey on QCD is given by W. MARCIANO and D. GROSS: *Phys. Rep. C*, **36**, 137 (1978).
(15) P. CALDIROLA, M. PAVŠIČ and E. RECAMI: *Nuovo Cimento B*, **48**, 205 (1978).

In terms of the hypothesis advanced here, the main difference between a black hole and a hadron is that, whereas the black hole is actually black (*i.e.* it is defined by a gravitational event horizon ([16]) which allows nothing to scape away from inside that black hole; therefore, in order to preserve the thermal equilibrium of the black hole with the surrounding heat bath, particle creation near the black hole ([17]) is required), the *strong event horizon* of hadron would allow the radiation of any particle which were not affected by strong interaction. There are of course many particles which do not feel strong interaction, so that thermal equilibrium does not require any particle creation outside the hadron. The suggested superstrong interaction would also define a *superstrong event horizon* which should be permeable to, at least, electromagnetic radiation, thus allowing the leptons to be *white*.

It is worth noting that the hadron event horizon also precludes particles feeling strong interaction to enter inside the hadron. The reason for that is not clear at all though it is no doubt connected with the very statistical properties of quarks in relation with their confinement.

([16]) P. C. W. DAVIES: *Rep. Prog. Phys.*, **41**, 1314 (1978).
([17]) YA. B. ZEL'DOVICH: *JETP Lett.*, **14**, 180 (1971).