Measurement at 4.2 K of the Brownian Noise in a 20 kg Gravitational Wave Antenna and Upper Limit for Gravitational Radiation at 8580 Hz.

E. AMALDI and C. COSMELLI
Istituto di Fisica dell'Università - Roma

F. BORDONI, P. BONIFAZI, U. GIOVANARDI and G. VANNARONI
Laboratorio Plasma Spazio del C.N.R. - Frascati

G. V. PALLOTTINO and G. PIZZELLA
Istituto di Fisica dell'Università - Roma
Laboratorio Plasma Spazio del C.N.R. - Frascati

I. MODENA
Laboratorio Elettronica dello Stato Solido del C.N.R. - Roma

(ricevuto il 22 Dicembre 1976)

The University of Rome in collaboration with the Consiglio Nazionale delle Ricerche (C.N.R.) has started a program aiming to the detection of gravitational waves (1-4). The final goal is the construction of a 5 ton aluminium antenna cooled to less than 10^{-2} K and operated in coincidence with similar antennae located at Louisiana State University and Stanford University.

In Rome we have planned to reach the final goal by successive steps, consisting in the construction and operation of two low-temperature antennae of smaller mass: an intermediate 400 kg antenna operated at 10^{-2} K and a small test antenna of 20 kg also operated at low temperature.

The test antenna has been already extensively used for designing the magnetic suspension (5) and for measuring the Q-value of the bar at low temperature (6).

(1) E. AMALDI and G. PIZZELLA: *The gravitational-wave experiment in Rome: progress report*, Nota Interna, No. 654 (Novembre 1975), Istituto di Fisica dell'Università, Roma.

(2) P. CARELLI, M. CERDONIO, U. GIOVANARDI, G. LUCANO and I. MODENA: *Low-temperature gravitational-radiation antenna: a progress report*, in *Ondes et radiation gravitationelle* (Paris, 1973).

(3) G. PIZZELLA: *Riv. Nuovo Cimento*, **5**, 369 (1975).

(4) G. PIZZELLA: *Estimated sensitivity of the low-temperature gravitational-wave antenna in Rome*, in *Proceedings of the International School ob Gravitational Waves, Erice, March 1975.*

(5) P. CARELLI, A. FOCO, U. GIOVANARDI and I. MODENA: *Magnetic levitation of a gravitational antenna at low temperature*, in *Cryogenics* (February 1976), p. 77.

(6) P. CARELLI, A. FOCO, U. GIOVANARDI, I. MODENA, D. BRAMANTI and G. PIZZELLA: *A measurement down to liquid helium temperatures for a gravitational-wave aluminium bar antenna*, in *Cryogenics* (July 1975), p. 406.

In this paper we report on measurements made with the 20 kg antenna at 4.2 K for testing the behaviour of the piezoelectric ceramics at low temperature, their mounting on the bar, as well as the electronic chain and the data analysis algorithm. We have found that the overall system operates very satisfactorily and that the data recorded, although for a small number of hours, allow us to give an upper limit for the gravitational radiation at the not-yet-explored frequency $\nu = 8580$ Hz.

A detailed description of the small antenna is given elsewere (7). Here we shall limit ourselves to point out the main features of the experimental set-up and the most interesting results of the measurements performed so far.

The antenna is an aluminium cylinder 30 cm long, 20 kg heavy, suspended with a wire through its centre-of-mass cross-section and cooled at 4.2 K. The dewar is acoustically isolated from the laboratory by means of metallic springs and layers of wood and rubber.

The vibrations are detected by means of a piezoelectric ceramic, Gulton G-1408-Lead Zirconate Titanate, which is located in a slot cut into the bar at the centre-of-mass cross-section. The slot has a width which, at room temperature, is 0.06 mm wider than that of the ceramic. By cooling to 4.2 K, the aluminium contracts more than the ceramic, the gap closes and a perfect coupling is obtained.

We have measured the following parameters of the equivalent electronic circuit at 4.2 K:

$$C_1 = 0.492\,\text{pF}\,,$$

$$C_2 = 670\,\text{pF}\,,$$

$$\text{tg}\,\delta = 0.0084\,,$$

$$\nu_0 = 8576.23\,\text{Hz}\,,$$

$$Q = 44\,000\,.$$

The voltage signal given by the piezoelectric ceramic is amplified by the amplification factor

$$A = 3.89\cdot 10^7$$

and sent to two-pha... sensitive detectors (PSD), in quadrature, with integration time constant

$$T_0 = 10\,\text{ms}$$

corresponding to a frequency band width

$$\Delta\nu = 50\,\text{Hz}\,.$$

The two PSD's provide, as customary, the quantities $x(t)$ and $y(t)$. The amplifier has a voltage noise

$$V_N = 0.7\,\frac{\text{nV}}{\sqrt{\text{Hz}}}\,,$$

(7) E. AMALDI *et al.*: *The gravitational-wave experiment in Rome*: *Progress report, 15 September 1976*, Nota Interna, No. 672 (Ottobre 1975), Istituto di Fisica dell'Università, Roma, presented at the *Experimental Gravitation, Pavia, September 17-20, 1976.*

and a current noise

$$I_N = 23.9 \frac{\text{fA}}{\sqrt{\text{Hz}}}$$

and an input capacity of 30 pF.

The quantities $x(t)$ and $y(t)$ are sampled at time intervals of 10 ms and recorded on magnetic tape in 8-bit words. From these two we can construct the stochastic variable

$$r^2(t) = x^2(t) + y^2(t)\,,$$

which, in the absence of nonthermal signals, has the distribution

$$F(r^2) = \frac{1}{2\sigma_0^2} \exp\left[-\frac{r^2}{2\sigma_0^2}\right],$$

where the parameter σ_0^2 is the sum of three quantities

$$\sigma_0^2 = V_B^2 + V_{RN}^2 + V_e^2\,,$$

which have the following meaning:

a)
$$V_B^2 = A^2 KT \frac{C_1}{C_2^2}$$

is due to the brownian noise of the antenna;

b)
$$V_{RN}^2 = A^2 \frac{I_N^2}{\omega_0^2 C_2^2}\left(\frac{C_1}{C_2} Q\right)^2 \frac{\pi}{2} \frac{\nu_0}{Q} = A^2 \frac{I_N^2 C_1^2 Q}{4\omega_0 C_2^4}$$

is the *resonant noise* due to the contribution of the current noise of the amplifier to the fundamental normal mode of the antenna. This noise cannot be distinguished from the Brownian noise V_B^2 since it has the same spectral behaviour. The only way to decrease its contribution is to have I_N as small as possible;

c)
$$V_e^2 = A^2\left(V_N^2 + \frac{I_N^2}{\omega_0^2 C_2^2}\right)\Delta\nu$$

is the wide-band noise due to the voltage and current noise of the amplifier integrated over the frequency band $\Delta\nu$.

The experimental frequency distribution for measurements taken during the period 19.40 h to 23.54 h of the 22nd July 1976 is shown in fig. 1. From these data we derive the experimental value

$$\sigma_0^2(\text{exp}) = 0.371\ \text{V}^2 \tag{1}$$

which must be compared to that computed from the measured parameters of the equivalent circuit and from the voltage and current noise of the amplifier.

We obtain:

(2)
$$\begin{cases} V_B^2 = 0.096\ \mathrm{V}^2\,, \\ V_{RN}^2 = 0.212\ \mathrm{V}^2\,, \\ V_e^2 = 0.070\ \mathrm{V}^2\,, \end{cases}$$

which add to give

(3)
$$\sigma_0^2(\text{theor}) = 0.378\ \mathrm{V}^2\,,$$

which is in very good agreement with the experimental value.

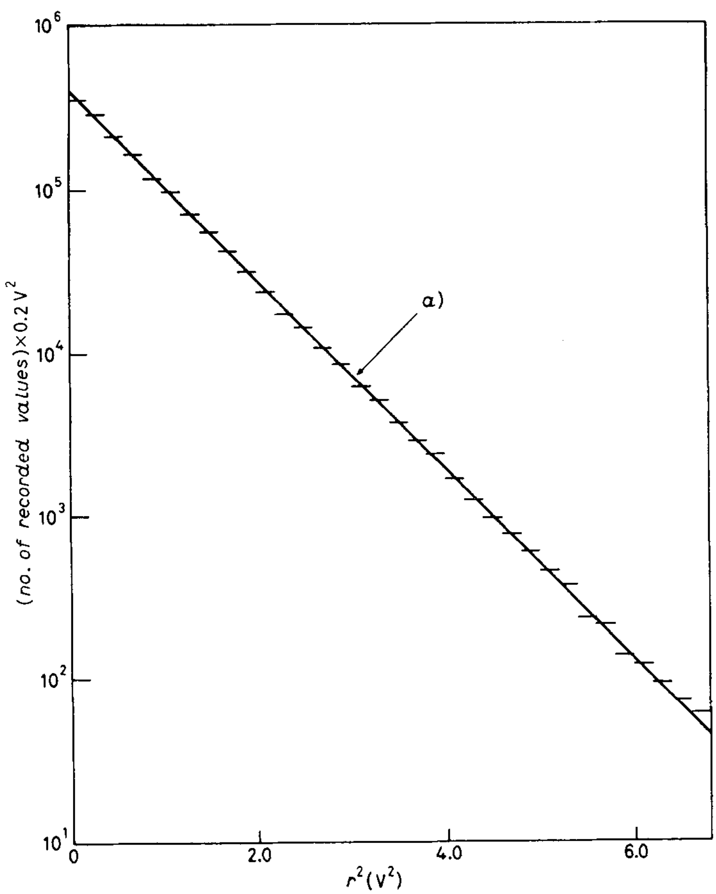

Fig. 1. – Hystogram of the observed frequency distribution of the variable $r^2 = x^2 + y^2$. $\sigma_0^2 = 0.371\ \mathrm{V}^2$. *a*) $\exp[-r^2/(2\sigma_0^2)]$.

We conclude that our small criogenic antenna is indeed observing the effective Brownian noise at 4.2 K. Furthermore we note from fig. 1 that no extra pulses are present, at least within the statistical fluctuations, which means that the mechanical and electrical filters we have used are sufficient to allow us to use this test antenna also for the search of gravitational waves, although its sensitivity is rather poor because of its small mass.

The value (2) of V_{RN}^2 is large since its effect is equivalent to an increase of the bar temperature of about 7 K. Such a large value derives from the use in our preamplifier of the FET use most groups working in this field (*i.e.* FET c 413 N).

It is possible, however, to use the FET BF 817 which has a smaller current noise, although V_N is slightly larger. Therefore we can rely for the near future on the possibility to construct an amplifier which will produce a resonant noise corresponding to an increase in temperature not larger than 1 K.

Only if a new FET with lower current noise will be realized or if a different kind of transducer (*i.e.* a SQUID transducer) is used it might be convenient to go to lower bar temperatures.

Even in the present conditions it is possible to improve the antenna sensibility by using, as customary, an optimum filter for the signal. The simplest filter consists in taking the variable [7,8]

$$\varrho^2(t, \Delta t) = [x(t+\Delta t) - x(t)R_{xx}(\Delta t)]^2 + [y(t+\Delta t) - y(t)R_{yy}(\Delta t)]^2 ,$$

where $R_{xx}(\Delta t)$ and $R_{yy}(\Delta t)$ are the normalized autocorrelation functions for the variables $x(t)$ and $y(t)$ computed at the time difference Δt. Δt is chosen in such a way to minimize

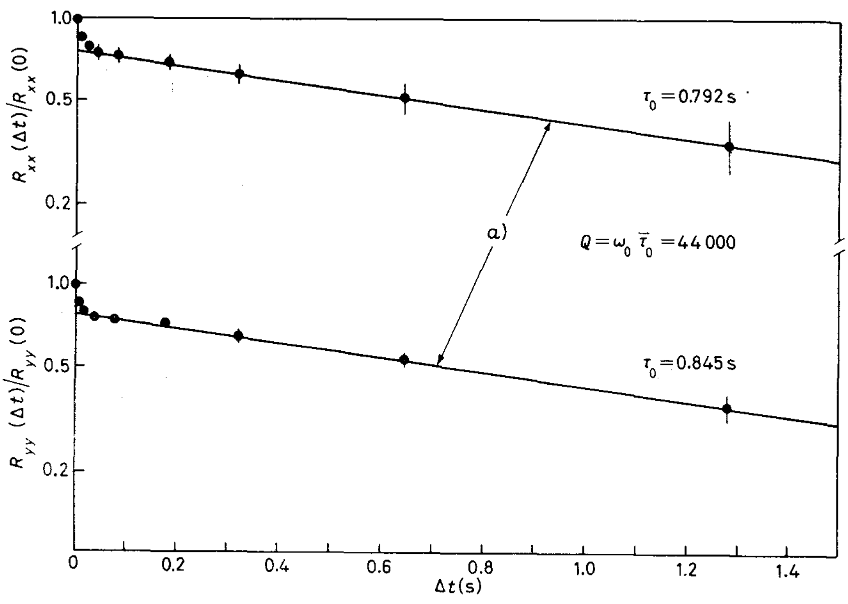

Fig. 2. – Normalized autocorrelation functions.

the standard deviation of the stochastic quantities

$$X = x(t+\Delta t) - x(t)R_{xx}(\Delta t) ,$$

$$Y = y(t+\Delta t) - y(t)R_{yy}(\Delta t) .$$

(8) J. L. LEVINE and R. L. GARWIN: *Phys. Rev. Lett.*, **31**, 173 (1973).

The frequency distribution of ϱ^2 turns out to be

$$F(\varrho^2, \Delta t) = \frac{1}{2\sigma^2}\exp[-\varrho^2/2\sigma^2]$$

with $\sigma^2(\Delta t)$ given approximatively (9) by

$$\sigma^2(\Delta t) \simeq (V_B^2 + V_{RN}^2)\frac{\Delta t}{\tau_0} + \frac{V_0^2}{\Delta t}.$$

The quantity τ_0 is the relaxation time of the antenna

$$\tau_0 = \frac{Q}{2\pi\nu_0}.$$

It can be estimated by measuring the damping time, or the resonance frequency band width or the autocorrelation function.

The measured autocorrelations $R_{xx}(\Delta t) = R_{yy}(\Delta t)$ are shown in fig. 2. It can be shown that

$$R_{xx}(\Delta t) = R_{yy}(\Delta t) = (V_B^2 + V_{RN}^2)\exp\left[-\frac{\Delta t}{2\tau_0}\right] + V_e^2\exp\left[-\frac{\Delta t}{T_0}\right].$$

In the figure the fast decrease for small values of Δt is due to the second term, *i.e.* to the electronic noise. From the behaviour at $\Delta t \gg T_0$ one can determine τ_0. Furthermore one can determine the ratio

$$\frac{V_B^2 + V_{RN}^2}{V_B^2 + V_{RN}^2 + V_e^2}.$$

From fig. 2 we deduce for this ratio the value 0.77, while from our previously computed values of V_B^2, V_{RN}^2 and V_e^2 we find the theoretical value 0.81; the agreement is satisfactory.

The quantity $\sigma^2(\Delta t)$ is minimum for

$$\Delta t \simeq V_0\sqrt{\frac{\tau_0}{V_B^2 + V_{RN}^2}}$$

which, in our case, amounts to $\Delta t = 0.12$ s.

In fig. 3 we show the frequency distribution of the variable ϱ^2. We note a good exponential decay corresponding to a temperature of 2.1 K. All data points except perhaps the last three appear to be within the statistical fluctuations.

We consider now worthwhile to attempt to give an upper limit to the flux of gravitational waves at 8580 Hz, in spite of the low sensitivity due to the small mass, because this frequency has not yet been explored.

---

(9) G. V. PALLOTTINO and G. PIZZELLA: *On the autocorrelation and prediction of averaged sequences with application to gravitational data analysis*, LPS 76-28 (October 1976).

This was done by computing, for each value of ϱ^2, the quantity $\sqrt{N(\varrho^2)}$, which is the statistical deviation for the number of gravitational-wave pulses producing, in the antenna, a displacement measured by ϱ^2. For pulses with energy flux per unit area I

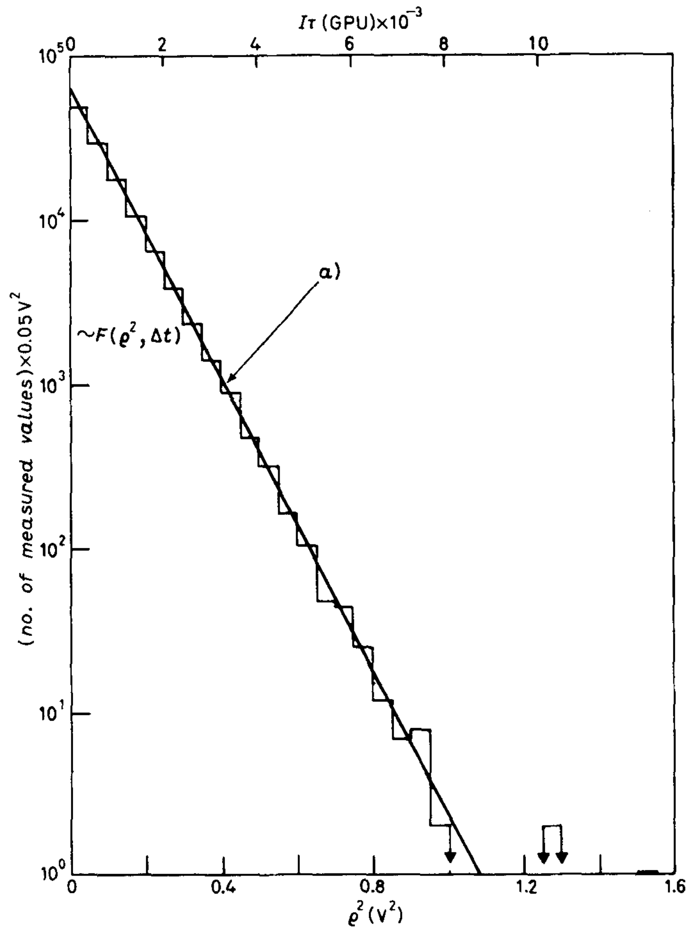

Fig. 3. – Hystogram of the observed frequency distribution of the variable $\varrho^2(t) = X^2(t) + Y^2(t)$. *a*) $\exp[-\varrho^2/(2\sigma^2(\Delta t))]$, for $\Delta t = 0.12$ s, $\sigma^2(\Delta t) = 0.050\ \text{V}^2$, $\rightarrow$ 2.1 K.

and duration τ much smaller than τ_0 we have the following relationship between $I\tau$ and ϱ^2 (7):

$$I\tau = \frac{C^3}{32\pi G}\,\frac{\pi^4\omega_0^2}{A^2 v^2 \alpha^2}\,\varrho^2$$

with the following meaning for the symbols: G = gravitational constant, C = velocity of light, v = velocity of sound in the aluminium bar, $\omega_0 = 2\pi\nu_0$ and $\alpha = (\omega_0/c_2)\sqrt{\frac{1}{2}mC_1}$ is the transducer coupling constant which relates the displacement in the bar to the ceramic output voltage.

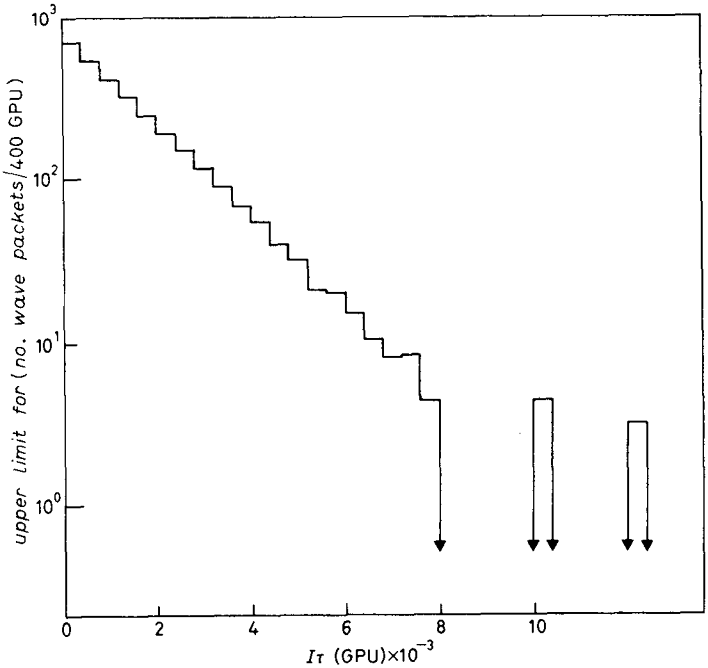

Fig. 4. – Upper limit for the number of possible gravitational pulses observed at $\nu = 8580$ Hz in 4 h 14 min. The two last points correspond to 2 and 1 recorded pulses. *a*) $\exp[-\Delta t/2\tau_c]$.

The quantity $3\sqrt{N(\varrho^2)}$ *vs.* $I\tau$ is plotted in fig. 4, as un upper limit with confidence greater than 99% for the number of gravitational-wave pulses. The unit for $I\tau$ is 1 G.P.U. $= 10^2$ J/(m² Hz).

* * *

We thank Prof. R. GIFFARD for pointing out to us the importance of the noise V^2_{RN}. This work has been made with the contribution of the Consiglio Nazionale delle Ricerche, Italy.

Direttore responsabile: CARLO CASTAGNOLI

Stampato in Bologna dalla Tipografia Compositori coi tipi della Tipografia Monograf
Questo fascicolo è stato licenziato dai torchi il 21-III-1977