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Is Loss-Free Counting Under Statistical Control?

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ABSTRACT

A new formula for the statistical uncertainty of "loss-free counting" (LFC) is presented. Its validity is demonstrated by comparing with experimental data obtained with a HPGe γ-ray spectrometer. Also, computer simulation data of nuclear counting with different types of count loss (pileup rejection, extending and nonextending dead time) are in agreement with the predicted counting uncertainty. The proposed formula for LFC uncertainty is applicable to spectrometers with a classical semi-Gaussian pulse-shaping amplifier as well as with a gated-integrator amplifier. Hence, achieving statistical control seems to be a feasible goal.

Index Entries: Nuclear counting; loss-free counting; statistics; pulse pileup; dead time; γ-ray spectrometry.

INTRODUCTION

Traceability is considered to be a major advantage of neutron activation analysis (NAA) as an analytical tool. It is often claimed that all sources of uncertainty are identifiable and under statistical control. Yet, this statement may be compromised with the introduction of an increasingly popular count-loss correction method in the NAA lab; the so-called "loss-free counting" technique *(1)* using the "virtual pulse generator" (VPG) method *(2–4)*. Loss-free counting (LFC) has already proven to be a powerful tool for correcting rate-related losses in gamma-spectrometry *(5)*. By performing add-*n* operations to the spectrum instead of add-1, LFC is able to restore the linearity of the spectrometer throughput. The main trade-off is an increase of the count scatter, which is roughly proportional to the average throughput correction factor. Heydorn and Damsgaard *(6)* were probably the first to recognize that the commonly adopted Westphal formula for the statistical accuracy of loss-free counting *(2,3)* does not always account for the count scatter in LFC spectra.

Recent work *(7–9)* shows that existing models for nuclear counting with rate-related loss (see, e.g., ref. *10*) do not properly account for the influence of pileup rejection on the statistical counting uncertainty. From a study of the interval-time density distribution of subsequently counted events, a simple expression was derived to account for the standard deviation of counting with pileup rejection *(11)*. Other research work *(5,12)* revealed that the main problem with the statistical control of loss-free counting originates from pileup rejection. Whereas other types of count loss, like extending and nonextending dead time, seem to comply with the Westphal prescription, the presence of pileup usually leads to a higher count scatter than expected. A new formula for loss-free counting uncertainty was presented by Pommé et al. *(5)*. In the present work, the performance of (a slightly modified version of) this formula is demonstrated for counters with different kinds of count loss, such as pileup rejection or dead time. The main goal of this work was to achieve a fundamentally correct treatment of counting statistics for spectrometers with pileup rejection, regardless of the type of pulse shape amplification (i.e., for gated-integrator amplifiers as well as for semi-Gaussian or triangular pulse shaping).

METHOD

The validity check of the presented LFC uncertainty formula is based on two tools: repeated measurements at a HPGe γ-ray spectrometer and computer simulations of such spectrometer with adjustable pulse shape and dead-time parameters.

The computer simulation program consists of a random generator of "Poisson distributed events" occurring in an imaginary detector. For each event, the program evaluates whether the electronics of the detector would accept the signal based on the time spacing between neighboring events and the time widths associated with the assumed count-loss generators. Also, the operation of a connected loss-free counting device is faithfully simulated.

Experimental proof is obtained from a series of γ-ray measurements on a 40% HPGe detector ("Panoramix"). To determine the $\sigma(N_{LFC})$ at particular count rates, typically 2000 spectra of 0.2 s "real time" were taken. The setup is connected to a loss-free counting device with a 5-MHz virtual pulse generator (CI-599). The only significant loss mechanism is pulse pileup in the shaping amplifier (timing constant set at 4 μs), because the ultrafast 800-ns fixed dead-time analog-to-digital convertor (CI-8715) adds no dead time to the system.

The mechanism of pileup rejection is therefore of particular interest to this work. Conventional γ-ray spectrometers have a limited time resolution, mainly because of the long pulse shaping time applied in the main amplifier. Hence, two pulses closely spaced in time risk being interpreted as one compound pulse. Spectrometers with pileup rejection use time information from a fast signal channel to decide whether or not to process the pulses in the slow amplitude-defining channel. The necessary action depends on the position of the second pulse with respect to the first: (1) If it falls within the leading edge (or pulse evolution time [PET]) of the first, the pileup rejection system excludes both events from the spectrum; (2) if it falls within the trailing edge, the first pulse is processed and the following one(s) discarded, exactly as in the case of extending dead time.

The applied setup and procedures are explained in more detail elsewhere *(5,9,11)*.

RESULTS

A New Formula for LFC Statistics

The time clustering of real and artificial counts in LFC spectra thoroughly destroys the Poisson nature of the observed event train. Hence, loss-free counting requires a specific statistical treatment. Pommé et al. *(5)* presented the following equation to calculate the standard deviation of the number of counts in a region of interest of a pulse-loss-corrected spectrum:

$$\left(\frac{\sigma(N_{\mathrm{LFC}})}{N_{\mathrm{LFC}}}\right) = \frac{r}{\sqrt{N}}\left[1 + \left(\frac{\sigma(n)}{< n >}\right)^2\right]^{\frac{1}{2}} \tag{1}$$

in which n is the count-loss correction factor, calculated per virtual pulse generator inspection period, and r = is the width of the count probability distribution per LFC inspection period (IP), relative to a Poisson distribution. The Westphal formula (2,3) corresponds to the particular case that Poisson statistics apply during fixed live-time periods (i.e., $r = 1$). The LFC method treats pileup rejection in a special way: To compensate the extra loss of signals by leading edge pileup, the "system busy" (and hence the VPG inspection period) is extended by one PET—the time necessary for one pulse to be detected correctly, without interference of another. This unavoidably broadens the distribution of valid counts in each VPG inspection period. In ref. 5, the following equation for r was suggested:

$$r \approx 1 + \frac{A}{2}\left(\frac{2 + A}{1 + A}\right)f \tag{2}$$

in which $A = (1 - e^{-\rho\mathrm{PET}})$, ρ is the (average) incoming count rate (s^{-1}), PET is the pulse evolution time (s) (also, leading edge of the pulse or linear gate time), and f is the considered fraction of the spectrum (f = 100% is the full spectrum). Note that r equates to 1 in the absence of leading-edge pileup (PET = 0) and the LFC uncertainty formula [Eq. (1)] reduces to the Westphal formula.

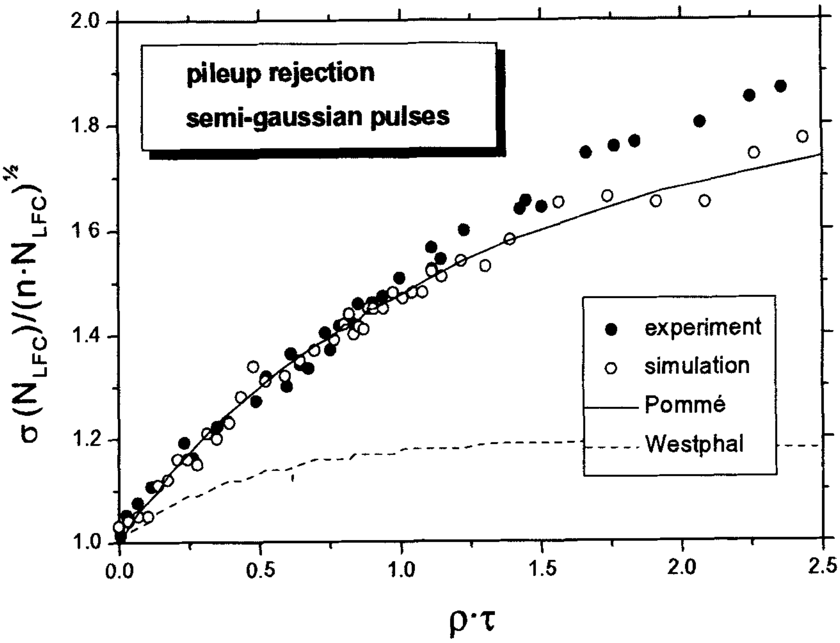

Fig. 1. Standard deviation of the count integral of full LFC spectra on γ-ray detector Panoramix, compared to theoretical expectations and computer simulation data. The "effective dead time" τ = PET + T_w was assumed to be 12.1 + 22.7 μs (see refs. *5* and *11*). The theoretical values were calculated with $\sigma(n)$ values derived from the simulations.

Counters with Pileup Rejection

As an example, the default setup of the Panoramix γ-ray spectrometer was tested by experiment and by computer simulation. The standard deviation on the count integral of the LFC spectra is presented as a function of incoming count rate in Fig. 1. One can see an excellent agreement between theory and experimental data up to at least $\rho = 1/\tau$, corresponding to 63% count loss. Slight deviations at higher count rates are attributed to the finite pulse pair resolution of the pileup rejection system, allowing an increasing number of compound pulses to slip through. The simulation data agree with Eq. (1) up to extremely high count rates, even close to 100% count loss.

The formula also works for a spectrometer equipped with a gated-integrator amplifier, which produces pulses in the shape of a cumulative Gaussian. The trailing edge is extremely short, and as a result, the pulse evolution corresponds to the duration of almost the entire pulse (i.e., PET ≈ T_w and $\tau \approx$ 2 PET. A typical example, obtained by simulation, of the corresponding loss-free counting statistics is presented in Fig. 2. Statistical control is clearly achieved, as it is the case for any ratio between the pulse evolution time and the total pulse width, PET/T_w. More importantly, the counting statistics of some part of the event spectrum can be determined accurately. The LFC uncertainty formula varies

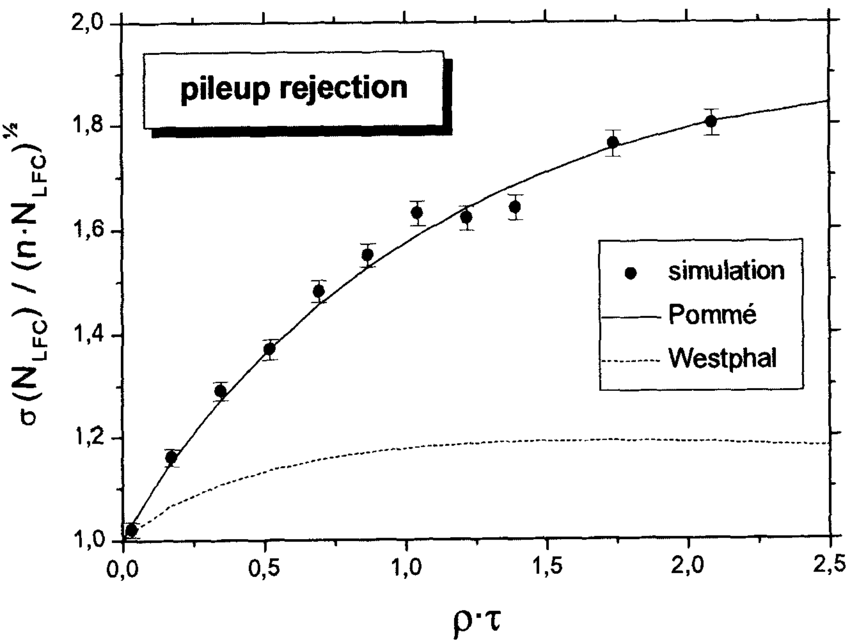

Fig. 2. Standard deviation of the count integral of full LFC spectra for a detector with gated-integrator amplifier, for which the pulse evolution time is equal to the total pulse width ($\tau/2$ = PET = T_w = 17.4 μs).

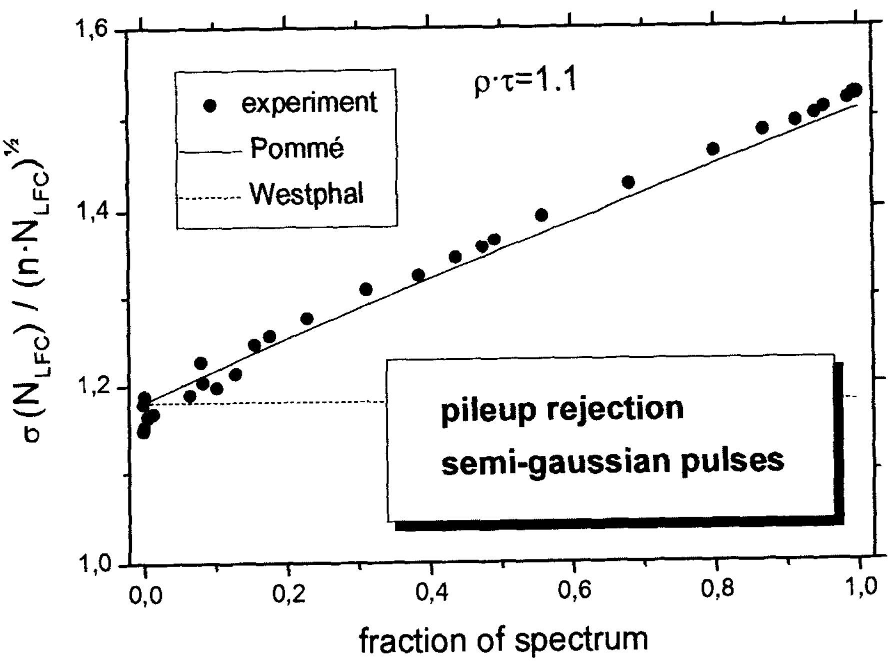

Fig. 3. Standard deviation of the count number in a region of interest of LFC spectra on γ-ray detector Panoramix (ρ = 32 kBq), compared to theoretical expectations [Eqs. (1) and (3)].

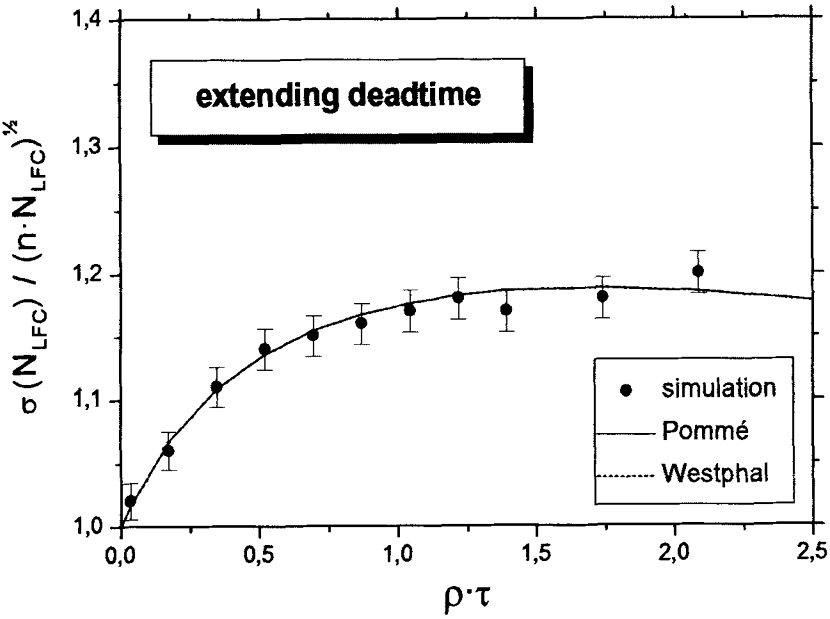

Fig. 4. Standard deviation of the count integral of LFC spectra on a counter with extending dead time (τ = 34.8 μs) as a function of incoming count rate.

according to the fraction of the spectrum represented by the selected region of interest. An example is shown in Fig. 3. The variation of $\sigma(N_{LFC})$ with f can be well approximated by a linear interpolation between the Pommé formula [Eq. (1), with f = 100%] and the Westphal formula ($r = 1$), as was already suggested by introducing Eq. (2). However, as in reality the curve is slightly bent—certainly at very high incoming count rates—Eq. (2) can be replaced by the following improved representation of r:

$$r \approx 1 + \frac{A}{2}\left(\frac{2 + A'}{1 + A'}\right)f \tag{3}$$

in which $A' = (1 - e^{-f\rho \mathrm{PET}})$ and A, f, PET, and ρ are as defined for Eq. (2).

As expected *(5,9)*, the Westphal formula is an excellent approximation when considering small fractions of the spectrum (e.g., one γ-ray peak in a complex activation spectrum).

Counters with Extending and Nonextending Dead Time

In the case that pileup is negligible and dead time is the main loss mechanism, the counting statistics are invariable to the observed fraction of the spectrum (i.e., the Westphal formula always applies). This is inferred from computer simulation data, as shown for extending dead time in Fig. 4. Similar results are available for nonextending dead time.

Remaining Problems

1. Only stationary counting processes were considered up to now. A further extension toward rapidly varying counting rates is still unexplored.
2. The validity of Eq. (1) has been checked for combinations of pileup and extended dead time only—not yet with an additional nonextending dead time component.
3. When applying Eq. (1), use was made of simulation results for the scatter on the LFC weighting factor, $\sigma(n)$. Whereas an exact expression for $\sigma(n)$ was found for the nonextending dead time, the case of pileup and extending dead time still needs further research. For the time being, one can consider using a first-order approximation: $\sigma(n) \approx \sqrt{n-1}$.

CONCLUSIONS

Pulse loss by pileup rejection is at the origin of an increased count variance in loss-free counting spectra. A new formula for LFC uncertainty is presented and its validity is demonstrated experimentally on a HPGe γ-ray detector setup. It replaces the existing Westphal formula, which is still valid for pulse loss by extending or nonextending dead time, or when considering only small parts of the spectrum in the case of pileup. Statistical control of loss-free counting is within reach.

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