CORRECTIONS TO "INVARIANT MEASURES AND ASYMPTOTICS FOR SOME SKEW PRODUCTS", Israel Journal of Mathematics, Vol. 128, 2002, pp. 93–134

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We refer to the statements of corollaries 2.7 and 2.8 in [ANSS]. In the proof of 2.7, it is wrongly stated (a) that ν must be non-atomic, and (b) that m is locally finite. The following gives correct versions. References not given here are listed in [ANSS].

Received May 5, 2003

2.7 Corollary: *Suppose that* $\Sigma = \Sigma_A$ *is a mixing SFT and that* $f\colon \Sigma \to \mathbb{Z}^d$ $(d \geq 1)$ *has finite memory. If* $\nu \in \mathcal{P}(\Sigma)$ *is* S_A^f*-invariant, ergodic, and with* $U := \operatorname{supp}\nu \subseteq \Sigma$ *clopen and* $\nu \circ \tau_U \sim \nu$, *then* $\nu \propto \mu_\alpha|_U$ *for some homomorphism* $\alpha\colon \mathbb{Z}^d \to \mathbb{R}$. *If (in addition)* $f\colon \Sigma \to \mathbb{Z}^d$ *is aperiodic, then* $U = \Sigma$.

Proof: Since ν is finite and τ_U-non-singular, it is non-atomic and the unique τ_{ϕ_f}-ergodic, invariant measure m on $\Sigma \times \mathbb{Z}^d$ so that $m(A \times \{0\}) = \nu(A)$ is locally finite.

In case f is aperiodic, theorem 2.2 shows that $m = m_\alpha$, $U = \Sigma$ and $\nu = \mu_\alpha$ for some homomorphism $\alpha\colon \mathbb{Z}^d \to \mathbb{R}$.

Otherwise (see, e.g., proposition 5.1 in [Pa-S] for $d = 1$) there is a subgroup $\mathbb{F} \subset \mathbb{Z}^d$ and

$$(\spadesuit) \qquad f = a + g - g \circ T + \overline{f},$$

where $a \in \mathbb{Z}^d$, $\overline{f}\colon \Sigma \to \mathbb{F}$ aperiodic and $g\colon \Sigma \to \mathbb{Z}^d$ both with memory no longer than that of f.

Thus $m \circ \pi^{-1}$ (where $\pi(x,z) := (x, z - g(x))$) is locally finite, $\tau_{\phi_{\overline{f}}}$-ergodic, invariant and supported on $\Sigma \times (z_0 + \mathbb{F})$ (some $z_0 \in \mathbb{Z}^d$). The result now follows from the aperiodic case. ∎

Remarks: (1) In the situation of Corollary 2.7, the ergodic decomposition of μ_α ($\alpha\colon \mathbb{Z}^d \to \mathbb{R}$ a homomorphism) is $\{[g \in b + \mathbb{F}]\colon b \in \mathbb{Z}^d\}$ (where g is as in $(\spadesuit)$). The proof of this uses [G-H] as in the proof of ergodicity in the aperiodic case.

(2) Suppose that Σ is a mixing SFT and that $\nu \in \mathcal{P}(\Sigma)$ is S_A^+-invariant and ergodic; then (for $s \in S$) $N_s := \sum_{n=0}^{\infty} 1_{[s]} \circ T^n$ is S_A^+-invariant, whence constant ν-a.e. Call $s \in S$ **ephemeral** if $1 \leq N_s < \infty$ ν-a.e., and **recurrent** if $N_s = \infty$ a.e. Let S_∞ and S_e denote the collections of recurrent and ephemeral states (respectively). Evidently $S_\infty \neq \emptyset$ and $N := \sum_{f \in S_e} N_f$ is constant and finite ($N := 0$ if $S_e = \emptyset$).

2.8 Corollary: *Suppose* $\Sigma = \Sigma_A$ *is a mixing SFT. If* $\nu \in \mathcal{P}(\Sigma)$ *is* S_A^+*-invariant and ergodic, then* $\exists$ *a cylinder* $f = [f_1, \ldots, f_N] \subset \Sigma$ *with* $f_1, \ldots, f_N \in S_e$; *a mixing SFT* $\Sigma' = \Sigma_{A'} \subset \Sigma \cap S_\infty^{\mathbb{N}}$; *a clopen,* $S_{A'}^+$*-invariant subset* $U \subset \Sigma'$; *a homomorphism* $\alpha\colon \mathbb{Z}^{S_\infty} \to \mathbb{R}$ *so that*

$$\nu = c \sum_{\pi \in S_N,\ \pi f \cap U \neq \emptyset} \delta_{\pi f} \times \mu|_{T^N \pi f \cap U}$$

where $\pi f := [f_{\pi(1)}, \ldots, f_{\pi(N)}]$, $\mu \in \mathcal{P}(\Sigma')$, $\frac{d\mu \circ T}{d\mu} = c' e^{\alpha \circ F^\#}$ *and* $c, c' > 0$.

Proof: Suppose first that $S_e = \emptyset$. We claim first that ν is the restriction of a Markov measure to a union of initial states. To see this, choose $s \in S_\infty$ with $\nu([s]) > 0$; then $\nu|_{[s]}$ is ergodic, invariant under finite permutations of inter-arrival words, whence by de Finetti's theorem (see, e.g., [D-F]) a product measure. By Proposition 15 of [D-F], $\nu|_{[s]}$ is the restriction of a stationary Markov measure to $[s]$, and by S_A^+-invariance and ergodicity, the transition matrix p does not depend on s, $\nu([s]) > 0$. It follows that $U := \operatorname{supp}\nu = \bigcup_{s\in S_\infty,\ \nu([s])>0}[s]$ is clopen in $\Sigma_{A'}$ ($A'_{s,t} = 1_{[p_{s,t}>0]}$), and that $\nu \circ \tau_U \sim \nu$ where τ is the adic transformation on $\Sigma_{A'}$. The result in case $S_e = \emptyset$ now follows from Corollary 2.7.

In general,

$$x_n \in \begin{cases} S_e, & 1 \le n \le N := \sum_{f\in S_e} N_f, \\ S_\infty, & n > N, \end{cases}$$

$\nu \circ T^{-N}$ is as above. The result follows from this. ∎

Remarks: Examples illustrating the various cases of Corollary 2.8 can be extracted from [P-S]. Corollary 2.8 now extends the one-sided version of theorem 6.2 in [P-S]. Theorems 2.9 and 2.11 there follow from it (by identification of possible Σ').

References

[ANSS] J. Aaronson, H. Nakada, O. Sarig and R. Solomyak, *Invariant measures and asymptotics for some skew products*, Israel Journal of Mathematics **128** (2002), 93–134.

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[Pa-S] W. Parry and K. Schmidt, *Natural coefficients and invariants for Markov-shifts*, Inventiones Mathematicae **76** (1984), 15–32.