# Minimal Set of Auxiliary Fields and $S$-Matrix for Extended Supergravity.

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Recently considerable progress has been achieved in understanding the structure of supergravity [1,2] due to the construction of the so-called tensor calculus [3,4]. This calculus, in its turn, is based on the minimal set of auxiliary fields [5-7].

Note, that the very interesting approach to supergravity by building the geometry in the right and left superspaces [8] gives probably a more adequate way to obtain these results.

However, the minimal set of auxiliary fields and, correspondingly, the tensor calculus was obtained only for the simplest model of supergravity namely the $SO_1$-model [1,2]. For the models of extended supergravity, in particular already for the $SO_2$ model [9-11] no set of auxiliary fields was known up to now that would provide the closure of the algebra. This circumstance is an obstacle against the generalization of the tensor calculus to the models of extended supergravity.

In the present paper the minimal set of auxiliary fields is obtained for the global $SO_2$-supermultiplet $(2, \frac{3}{2}, \frac{3}{2}, 1)$ which corresponds to the linearized limit of the $SO_2$-model of the extended supergravity. This set of auxiliary fields contains two spinorial fields (*) $\chi^i_{(\alpha)}$, $\varphi^i_{(\alpha)}$ and a number of boson fields which may be derived for convenience into three groups

1) Scalars under the $SO_2$ group. These are a vector $v_\varrho$, an axial vector $a_\varrho$ and a scalar $s$.

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(*) The indices $i, j, k \ldots = 1, 2$ serve the vector representation of the $SO_2$-group. The bracketed indices $(\alpha), (\beta) \ldots$ are spinorial ones. (All the spinors are understood as Majorana's: $\bar{\chi}^i_{(\alpha)} = C^{-1}_{(\alpha)(\beta)}\chi^i_{(\beta)}$.)

2) Symmetrical traceless tensors with respect to the internal indices $i$, $j$. These are an axial vector $a_\varrho^{ij}$ and a pseudoscalar $p^{ij}$.

3) Antisymmetrical tensors with respect to the indices $i$, $j$. These are an antisymmetrical tensor $\omega_{\nu\mu}^{ij} = -\omega_{\mu\nu}^{ij}$, a vector $v_\nu^{ij}$, a scalar $u^{ij}$ and a pseudoscalar $t^{ij}$.

An essential qualitative feature of the minimal set of auxiliary fields in the $SO_2$ model is the fact that it does not match the minimal set of auxiliary fields when one passes over to the $SO_1$ model. In other words, in the « $SO_1$ limit » one comes to the set of auxiliary fields that does not contain the three fields $s$, $p$, $a_\nu$, but a larger amount of fields $s$, $p$, $a'_\varrho$, $b_\varrho$, $v_\varrho$, $\chi_{(\alpha)}$; $\varphi_{(\alpha)}$. (Here $a'_\varrho$ and $b_\varrho$ are combinations of the fields $a_\varrho$ and $a_\varrho^{ij}$).

This circumstance is due to the fact that the way to introduce the auxiliary fields $s$, $p$, $a'_\varrho$, $b_\varrho$, $v_\varrho$, $\chi_{(\alpha)}$, $\varphi_{(\alpha)}$ which provide the closure of the algebra in the $SO_1$-model is not unique but contains one-parameter arbitrariness and there exists such a value of the free parameter ($\alpha = 0$) for which the fields $v_\varrho$, $b_\varrho$, $\varphi_{(\alpha)}$, $\chi_{(\alpha)}$ fall out of the consideration. It is this case that corresponds to the minimal set of auxiliary fields $s$, $p$, $a_\nu$ in the $SO_1$ supergravity (5-7). On the contrary, the minimal set of auxiliary fields in the $SO_2$ model may be built only if the value of the free parameter $\alpha$ corresponding to it in the $SO_1$ model is $\alpha = 1$ (and not $\alpha = 0$ which minimized the set of auxiliary fields in the $SO_1$ model).

Below we present the final results for the minimal set of auxiliary fields in the $SO_2$ model and also for the above-pointed one-parameter extension of the minimal set of auxiliary fields in the $SO_1$ model. We present, besides, the final expression for the generating functional in the $SO_2$ supergravity. Finally we discuss the basic points of the method used by us to obtain the minimal set of auxiliary fields in the linearized $SO_2$ supergravity.

We proceed now with the description of the minimal set of auxiliary fields in the linearized $SO_2$ supergravity. The action and the supertransformations in this case have the form (*)

$$S_2 = \frac{1}{4}[\omega_{\nu,\mu\varrho}\,\omega_{\mu,\nu\varrho} - (\omega_{\nu,\nu\mu})^2] - \frac{1}{4}(F_{\nu\mu})^2 + \frac{i}{2}\,\mathcal{E}^{\nu\mu\varrho\sigma}(\bar\psi_\nu^i\gamma_5\gamma_\mu\partial_\varrho\psi_\sigma^i)\,, \tag{1}$$

$$\delta h_{\nu\mu} = \frac{i}{2}[(\bar\psi_\nu^i\gamma_\mu\varepsilon^i) + (\bar\psi_\mu^i\gamma_\nu\varepsilon^i)]\,, \tag{2}$$

$$\delta A_\nu = -\,\mathcal{E}^{ij}(\bar\psi_\nu^i\varepsilon^j)\,, \tag{3}$$

$$\delta\psi_\nu^i = \frac{1}{2}\,\omega_{\nu,\varrho\sigma}\sigma_{\varrho\sigma}\varepsilon^i + \frac{i}{2}\,\mathcal{E}^{ij}F_{\varrho\sigma}\sigma_{\varrho\sigma}\gamma_\nu\varepsilon^j\,. \tag{4}$$

Here

$$\omega_{\varrho,\nu\mu} = \partial_\mu h_{\nu\varrho} - \partial_\nu h_{\mu\varrho}\,, \qquad F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu\,, \tag{5}$$

$$\mathcal{E}^{ij} = -\,\mathcal{E}^{ji}\,, \qquad \mathcal{E}^{12} = 1\,. \tag{6}$$

(*) The metrics used is $(+\,-\,-\,-)$; $[\gamma_\mu, \gamma_\nu]_+ = 2\eta_{\nu\mu}$; $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$; $\mathcal{E}^{0123} = 1$. The complex notations used in ref. (11) are connected with those used below in the following way: $\psi_\nu = \psi_\nu^1 + i\psi_\nu^2$, $\varepsilon = \varepsilon^1 + i\varepsilon^2$. Note that in the linearized limit the antisymmetric part of the tensor $e_\nu^a$ falls out of the equations and the gravitational field is described by the symmetric tensor $h_{\mu\nu}$.

Apart from the invariance under the global supertransformations (2)-(4) the action (1) is invariant with respect to the global translations, global Lorentz and global $SO_2$ transformations. The action (1) is invariant besides, under the three kinds of local transformations

$$\delta h_{\nu\mu} = \partial_\nu \xi_\mu + \partial_\mu \xi_\nu\,, \qquad \delta A_\nu = 0\,, \qquad \delta\psi_\nu = 0\,, \tag{7}$$

$$\delta h_{\nu\mu} = 0\,, \qquad \delta A_\nu = \partial_\nu \varphi\,, \qquad \delta\psi_\nu = 0\,, \tag{8}$$

$$\delta h_{\nu\mu} = 0\,, \qquad \delta A_\nu = 0\,, \qquad \delta\psi_\nu = \partial_\nu \lambda\,. \tag{9}$$

As it was shown in ([11]) in the arbitrary theory fixed by the set of variables $q^A$ and the action $S(q^A)$ which is invariant under the transformations with the parameters $\varepsilon^\alpha$

$$\delta q^A = R^A_\alpha \varepsilon^\alpha \tag{10}$$

the commutation relation between two transformations has the form

$$R^A_{\alpha,B} R^B_\beta - (-1)^{\eta^\alpha\eta^\beta} R^A_{\beta,B} R^B_\alpha = R^A_\gamma U^\gamma_{\alpha\beta} + S_{,B} B^{AB}_{\alpha\beta}\,. \tag{11}$$

Here

$$R^A_{\alpha,B} = \frac{\partial^r R^A_\alpha}{\partial q^B}\,, \qquad S_{,A} = \frac{\partial^r S}{\partial q^A}\,, \qquad B^{AB}_{\alpha\beta} = -(-1)^{\eta^A\eta^B} B^{BA}_{\alpha\beta} \tag{12}$$

and $\eta^\alpha = 0$ if $\alpha$ corresponds to a Bose quantity, whereas otherwise $\eta^\alpha = 1$.

For the $SO_2$ supergravity the structure coefficients $B^{AB}_{\alpha\beta}$ were obtained in ref. ([11]). They are different from zero only when the indices $A$, $B$ indicate the field $\psi^i_{\nu(\alpha)}$ and the indices $\alpha$, $\beta$ correspond to the supertransformations (2)-(4). After going to the present notations and fulfilling some Fierz transpositions the corresponding quantities $B^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\delta)]}$ can be reduced to the form

$$\begin{aligned} B^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\delta)]} = {} & \tfrac{1}{8}\, C_{(\varrho)(\beta)} C^{-1}_{(\sigma)(\gamma)} \{\tfrac{1}{2}\, \mathcal{E}^{nm} \mathcal{E}^{ij} [2\sigma_{\nu\mu(\alpha)(\varrho)}\, \delta_{(\sigma)(\delta)} + \\ & + 3\gamma_5(\gamma_\mu\gamma_\nu + \eta_{\nu\mu})_{(\alpha)(\varrho)} \gamma_{5(\sigma)(\delta)} - \gamma_5(\gamma_\mu\gamma_\eta\gamma_\nu + \eta_{\nu\mu}\gamma_\eta)_{(\alpha)(\varrho)} (\gamma_5\gamma_\eta)_{(\sigma)(\delta)}] \\ & + \delta^{ij}\delta^{nm}[(\gamma_\nu\gamma_\eta\gamma_\mu - 2\gamma_\mu\gamma_\eta\gamma_\nu)_{(\alpha)(\varrho)} \gamma_{\eta(\sigma)(\delta)} + \\ & + 8(\eta_{\mu\eta} - \tfrac{1}{4}\gamma_\eta\gamma_\mu)_{(\alpha)(\varrho)} \sigma_{\nu\eta(\sigma)(\delta)} + 8\gamma_5(\eta_{\mu\eta} - \tfrac{1}{4}\gamma_\eta\gamma_\mu)_{(\alpha)(\varrho)} (\sigma_{\nu\eta}\gamma_5)_{(\sigma)(\delta)}] + \\ & + \tfrac{1}{2}(\delta^{nj}\delta^{mi} + \delta^{ni}\delta^{mj})[(\gamma_\mu\gamma_\eta\gamma_\nu + \eta_{\nu\mu}\gamma_\eta)_{(\alpha)(\varrho)} \gamma_{\eta(\sigma)(\delta)} + \\ & + 2\gamma_5(\gamma_\mu\sigma_{\eta\varrho}\gamma_\nu - \eta_{\nu\mu}\sigma_{\eta\varrho})_{(\alpha)(\varrho)} (\gamma_5\sigma_{\eta\varrho})_{(\sigma)(\delta)} - 8\gamma_{5(\alpha)(\varrho)} (\gamma_5\sigma_{\nu\mu})_{(\sigma)(\delta)}]\}\,. \end{aligned} \tag{13}$$

The main result of the present paper is that one can achieve the closure of the algebra in the linearized $SO_2$ supergravity (*i.e.* the nullification of the coefficients $B^{AB}_{\alpha\beta}$ in (11) by introducing the auxiliary fields $a_\varrho$, $v_\varrho$, $s$, $p^{ij}$, $a^{ij}_\varrho$, $\omega^{ij}_{\nu\mu}$, $v^{ij}_\varrho$, $u^{ij}$, $t^{ij}$, $\chi^i_{(\alpha)}$, $\varphi^i_{(\alpha)}$ whose structure was described above and by modifying the action and the global supertransformations as follows:

$$S_2^* = S_2 - \tfrac{1}{2}a_\varrho^2 + v_\varrho^2 + \tfrac{1}{2}s^2 + \tfrac{1}{4}(p^{ij})^2 - \tfrac{1}{2}(a_\varrho^{ij})^2 - \tfrac{1}{2}(v_\varrho^{ij})^2 - \tfrac{1}{4}(\omega_{\nu\mu}^{ij})^2 + \tfrac{1}{2}(u^{ij})^2 + \tfrac{1}{2}(t^{ij})^2 + (\bar\varphi^i\chi^i)\,, \tag{14}$$

$$\delta^* h_{\nu\mu} = \delta h_{\nu\mu} + \eta_{\nu\mu}(\bar\varepsilon^i\chi^i)\,, \tag{15}$$

$$\delta^* A_\nu = \delta A_\nu + i\mathcal{E}^{ij}(\bar\varepsilon^i\gamma_\nu\chi^j)\,, \tag{16}$$

$$\delta^*\psi_\nu^i = \delta\psi_\nu^i - \tfrac{1}{3}[\tfrac{3}{2}\gamma_5\sigma_{\nu\varrho}^{-1}\varepsilon^j(a_\varrho^{ij} + \tfrac{1}{2}\delta^{ij}a_\varrho) + \tfrac{1}{2}\gamma_\nu\varepsilon^i s + \tfrac{1}{2}\gamma_5\gamma_\nu\varepsilon^j p^{ij} + \gamma_\nu\gamma_\varrho\varepsilon^i v_\varrho + \tfrac{3}{2}\sigma_{\nu\varrho}^{-1}\varepsilon^j v_\varrho^{ij} + \gamma_\nu\varepsilon^j u^{ij} + \gamma_5\gamma_\nu\varepsilon^j t^{ij} - \tfrac{1}{2}(\gamma_\nu\sigma_{\mu\varrho} + 3\sigma_{\mu\varrho}\gamma_\nu)\varepsilon^j\omega_{\mu\varrho}^{ij}]\,, \tag{17}$$

$$\delta^*\chi^i = \frac{1}{3i}[\gamma_5\gamma_\varrho\varepsilon^j(\delta^{ij}a_\varrho - a_\varrho^{ij}) + \gamma_\varrho\varepsilon^i v_\varrho + \gamma_5\varepsilon^j p^{ij} - \varepsilon^i s + \gamma_\varrho\varepsilon^j v_\varrho^{ij} + \sigma_{\nu\mu}\varepsilon^j\omega_{\nu\mu}^{ij} + \varepsilon^j u^{ij} - \gamma_5\varepsilon^i t^{ij}]\,, \tag{18}$$

$$\delta^*\varphi^i = -\frac{\delta S_2}{\delta h_{\nu\mu}}\eta_{\nu\mu}\varepsilon^i - i\frac{\delta S_2}{\delta A_\nu}\gamma_\nu\varepsilon^j\mathcal{E}^{ij} + \frac{1}{2}\partial_\mu[\gamma_5\gamma_\varrho\gamma_\mu\varepsilon^j(\delta^{ij}a_\varrho - a_\varrho^{ij}) + (6\eta_{\mu\varrho} - \gamma_\mu\gamma_\varrho)\varepsilon^i v_\varrho + \gamma_\mu\varepsilon^i s + \gamma_5\gamma_\mu\varepsilon^j p^{ij} - \gamma_\mu\varepsilon^j u^{ij} - \gamma_5\gamma_\mu\varepsilon^j t^{ij} + (2\sigma_{\nu\varrho}\gamma_\mu - \gamma_\mu\sigma_{\nu\varrho})\varepsilon^j\omega_{\nu\varrho}^{ij} - \gamma_\varrho\gamma_\mu\varepsilon^j v_\varrho^{ij}]\,, \tag{19}$$

$$\delta^* a_\varrho = -\frac{1}{6}\left[\frac{3}{2}\left(\frac{\delta^r S_2}{\delta\psi_\nu^i}\gamma_5\sigma_{\nu\varrho}^{-1}\varepsilon^i\right) + 2i(\bar\varphi^j\gamma_5\gamma_\varrho\varepsilon^i) + 3\partial_\mu(\bar\chi^i\gamma_5\gamma_\varrho\gamma_\mu\varepsilon^i)\right], \tag{20}$$

$$\delta^* s = \frac{1}{6}\left[\left(\frac{\delta^r S_2}{\delta\psi_\nu^i}\gamma_\nu\varepsilon^i\right) - 2i(\bar\varphi^i\varepsilon^i) + 3\partial_\nu(\bar\chi^i\gamma_\nu\varepsilon^i)\right], \tag{21}$$

$$\delta^* v_\varrho = \frac{1}{6}\left[\left(\frac{\delta^r S_2}{\delta\psi_\nu^i}\gamma_\nu\gamma_\varrho\varepsilon^i\right) + i(\bar\varphi^i\gamma_\varrho\varepsilon^i) + 9\partial_\varrho(\bar\chi^i\varepsilon^i) - \frac{3}{2}\partial_\nu(\bar\chi^i\gamma_\nu\gamma_\varrho\varepsilon^i)\right], \tag{22}$$

$$\delta^* a_\varrho^{ij} = \frac{1}{6}(\delta^{in}\delta^{jm} + \delta^{im}\delta^{jn} - \delta^{ij}\delta^{nm})\left[-\frac{3}{2}\left(\frac{\delta^r S_2}{\delta\psi_\nu^n}\gamma_5\sigma_{\nu\varrho}^{-1}\varepsilon^m\right) - i(\bar\varphi^n\gamma_\varrho\gamma_5\varepsilon^m) + \frac{3}{2}\partial_\mu(\bar\chi^n\gamma_5\gamma_\varrho\gamma_\mu\varepsilon^m)\right], \tag{23}$$

$$\delta^* p^{ij} = \frac{1}{6}(\delta^{in}\delta^{jm} + \delta^{im}\delta^{jn} - \delta^{ij}\delta^{nm})\left[\left(\frac{\delta^r S_2}{\delta\psi_\nu^n}\gamma_5\gamma_\nu\varepsilon^m\right) + 2i(\bar\varphi^n\gamma_5\varepsilon^m) + 3\partial_\varrho(\bar\chi^n\gamma_5\gamma_\varrho\varepsilon^m)\right], \tag{24}$$

$$\delta^*\omega_{\varrho\mu}^{ij} = \frac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\left[\left(\frac{\delta^r S_2}{\delta\psi_\nu^n}(\gamma_\nu\sigma_{\varrho\mu} + 3\sigma_{\varrho\mu}\gamma_\nu)\varepsilon^m\right) - 2i(\bar\varphi^n\sigma_{\varrho\mu}\varepsilon^m) + 3\partial_\nu(\bar\chi^n(\gamma_\nu\sigma_{\varrho\mu} - 2\sigma_{\varrho\mu}\gamma_\nu)\varepsilon^m)\right], \tag{25}$$

$$\delta^* v^{ij}_\varrho = \frac{1}{6}\varepsilon^{ij}\varepsilon^{nm}\left[-\frac{3}{2}\left(\frac{\delta^r S_2}{\delta\psi^n_\nu}\sigma^{-1}_{\nu\varrho}\varepsilon^m\right) - i(\bar\varphi^n,\gamma_\varrho\varepsilon^m) + \frac{3}{2}\partial_\mu(\bar\chi^n_\mu\gamma_\varrho\gamma_\mu\varepsilon^m)\right], \tag{26}$$

$$\delta^* u^{ij} = \frac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\left[\left(\frac{\delta^r S_2}{\delta\psi^n_\nu}\gamma_\nu\varepsilon^m\right) + i(\bar\varphi^n\varepsilon^m) - \frac{3}{2}\partial_\varrho(\bar\chi^n\gamma_\varrho\varepsilon^m)\right], \tag{27}$$

$$\delta^* t^{ij} = \frac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\left[\left(\frac{\delta^r S_2}{\delta\psi^n_\nu}\gamma_5\gamma_\nu\varepsilon^m\right) - i(\bar\varphi^n\gamma_5\varepsilon^m) - \frac{3}{2}\partial_\varrho(\bar\chi^n\gamma_5\gamma_\varrho\varepsilon^m)\right]. \tag{28}$$

Here the action $S_2$ and the transformation laws $\delta h_{\nu\mu}$, $\delta A_\nu$ and $\delta\psi_\nu$ are defined by relations (1)-(4). The use is also made of the notation

$$\sigma^{-1}_{\nu\mu} = -\tfrac{2}{3}(\gamma_\mu\gamma_\nu + \eta_{\nu\mu}), \qquad \sigma^{-1}_{\mu\nu}\sigma_{\mu\varrho} = \eta_{\nu\varrho}. \tag{29}$$

The action $S_2^*$ (14) is invariant under the global supertransformations (15)-(28). The commutator between two supertransformations $[\delta^*(\varepsilon^i_2), \delta^*(\varepsilon^j_1)]_-$ with the parameters $\varepsilon^j_1$ and $\varepsilon^i_2$ leads to the global translations with the parameters $\eta_\mu = i(\bar\varepsilon^i_2\gamma_\mu\varepsilon^i_1)$, « local translations » (7) with the parameters $\xi_\nu = -\frac{1}{2}\eta_\mu h_{\nu\mu}$, « local $SO_2$ transformations » (8) with the parameter $\varphi = -\eta_\mu A_\mu$ and, finally, «local supertransformation » with the parameters $\xi^i_{(\alpha)}$

$$\xi^i = -\eta_\mu\left(\psi^i_\mu - \frac{i}{2}\gamma_\mu\chi^i\right) + \frac{1}{2}\Big\{i(\bar\varepsilon^j_2\gamma_\varrho\psi^j_\sigma)\sigma_{\sigma\varrho}\varepsilon^i_1 - \varepsilon^i_1(\bar\varepsilon^j_2\chi^j) + \gamma_5\chi^j(\bar\varepsilon^j_2\gamma_5\varepsilon^i_1) - \chi^j(\bar\varepsilon^j_2\varepsilon^i_1) + $$
$$+ \frac{1}{2}\gamma_5\gamma_\nu\chi^j(\bar\varepsilon^j_2\gamma_5\gamma_\nu\varepsilon^i_1) + \frac{1}{2}\gamma_\nu\chi^j(\bar\varepsilon^j_2\gamma_\nu\varepsilon^i_1)\Big\} - \{1 \leftrightarrow 2\}. \tag{30}$$

(Note, that the first term in the curly bracket appears owing to the fact that the antisymmetric part of the vierbein $e_{\nu\alpha}$ was taken identically equal to zero.)

Now consider the « $SO_1$ limit » for the relations (14)-(28). In order to obtain the $SO_1$ model put $\varepsilon^2 = 0$ in relations (14)-(28). In this case the fields

$$\psi^2_\nu,\quad A_\nu,\quad \chi^2,\quad \varphi^2,\quad a^{12}_\varrho = a^{21}_\varrho,\quad p^{12} = p^{21},\quad \omega^{ij}_{\nu\mu},\quad v^{ij}_\varrho,\quad u^{ij},\quad t^{ij} \tag{31}$$

transform only amidst themselves. Analogously the fields

$$\psi^1_\nu,\quad h_{\nu\mu},\quad \chi^1,\quad \varphi^1,\quad a_\varrho,\quad v_\varrho,\quad s,\quad p^{11} = -p^{22},\quad a^{11}_\varrho = -a^{22}_\varrho \tag{32}$$

also transform through themselves. If one puts now all the fields (31) equal to zero, one comes to the $SO_1$ model (with the closed algebra), given in terms of the fields (32).

The action and the supertransformations that result from the above-described procedure turn out to correspond only to one particular possibility out of the one-parameter class of ways to close the $SO_1$ algebra within the set of fields (32).

Indeed let us write the action for the linearized $SO_1$ supergravity in the form

$$S_1^* = \frac{1}{4}(\omega_{\nu,\mu\varrho}\omega_{\mu,\nu\varrho} - (\omega_{\nu,\nu\varrho})^2) + \frac{i}{2}\mathcal{E}^{\nu\mu\varrho\sigma}(\bar\psi_\nu\gamma_5\gamma_\mu\partial_\varrho\psi_\sigma) + $$
$$+ \alpha v^2_\varrho + \frac{6\alpha}{\beta}b^2_\varrho - \frac{3}{2}(a'_\varrho)^2 - \beta(s^2 + p^2) + \alpha(\bar\varphi\chi), \tag{33}$$

where $\omega_{\varrho,\nu\mu}$ is defined in (5) and the fields $s$, $p$, $a'_\varrho$, $b_\varrho$, $v_\varrho$, $\chi_{(\alpha)}$, $\varphi_{(\alpha)}$ are auxiliary ($s$, $p$, $v_\varrho$ are scalar, pseudoscalar and vector fields, respectively, $a'_\varrho$ and $b_\varrho$ are axial vectors, and $\chi_{(\alpha)}$ and $\varphi_{(\alpha)}$ are spinors), $\alpha$ is a free parameter and $\beta = \alpha - \frac{3}{2}$. Then one can see that $S_1^*$ (33) is invariant under the following transformations that form closed algebra:

$$\delta^* h_{\nu\mu} = \frac{i}{2}[(\bar\psi_\nu\gamma_\mu\varepsilon) + (\bar\psi_\mu\gamma_\nu\varepsilon)] + \alpha\eta_{\nu\mu}(\bar\chi\varepsilon)\,, \tag{34}$$

$$\delta^*\psi_\nu = \frac{1}{2}\,\omega_{\nu,\varrho\sigma}\,\sigma_{\varrho\sigma}\,\varepsilon - \frac{1}{3}[\alpha\gamma_\nu\gamma_\varrho\,\varepsilon v_\varrho + \frac{9}{4}\gamma_5\sigma_{\nu\varrho}^{-1}\,\varepsilon a'_\varrho - \beta(\gamma_\nu\varepsilon s + \gamma_5\gamma_\nu\varepsilon p)]\,, \tag{35}$$

$$\delta^*\chi = -\frac{i}{3}\left[\gamma_\varrho\,\varepsilon v_\varrho + \gamma_5\,\varepsilon p - \varepsilon s + \frac{3}{\beta}\gamma_5\gamma_\varrho\,\varepsilon b_\varrho\right], \tag{36}$$

$$\delta^*\varphi = -\frac{\delta S_1^*}{\delta h_{\nu\mu}}\,\eta_{\nu\mu}\,\varepsilon + \beta\partial_\mu\gamma_\mu[\gamma_\varrho\,\varepsilon v_\varrho + \gamma_5\,\varepsilon p - \varepsilon s] + 3\partial_\mu[\varepsilon v_\mu + \gamma_5\gamma_\mu\gamma_\nu\,\varepsilon b_\nu - 2\gamma_5\,\varepsilon b_\mu]\,, \tag{37}$$

$$\delta^* a'_\varrho = -\frac{1}{4}\left(\frac{\delta^r S_1^*}{\delta\psi_\nu}\,\gamma_5\,\sigma_{\nu\varrho}^{-1}\,\varepsilon\right), \tag{38}$$

$$\delta^* s = \frac{1}{6}\left[\left(\frac{\delta^r S_1^*}{\delta\psi_\nu}\,\gamma_\nu\,\varepsilon\right) + \frac{i\alpha}{\beta}(\bar\varphi\varepsilon) + 3\alpha\partial_\nu(\bar\chi\gamma_\nu\,\varepsilon)\right], \tag{39}$$

$$\delta^* p = -\frac{1}{6}\left[\left(\frac{\delta^r S_1^*}{\delta\psi_\nu}\,\gamma_\nu\gamma_5\,\varepsilon + \frac{i\alpha}{\beta}(\bar\varphi\gamma_5\,\varepsilon) + 3\alpha\partial_\nu(\bar\chi\gamma_\nu\gamma_5\,\varepsilon)\right)\right], \tag{40}$$

$$\delta^* v_\varrho = \frac{1}{6}\left[\left(\frac{\delta^r S_1^*}{\delta\psi_\nu}\,\gamma_\nu\gamma_\varrho\,\varepsilon\right) + i(\bar\varphi\gamma_\varrho\,\varepsilon) + 9\partial_\varrho(\bar\chi\varepsilon) + 3\beta\partial_\nu(\bar\chi\gamma_\nu\gamma_\varrho\,\varepsilon)\right], \tag{41}$$

$$\delta^* b_\varrho = \frac{1}{6}\left[\frac{i}{2}(\bar\varphi\gamma_5\gamma_\varrho\,\varepsilon) - \frac{3}{2}\beta\partial_\nu(\bar\chi\gamma_\varrho\gamma_\nu\gamma_5\,\varepsilon)\right]. \tag{42}$$

Consider now two particular cases for different choices of the free parameter $\alpha$. If (and only if) $\alpha = 0$ the fields $\psi_\nu$, $h_{\nu\mu}$, $s$, $p$, $a'_\varrho$ transform only through themselves and one may set $\chi_{(\alpha)} = \varphi_{(\alpha)} = 0$, $v_\varrho = b_\varrho = 0$ without breaking the invariance of the action and the closedness of the algebra. We have come exactly to the minimal set of auxiliary fields in the $SO_1$ supergravity [5-7].

It is essential that the minimal set of auxiliary fields in the $SO_2$ supergravity corresponds in the limit of $SO_1$ model to a different value of the free parameter namely $\alpha = 1$. One may easily verify this by comparing relations (14)-(28) with (33)-(42) if one sets

$$a'_\varrho = \tfrac{1}{3}(a_\varrho + 2a_\varrho^{11})\,, \qquad b_\varrho = \tfrac{1}{6}(a_\varrho^{11} - a_\varrho)\,, \qquad p = p^{11}\,. \tag{43}$$

Therefore, the minimal set of auxiliary fields for the $SO_2$ supergravity is not a direct extension of that for the $SO_1$ supergravity and essentially differs from the latter in its structure.

The results of the present work refer only to the global supertransformations corresponding to the linearized actions for the $SO_2$ and $SO_1$ models. The problem of generalizing these results to the locally supersymmetric models $SO_1$ and $SO_2$ supergravity is now being studied. We do not doubt however that the corresponding generalization is possible, indeed. This is confirmed, in particular, also by the fact that (as one may see without using the auxiliary fields, as well) the generating functional in the $SO_2$ extended supergravity has the standard form characteristic of the theory of the rank 2

$$Z = \int \mathcal{D}\psi_\nu \, \mathcal{D}e^a_\nu \, \mathcal{D}\bar{b}_\alpha \, \mathcal{D}c^\alpha g^{-11/4}\delta(\chi^\alpha) \exp\left[ i \left[ S_2 + \bar{b}_\alpha \frac{\delta^r \chi^\alpha}{\delta q^A} R^A_\beta c^\beta - \right.\right.$$
$$\left.\left. - \frac{1}{4} \bar{b}_\alpha \frac{\delta^r \chi^\alpha}{\delta \psi^i_{\nu(\alpha)}} \bar{b}_\beta \frac{\delta^r \chi^\beta}{\delta \psi^i_{\mu(\beta)}} c^n_{(\gamma)} c^m_{(\delta)} B^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\delta)]} \right]\right], \tag{44}$$

where $g = -\det|g_{\nu\mu}|$, $c^\alpha$ and $\bar{b}_\alpha$ are ghost fields, $\chi^\alpha$ are gauge conditions and $B^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\delta)]}$ are structure coefficients (13). For the particular gauge $\chi^i_{(\alpha)} = i\gamma_\nu \psi^i_\nu$ the four-ghost term in (44) becomes

$$- \frac{1}{16\sqrt{g}} [(\bar{b}^i \gamma_a b^j)(\bar{c}^j \gamma^a c^i) + 4(\bar{b}^i \gamma_a b^i)(\bar{c}^j \gamma^a c^j) -$$
$$- 6[(\bar{b}^i b^j)(\bar{c}^j c^i) + (\bar{b}^i \gamma_5 b^j)(\bar{c}^j \gamma_5 c^i)] - (\bar{b}^i \gamma_5 \gamma_a b^j)(\bar{c}^j \gamma_5 \gamma^a c^i)] . \tag{45}$$

In conclusion we dwell upon the main points of the method used for finding the structure of the minimal set in the linearized $SO_2$ model.

In ref. ([7]) it was shown that if one confines oneself only to the auxiliary fields needed to compensate the structure coefficients (*) $B^{AB}_{\alpha\beta}$ (11), the action $S^*$ and the transformation laws of the dynamical $q^A$ and the auxiliary $\omega^\alpha_A$ fields in the general case become

$$S^* = S + \tfrac{1}{4}(-1)^{\eta^A(\eta^B+\eta^\beta)} \hat{B}^{BA}_{\beta\alpha} \omega^\alpha_A \omega^\beta_B , \tag{46}$$

$$\delta^* q^A = R^A_\alpha \varepsilon^\alpha - \tfrac{1}{2} \hat{B}^{BA}_{\beta\alpha} \varepsilon^\alpha \omega^\beta_B , \tag{47}$$

$$\delta^* \omega^\beta_B = (-1)^{\eta^B(\eta^\beta+1)} \frac{\delta^r S}{\delta q^B} \varepsilon^\beta + \text{terms} \sim \omega \tag{48}$$

and the matrix $\hat{B}^{AB}_{\alpha\beta}$ must obey the relations

$$\hat{B}^{AB}_{\alpha\beta} - (-1)^{\eta_\alpha \eta_\beta} \hat{B}^{AB}_{\beta\alpha} = \hat{B}^{AB}_{\alpha\beta} - (-1)^{\eta^A \eta^B} \hat{B}^{BA}_{\alpha\beta} = 2B^{AB}_{\alpha\beta} . \tag{49}$$

The fulfillment of the relations (46)-(49) provides the invariance of the action $S^*$ with the accuracy to the terms of order $\omega^2$ and the closedness of the algebra of transformations acting on the physical fields $q^A$ up to the terms of order $\omega$. Note that the matrix $\hat{B}^{AB}_{\alpha\beta}$ may be degenerate in which case not all of the fields $\omega^\alpha_A$ appear in the theory as a matter of fact.

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(*) *I.e.* if one considers only auxiliary fields that are involved in the transformation laws of the dynamical variables $\delta^* q^A$ and whose transformation laws $\delta^* \omega$ contain the equations of motion for the dynamical variables $\delta^r S/\delta q^A$. All these fields may be treated as components of the field $\omega^\alpha_A$. Not all of the components of the field $\omega^\alpha_A$ may, however, really appear in the theory.

When considering the $SO_2$ model we shall show first of all that as distinct from the $SO_1$ model, it is impossible in this case to achieve the closure of the algebra without introducing at least two auxiliary Fermi-fields. Indeed, without fermionic auxiliary fields the transformation laws of the dynamical Bose-fields $\delta h_{\nu\mu}$ and $\delta A_\nu$ remain unchanged after the bosonic auxiliary fields are introduced (linearized model). The requirement that the algebra for the fields $h_{\nu\mu}$ and $A_\nu$ should be closed imposes a number of restrictions on the structure of the matrix $\hat{B}^{AB}_{\alpha\beta}$. To demonstrate how these restrictions arise consider an example. Let

$$\hat{B}^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\delta)]} = \varkappa \eta_{\nu\mu} \delta^{in} \delta^{jn} \delta^{(\alpha)}_{\gamma} \delta^{(\beta)}_{\delta} + \dots \tag{50}$$

Then the transformation laws (47) have the form

$$\delta^* \psi^i_\nu = \delta\psi^i_\nu - \tfrac{1}{2}\varkappa \varepsilon^i \omega_\nu + \dots , \tag{51}$$

where $\omega_\nu = \omega^{[i(\alpha)]}_{[\nu i(\alpha)]}$. On the other hand

$$[\delta^*_1, \delta^*_2] h_{\nu\mu} = \frac{i\varkappa}{2} [(\bar{\varepsilon}^i_1 \gamma_\nu \varepsilon^i_2)\, \omega_\mu + (\bar{\varepsilon}^i_1 \gamma_\mu \varepsilon^i_2)\, \omega_\nu] + \dots \neq 0 . \tag{52}$$

Therefore the condition for the closedness of the algebra for the field $h_{\nu\mu}$ (52) requires that, in particular, $\varkappa = 0$ in (50).

All the restrictions of this nature combined together allow us to determine the matrix $\hat{B}^{AB}_{2\alpha\beta}$ in the $SO_2$ model in a unique way. It turns out nevertheless that, with the fields $\omega^\alpha_A$ corresponding to this form of the matrix $\hat{B}^{AB}_{2\alpha\beta}$ the complete algebra with the auxiliary fields included cannot be closed. This follows from the fact that in the « $SO_1$ limit » $B^{AB}_{2\alpha\beta} \to B^{AB}_{1\alpha\beta}$ we obtain an expression for $B^{AB}_{1\alpha\beta}$ that does not admit the closure of the algebra of the $SO_1$-model (*) (until fermionic auxiliary fields are not used).

Thus, any set of auxiliary fields in the $SO_2$ model must contain Fermi-fields. Consider the simplest case when the modified transformation laws of the Bose-fields $h_{\nu\mu}$ and $A_\nu$ involve only one spinorial auxiliary field $\chi^i_{(\alpha)}$

$$\delta^* h_{\nu\mu} = \delta h_{\nu\mu} + \lambda_1 \eta_{\nu\mu} (\bar{\varepsilon}^i \chi^i) , \qquad \delta^* A_\nu = \delta A_\nu + i\lambda_2 (\bar{\varepsilon}^i \gamma_\nu \chi^j)\, \varepsilon^{ij} . \tag{53}$$

The dimensionality of the field $\chi^i_{(\alpha)}$ evidently is $[\chi^i] = \text{cm}^{-\frac{3}{2}}$. Since there are no dimensional parameters in the theory (linearized limit) one must introduce another spinorial auxiliary field $\varphi^i_{(\alpha)}$ with the dimensionality $[\varphi^i] = \text{cm}^{-\frac{5}{2}}$ in order to be able to build the action, containing the field $\chi^i : \Delta S = \int (\bar{\varphi}^i \chi^i)\, d^4x$. (Note that in the theories without dimensional parameters the auxiliary Fermi fields involved into the law of transformations of the dynamical Bose-fields should always have partners of different dimensionalities.)

When building the minimal set of auxiliary fields in the $SO_2$ model we confined ourselves to the simplest case when there are only two auxiliary Fermi fields $\chi^i_{(\alpha)}$ and $\varphi^i_{(\alpha)}$. As for the structure of the auxiliary Bose fields we consider, as before, only those fields that are needed for compensating the structural coefficients $B^{AB}_{\alpha\beta}$ (13) in relations (11),

(*) Note, that in the $SO_1$-case the requirement that auxiliary Fermi-fields should be absent leads to the matrix $B^{AB}_{1\alpha\beta}$ determined up to one free parameter $b : B^{AB}_{1\alpha\beta}(b)$. One manages to close the algebra only in $b = \frac{1}{3}$ (7). The matrix $B^{AB}_{2\alpha\beta}$ in the $SO_1$ limit corresponds to the value $b \neq \frac{1}{3}$.

*i.e.* the fields $\omega_A^\alpha$ involved in eqs. (46)-(48). However, now one must add terms proportional to the fields $\chi^i_{(\alpha)}$ and $\varphi^i_{(\alpha)}$ into eqs. (46)-(48). Taking the statistics and dimensionalities of the fields into account one can easily see that the modified action and transformation laws must have the following structure (see also (53)):

$$S^* = S + \tfrac{1}{4}\hat{B}^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\sigma)]}\,\omega^{[n(\gamma)]}_{[\nu i(\alpha)]}\omega^{[m(\sigma)]}_{[\mu j(\beta)]} + (\bar{\varphi}^i\chi^i)\,, \tag{54}$$

$$\delta^*\psi^i_{\nu(\alpha)} = \delta\psi^i_{\nu(\alpha)} - \tfrac{1}{2}B^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m(\sigma)]}\,\omega^{[m(\sigma)]}_{[\mu j(\beta)]}\,\varepsilon^n_{(\gamma)}\,, \tag{55}$$

$$\delta^*\omega^{[n(\beta)]}_{[\nu i(\alpha)]} = \frac{\delta^r S}{\delta\psi^i_{\nu(\alpha)}}\,\varepsilon^n_{(\beta)} + \text{terms} \sim \varphi\varepsilon + \text{terms} \sim \partial_\nu(\chi)\,\varepsilon\,, \tag{56}$$

$$\delta^*\chi^i_{(\alpha)} \sim \omega\varepsilon\,, \tag{57}$$

$$\delta^*\bar{\varphi}^i_{(\alpha)} = -\lambda_1\frac{\delta S}{\delta h_{\nu\mu}}\,\eta_{\nu\mu}\,\bar{\varepsilon}^i_{(\alpha)} - i\lambda_2\frac{\delta S}{\delta A_\nu}\,(\bar{\varepsilon}^j\gamma_\nu)_{(\alpha)}\,\mathcal{E}^{ji} + \text{terms} \sim \partial_\nu(\omega)\,\varepsilon\,. \tag{58}$$

The requirement that the commutation relation of two supertransformations acting on the fields $h_{\nu\mu}$ and $A_\nu$ be closed restricts the structure of the matrix $\hat{B}^{AB}_{\alpha\beta}$ (*e.g.* the structure (50) can be readily checked, to be again forbidden) and allows us to determine its form up to seven free parameters.

The conditions for invariance of the action and closedness of the algebra for the auxiliary Bose-fields $\omega^{[n(\beta)]}_{[\nu i(\alpha)]}$ being imposed the remaining arbitrariness in relations (53)-(58) may be completely fixed. Finally, the matrix acquires the form

$$\begin{aligned}\hat{B}^{[\nu i(\alpha)][\mu n(\sigma)]}_{[j(\beta)][m(\delta)]} = -\tfrac{1}{9}\{&\mathcal{E}^{ij}\mathcal{E}^{nm}[\tfrac{9}{4}\sigma_{\nu\varrho}^{-1(\alpha)}{}_{(\beta)}\sigma_{\mu\varrho}^{-1(\sigma)}{}_{(\delta)} - (\gamma_5\gamma_\nu)^{(\alpha)}{}_{(\beta)}(\gamma_5\gamma_\mu)^{(\sigma)}{}_{(\delta)} - \\ &-\gamma_\nu{}^{(\alpha)}{}_{(\beta)}\gamma_\mu{}^{(\sigma)}{}_{(\delta)} + \tfrac{1}{2}(\gamma_\nu\sigma_{\varrho\sigma} + 3\sigma_{\varrho\sigma}\gamma_\nu)^{(\alpha)}{}_{(\beta)}(\gamma_\mu\sigma_{\varrho\sigma} + 3\sigma_{\varrho\sigma}\gamma_\mu)^{(\sigma)}{}_{(\delta)}] + \\ &+(\delta^{in}\delta^{jm} + \delta^{im}\delta^{jn} - \delta^{ij}\delta^{nm})[\tfrac{9}{4}(\gamma_5\sigma^{-1}_{\nu\varrho})^{(\alpha)}{}_{(\beta)}(\gamma_5\sigma^{-1}_{\mu\varrho})^{(\sigma)}{}_{(\delta)} - \tfrac{1}{2}(\gamma_5\gamma_\nu)^{(\alpha)}{}_{(\beta)}(\gamma_5\gamma_\mu)^{(\sigma}{}_{(\delta)}] + \\ &+\delta^{ij}\delta^{nm}[\tfrac{9}{8}(\gamma_5\sigma^{-1}_{\nu\varrho})^{(\alpha)}{}_{(\beta)}(\gamma_5\sigma^{-1}_{\mu\varrho})^{(\sigma)}{}_{(\delta)} - (\gamma_\nu\gamma_\varrho)^{(\alpha)}{}_{(\beta)}(\gamma_\mu\gamma_\varrho)^{(\sigma)}{}_{(\delta)} - \tfrac{1}{2}\gamma_\nu{}^{(\alpha)}{}_{(\beta)}\gamma_\mu{}^{(\sigma)}{}_{(\delta)}]\}\,.\end{aligned} \tag{59}$$

The matrix $\hat{B}^{[\nu i(\alpha)][\mu\eta(\sigma)]}_{[j(\beta)][m(\delta)]}$ (59) is so arranged that only the following components of the fields $\omega^{[m(\beta)]}_{[\nu n(\alpha)]}$ enter into the action (54) and into the transformation laws $\delta^*\psi^i_\nu$ (55):

$$\left\{\begin{aligned} &a_\varrho = -\tfrac{1}{4}\,\omega^{[n(\beta)]}_{[\nu n(\alpha)]}(\gamma_5\sigma^{-1}_{\nu\varrho})^{(\alpha)}{}_{(\beta)}\,, \qquad v_\varrho = \tfrac{1}{6}\,\omega^{[n(\beta)]}_{[\nu n(\alpha)]}(\gamma_\nu\gamma_\varrho)^{(\alpha)}{}_{(\beta)}\,,\\ &s = \tfrac{1}{6}\,\omega^{[n(\beta)]}_{[\nu n(\alpha)]}\gamma_\nu{}^{(\alpha)}{}_{(\beta)}\,, \qquad a^{ij}_\varrho = -\tfrac{1}{4}(\delta^{in}\delta^{jm} + \delta^{im}\delta^{jn} - \delta^{ij}\delta^{nm})\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}(\gamma_5\sigma^{-1}_{\nu\varrho})^{(\alpha)}{}_{(\beta)}\,,\\ &p^{ij} = \tfrac{1}{6}(\delta^{in}\delta^{jm} + \delta^{im}\delta^{jn} - \delta^{ij}\delta^{nm})\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}(\gamma_\nu)^{(\alpha)}{}_{(\beta)}\,,\\ &v^{ij}_\varrho = -\tfrac{1}{4}\mathcal{E}^{ij}\mathcal{E}^{nm}\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}\,\sigma_{\nu\varrho}^{-1(\alpha)}{}_{(\beta)}\,,\\ &\omega^{ij}_{\varrho\sigma} = \tfrac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}(\gamma_\nu\sigma_{\varrho\sigma} + 3\sigma_{\varrho\sigma}\gamma_\nu)^{(\alpha)}{}_{(\beta)}\,,\\ &u^{ij} = \tfrac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}\gamma_\nu{}^{(\alpha)}{}_{(\beta)}\,,\\ &t^{ij} = \tfrac{1}{6}\mathcal{E}^{ij}\mathcal{E}^{nm}\,\omega^{[m(\beta)]}_{[\nu n(\alpha)]}(\gamma_5\gamma_\nu)^{(\alpha)}{}_{(\beta)}\,. \end{aligned}\right. \tag{60}$$

The rest of the fields $\omega^{[m(\beta)]}_{[\nu n(\alpha)]}$ fall out of consideration. To be more exact, the free parameters in the matrix $\hat{B}^{[\nu i(\alpha)][\mu j(\beta)]}_{[n(\gamma)][m)\delta)]}$ have been just found from the condition that the fields that hinder the closure of the algebra should be excluded from the theory.

One may make directly that the resulting transformation laws (15)-(28) leads to closing the algebra completely with respect to all the fields involved (both dynamical and auxiliary).