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On a new method to measure the gravitomagnetic field using two orbiting satellites

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Summary. — We describe a new method to obtain the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct detection of the gravitomagnetic field. This method is based on the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II. By this new approach one achieves a measurement of the gravitomagnetic field with accuracy of about 25%, or less, of the Lense-Thirring effect in general relativity.

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1. – The gravitomagnetic field, its invariant characterization and past attempts to measure it

Einstein's theory of general relativity [1, 2] predicts the occurrence of a «new» field generated by mass-energy currents, not present in classical Galilei-Newton mechanics. This field is called the gravitomagnetic field for its analogies with the magnetic field in electrodynamics.

In general relativity, for a stationary mass-energy current distribution $\varrho_m \boldsymbol{v}$, in the weak-field and slow-motion limit, one can write [2] the Einstein equation in the Lorentz gauge: $\Delta \boldsymbol{h} \cong 16\pi\varrho_m \boldsymbol{v}$, where $\boldsymbol{h} \equiv (h_{01}, h_{02}, h_{03})$ are the $(0i)$-components of the metric tensor; $\boldsymbol{h}$ is called the gravitomagnetic potential. For a localized, stationary mass-energy distribution, in the weak-field and slow-motion limit, we can then write: $\boldsymbol{h} \cong -2((\boldsymbol{J} \times \boldsymbol{x})/r^3)$, where $\boldsymbol{J}$ is the angular momentum of the central body. In general relativity, one can also define [2] a gravitomagnetic field $\boldsymbol{H}$ given by $\boldsymbol{H} = \nabla \times \boldsymbol{h}$.

The Lense-Thirring effect is a consequence of the gravitomagnetic field and consists of a tiny perturbation of the orbital elements of a test particle due to the angular momentum of the central body. To characterize the gravitomagnetic field generated by the angular momentum of a body, and the Lense-Thirring effect, and distinguish it from other relativistic phenomena, such as the de Sitter effect, due to the motion of a gyroscope in a static gravitational field, one can give [2,3] a description of the gravitomagnetic field by spacetime-curvature invariants. The pseudoinvariant $^{*}\boldsymbol{R}\cdot\boldsymbol{R}$, that is: $(1/2)\,\varepsilon^{\alpha\beta\sigma\varrho}R_{\sigma\varrho}{}^{\mu\nu}R_{\alpha\beta\mu\nu}$ (where $\varepsilon^{\alpha\beta\sigma\varrho}$ is the Levi-Civita pseudotensor) built the Riemann tensor $\boldsymbol{R}$ and its dual $^{*}\boldsymbol{R}$, gives an invariant characterization of gravitomagnetism since it is nonzero in the field of a central body if and only if the body is rotating. Indeed the invariant $^{*}\boldsymbol{R}\cdot\boldsymbol{R}$ is proportional to the angular momentum of the central body.

Thus, one may describe gravitomagnetism as that phenomenon of nature such that *spacetime curvature is generated by the spin of a body*. This phenomenon has never been measured or detected, until the method described in this paper has been used to directly detect it for the first time.

Several perturbations are due to the gravitomagnetic field $\boldsymbol{H}$.

A test gyroscope precesses with respect to «an asymptotic inertial frame» with angular velocity: $\dot{\boldsymbol{\Omega}} = -(1/2)\,\boldsymbol{H} = [-\boldsymbol{J} + 3(\boldsymbol{J}\cdot\hat{\boldsymbol{x}})\,\hat{\boldsymbol{x}}]/|\boldsymbol{x}|^3$, where $\boldsymbol{J}$ is the angular momentum of the central object. This phenomenon is the «dragging of gyroscopes» or «dragging of inertial frames», of which the gyroscopes define the axes [1, 2, 4].

The orbit of a test particle around a central body with angular momentum $\boldsymbol{J}$ has a secular rate of change of the longitude of the nodes (intersection between the orbital plane of the test particle and the equatorial plane of the central object), discovered by Lense-Thirring (1918) [5]; see eqs. (1) and (3) below.

Similarly, by integrating the equation of motion of a test particle, one can find the formulae for the secular rates of change of the argument of pericenter $\dot{\omega}$, (defining the Lenz vector), eqs. (9) and (10) below, and of the mean orbital longitude $\dot{L}_0$.

Since 1896 many experiments have been discussed and proposed to measure the dragging of inertial frames [2, 4]. Of special interest for their extensive feasibility studies are the NASA Gravity Probe-B experiment, to measure the gravitomagnetic precession of a gyroscope orbiting Earth, and the LAGEOS III experiment. We just briefly review here the main proposals.

In 1896, B. and I. Friedländer tried to measure the dragging effect due to a rapidly rotating, heavy fly-wheel on a torsion balance. In 1904, A. Föppl tried to measure on a gyroscope the dragging effect due to the rotation of Earth, he reached an accuracy of about 2% of the Earth's angular velocity. However, the general relativistic dragging effect on a gyroscope at the surface of Earth (at a US or European latitude) is about $2\cdot 10^{-10}$ of its rotation rate. In 1916, de Sitter calculated the tiny shift of the perihelion of Mercury due to the rotation of the Sun, a particular case of the shift of the pericenter of an orbiting test particle due to the angular momentum of the central body. However, this shift is of the order of $-0.002''$/century, about $5\cdot 10^{-5}$ times smaller than the Mercury standard general relativistic precession of $\cong 43''$/century. In 1918 Lense and Thirring calculated the gravitomagnetic secular perturbations of the moons of various planets, in particular the V moon of Jupiter has a considerable gravitomagnetic secular precession, however the observations do not yet allow separation and measurement of this effect. In 1959 Yilmaz proposed to use polar satellites to detect the gravitomagnetic field, avoiding in such a way the effects due to the nonsphericity of the Earth's gravity field. In 1976 Van Patten and Everitt proposed measuring the Lense-Thirring nodal precession using two drag-free, guided satellites, counterrotating in the same polar plane. The reason for proposing two counterrotating satellites was to avoid the error associated with the determination of the inclination. In 1984, we proposed [6, 7] the LAGEOS III experiment, to detect the gravitomagnetic field by measuring its orbital drag on nonpolar, passive, laser-ranged satellites. We can decompose the fundamental idea [6] of the LAGEOS III experiment into two parts. Position measurements of laser-ranged satellites, of LAGEOS (1976) type (see below), are accurate enough to detect the very tiny effect due to the gravitomagnetic field, the Lense-Thirring precession; and to «cancel out» all the enormous perturbations due to the nonsphericity of the Earth's gravity field, we need a new satellite: LAGEOS III, with inclination supplementary to that of LAGEOS, and with the other orbital parameters, a and e, equal to those of LAGEOS. With the LAGEOS III experiment one may achieve a measurement of the Lense-Thirring effect with accuracy of about 3%, or less. Finally, we have the well known NASA-Stanford Gravity Probe-B, GP-B, experiment [8], to measure the precession of the spin axis of a gyroscope orbiting Earth, with accuracy of 1%, or less.

2. – A new method to detect the gravitomagnetic field using existing data from orbiting satellites

One of the relativistic effects best measurable on the orbital parameters of satellites of LAGEOS type, with $e \ll 1$, is the precession of the nodal lines. For LAGEOS, the observed total nodal precession is $\dot{\Omega}^{\text{obs}}_{\text{LAGEOS}} \cong 126°/\text{year}$, and the Lense-Thirring precession is [2, 6]:

$$\dot{\Omega}^{\text{Lense-Thirring}}_{\text{I}} = \frac{2J}{a_{\text{I}}^3 (1 - e_{\text{I}}^2)^{3/2}} \cong 31 \text{ milliarcsec/year}\,, \tag{1}$$

where $J_{\oplus} \cong 5.9 \cdot 10^{40}\ \text{g cm}^2/\text{s} \cong 145\ \text{cm}^2$ (in geometrized units) is the angular momentum of Earth.

The total nodal precession can be measured on LAGEOS with an accuracy of the order of 0.5 milliarcsec/year.

Unfortunately, the Lense-Thirring precession cannot be extracted from the experimental value of $\dot{\Omega}^{\text{obs}}_{\text{LAGEOS}}$ because of the uncertainty in the value of the classical precession [9]:

$$\dot{\Omega}^{\text{Class}} = -\frac{3}{2} n \left(\frac{R_{\oplus}}{a}\right)^2 \frac{\cos I}{(1 - e^2)^2} \cdot \left\{ J_2 + J_4 \left[\frac{5}{8}\left(\frac{R_{\oplus}}{a}\right)^2 (7 \sin^2 I - 4)\, \frac{1 + (3/2)\, e^2}{(1 - e^2)^2}\right] + \Sigma N_{2n} \times J_{2n} \right\}, \tag{2}$$

where the J_{2n} are the nonnormalized even zonal harmonic coefficients, $J_{2n} \equiv -\sqrt{4n+1}\, C_{2n0}$, the C_{2n0} are the normalized even zonal harmonic coefficients, N_{2n} are the coefficients (in the equation for the nodal rate) of the J_{2n}, $n = 2\pi/P$ is the orbital mean motion, and $R_{\oplus}$ is the Earth's equatorial radius. This classical precession is due to the quadrupole and higher multipole mass moments of Earth, measured by the coefficients C_{2n0}. The orbital parameters n, a and e in formula (2) are determined with sufficient accuracy via the LAGEOS laser ranging [10] and one can determine the average inclination angle I with sufficient accuracy over a long enough period of time (see below). Any other quantity in eq. (2) can be determined or is known with sufficient accuracy, apart from the C_{2n0}. Indeed, the largest uncertainty in the classical precession $\dot{\Omega}^{\text{Class}}_{\text{LAGEOS}}$ arises from the uncertainty in the coefficients C_{2n0}. This uncertainty, relative to C_{20}, is of the order of [11, 12]: $\delta C_{2n0}/C_{20} \sim 10^{-6}$ to 10^{-7}. For C_{20}, this corresponds, from formula (2), to an uncertainty in the nodal precession of about 450 to 45 milliarcsec/year, in addition we have the uncertainty due to the higher C_{2n0} coefficients. Therefore, the uncertainty in the modeling of $\dot{\Omega}^{\text{Class}}_{\text{LAGEOS}}$ is larger than the Lense-Thirring precession.

A solution would be to orbit several high–altitude, laser–ranged satellites, similar to LAGEOS, to measure C_{20}, C_{40}, C_{60}, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero.

A third solution [6, 7] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the *inclination supplementary* to that of LAGEOS. Therefore, «LAGEOS III» should have the following orbital parameters: $I^{\text{III}} = 180° - I^{\text{I}} \cong 70°$, $a^{\text{III}} = a^{\text{I}}$, and $e^{\text{III}} = e^{\text{I}}$. With this choice, since the classical precession $\dot{\Omega}^{\text{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\text{Class}}$ will be equal and opposite for the two satellites. By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$, eq. (1), is independent of the inclination, $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites. Then, by properly combining the measured nodal precessions, we will get the Lense-Thirring precession. Several investigations have been published on the LAGEOS III experiment, together with comprehensive and extensive error analyses, computer simulations of the experiment and «blind tests», performed in joint studies by ASI (Italian Space Agency), NASA, University of Texas at Austin, US Air Force, and several other universities and research centers in the US and Italy [7, 13, 14]. Present error analyses and other studies show a total error in the LAGEOS III gravitomagnetic measurement of about 3%, or less, of the Lense-Thirring effect, over a three-year period [3, 15]. However, this measurement cannot be performed until the LAGEOS III satellite will be constructed and launched.

Let us now describe the new method that is leading [16] to the first detection of the Lense-Thirring effect.

In October 1992, the LAGEOS II satellite (built by Alenia), a copy of LAGEOS, was successfully launched by NASA and by the Italian Space Agency, ASI. The LAGEOS II semimajor axis is $a \cong 12,163$ km, the eccentricity $e \cong 0.014$, and the inclination $I \cong 52.65°$. Similarly to LAGEOS, one of the relativistic effects best measurable on the orbital parameters of LAGEOS II is the precession of the nodal lines. For LAGEOS II, the total nodal precession is: $\dot{\Omega}^{\text{obs}}_{\text{LAGEOS II}} \cong -231°/\text{year}$, and the Lense-Thirring precession is for LAGEOS II:

$$\dot{\Omega}^{\text{Lense-Thirring}}_{\text{II}} = \frac{2J}{a^3_{\text{II}}(1 - e^2_{\text{II}})^{3/2}} \cong 31.5 \text{ milliarcsec/year}\,, \tag{3}$$

However, in spite of the highly accurate gravity field solutions today available, the uncertainties in the Earth's even zonal harmonics do not allow the measurement of the Lense-Thirring effect using the two observable quantities $\dot{\Omega}_{\text{LAGEOS}}$ and $\dot{\Omega}_{\text{LAGEOS II}}$, nodal rates of LAGEOS and LAGEOS II.

Today one of the most accurate Earth's gravity field solutions available is JGM-3, jointly developed by NASA-Goddard and by the Center for Space Research (CSR) of the University of Texas at Austin. Furthermore, a new improved Earth gravity field solution has been recently developed in US. Let us first analyse the gravity field solution JGM-3. The values of the main five even zonal harmonic coefficients C_{2n0} of the

JGM-3 solution are:

$$(4)\qquad \begin{cases} C_{20} \cong -484.1653754 \cdot 10^{-6}\,, \\ C_{40} \cong 0.5397771 \cdot 10^{-6}\,, \\ C_{60} \cong -0.1496716 \cdot 10^{-6}\,, \\ C_{80} \cong 0.0491180 \cdot 10^{-6}\,, \\ C_{10,0} \cong 0.0541304 \cdot 10^{-6}\,. \end{cases}$$

The corresponding estimated errors associated with each of these zonal harmonics coefficients are of the order of:

$$(5)\qquad \begin{cases} \delta C_{20} \cong 0.05 \cdot 10^{-9}\,, \\ \delta C_{40} \cong 0.1 \cdot 10^{-9}\,, \\ \delta C_{60} \cong 0.2 \cdot 10^{-9}\,, \\ \delta C_{80} \cong 0.2 \cdot 10^{-9}\,, \\ \delta C_{10,0} \cong 0.2 \cdot 10^{-9}\,. \end{cases}$$

Now, there is a basic problem to evaluate if these estimated errors in the spherical harmonic coefficients of the Earth's gravity field solution are consistent with the *true* errors in the value of these coefficients. To see where the main errors are likely to be concentrated in the estimated C_{2n0} coefficients one might take the difference between two different gravity field solutions. This method has been, for example, applied by Lerch *et al.* [17] to the solution GEMT-2. Let us then consider [11] the older gravity field solution GEMT-3S, computed without altimeter and surface gravity information, using exclusively satellite tracking data up to 1988, without LAGEOS II data. The values of the main five even zonal harmonic coefficients, C_{2n0}, of the GEMT-3S solution are:

$$(6)\qquad \begin{cases} C_{20} \cong -484.1650994 \cdot 10^{-6}\,, \\ C_{40} \cong 0.5395212 \cdot 10^{-6}\,, \\ C_{60} \cong -0.1495135 \cdot 10^{-6}\,, \\ C_{80} \cong 0.0488832 \cdot 10^{-6}\,, \\ C_{10,0} \cong 0.0540650 \cdot 10^{-6}\,. \end{cases}$$

The corresponding estimated errors associated with each of these zonal harmonic coefficients are of the order of:

$$(7)\qquad \begin{cases} \delta C_{20} \cong 0.2 \cdot 10^{-9}\,, \\ \delta C_{40} \cong 1 \cdot 10^{-9}\,, \\ \delta C_{60} \cong 1 \cdot 10^{-9}\,, \\ \delta C_{80} \cong 2 \cdot 10^{-9}\,, \\ \delta C_{10,0} \cong 2 \cdot 10^{-9}\,. \end{cases}$$

Let us then compare the differences ΔC_{2n0} between the corresponding even zonal harmonic coefficients of the solutions GEMT-3S and JGM-3, and the estimated errors given by GEMT-3S and JGM-3,

$$(8)\quad \begin{cases} |\Delta C_{20}| \cong 0.276\cdot 10^{-9}\,; & \delta C_{20}^{\mathrm{GEMT-3}S} \cong 0.2\cdot 10^{-9}\,; & \delta C_{20}^{\mathrm{JGM-3}} \cong 0.05\cdot 10^{-9}\,,\\ |\Delta C_{40}| \cong 0.256\cdot 10^{-9}\,; & \delta C_{40}^{\mathrm{GEMT-3}S} \cong 1\cdot 10^{-9}\,; & \delta C_{40}^{\mathrm{JGM-3}} \cong 0.1\cdot 10^{-9}\,,\\ |\Delta C_{60}| \cong 0.158\cdot 10^{-9}\,; & \delta C_{60}^{\mathrm{GEMT-3}S} \cong 1\cdot 10^{-9}\,; & \delta C_{60}^{\mathrm{JGM-3}} \cong 0.2\cdot 10^{-9}\,,\\ |\Delta C_{80}| \cong 0.235\cdot 10^{-9}\,; & \delta C_{80}^{\mathrm{GEMT-3}S} \cong 2\cdot 10^{-9}\,; & \delta C_{80}^{\mathrm{JGM-3}} \cong 0.2\cdot 10^{-9}\,,\\ |\Delta C_{10,0}| \cong 0.0654\cdot 10^{-9}\,; & \delta C_{10,0}^{\mathrm{GEMT-3}S} \cong 2\cdot 10^{-9}\,; & \delta C_{10,0}^{\mathrm{JGM-3}} \cong 0.2\cdot 10^{-9}\,. \end{cases}$$

However, in order to measure the Lense-Thirring effect, one has to investigate how the errors in the C_{2n0} coefficients propagate in the two observable quantities, the nodal rates of LAGEOS and LAGEOS II, and compare these nodal errors with the corresponding size of the Lense-Thirring effect on the nodes. Thus, for the nodes of LAGEOS and LAGEOS II, one has (in units of $\dot{\Omega}_{\mathrm{I}}^{\mathrm{L\text{-}T}}$ and $\dot{\Omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$):

	$\delta\dot{\Omega}_{\mathrm{I}}/\dot{\Omega}_{\mathrm{I}}^{\mathrm{L\text{-}T}}$ due to JGM3 estimated errors	$\delta\dot{\Omega}_{\mathrm{I}}/\dot{\Omega}_{\mathrm{I}}^{\mathrm{L\text{-}T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 1.5	~ 8.3
δC_{40}	~ 1.5	~ 3.8
δC_{60}	~ 0.76	~ 0.6
δC_{80}	~ 0.06	~ 0.07
$\delta C_{10,0}$	~ 0.04	~ 0.01

and

	$\delta\dot{\Omega}_{\mathrm{II}}/\dot{\Omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to JGM3 estimated errors	$\delta\dot{\Omega}_{\mathrm{II}}/\dot{\Omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 2.7	~ 15
δC_{40}	~ 0.51	~ 1.3
δC_{60}	~ 1.2	~ 0.92
δC_{80}	~ 0.28	~ 0.33
$\delta C_{10,0}$	~ 0.07	~ 0.02

From these uncertainties in the nodal rates of LAGEOS and LAGEOS II, both by considering the estimated errors and the differences between the two Earth gravity field solutions (that should provide upper limits to the true errors in the modeling of the C_{2n0}), it is manifest that the dominant error sources are due to the uncertainties in C_{20} and C_{40}, and in part to the uncertainty in C_{60}. However, much smaller errors, compared to the Lense-Thirring effect, are due to the higher even zonal harmonics. Therefore, in order to get a measurement of the Lense-Thirring effect one needs at least to eliminate the errors arising from C_{20} and C_{40}. This cancellation must also include the uncertainties in the temporal and seasonal variations of the even zonal harmonics: to get a measurement of the Lense-Thirring effect one needs at least to eliminate the uncertainties in $\dot{C}_{20}$. Indeed, it is necessary to take into account that if one uses the JGM-3 covariance matrix, by considering the correlations of the errors in the even zonal harmonics higher than C_{40}, the estimated error in the measurement of the Lense-Thirring effect is further reduced (see below). Then, summarizing, one has at least three unknowns: δC_{20}, δC_{40} and Lense-Thirring effect, but only two observable quantities: $\dot{\Omega}_{\mathrm{LAGEOS}}$ and $\dot{\Omega}_{\mathrm{LAGEOS\,II}}$. At this point it is important to observe that the orbital eccentricity of LAGEOS II is larger than the orbital eccentricity of LAGEOS. Indeed, in regard to the perigee, the observable quantity is $ea\dot{\omega}$, where e is the orbital eccentricity of the satellite. Thus, since the LAGEOS eccentricity is about $4 \cdot 10^{-3}$, the perigee precession, $\dot{\omega}$, is a quantity extremely difficult to be measured for LAGEOS. However, the LAGEOS II orbit is more eccentric, its eccentricity is about 0.014, and, in addition, the Lense-Thirring effect on the LAGEOS II perigee is almost twice larger, in magnitude, than the Lense-Thirring effect on the LAGEOS perigee. Indeed, the argument of pericenter (perigee in our analysis), ω, of a test particle, that is the angle on the orbital plane measuring the departure of the pericenter from the equatorial plane, has a secular perturbation due to the gravitomagnetic field [2]. One has for LAGEOS I:

$$\dot{\omega}_{\mathrm{I}}^{\text{Lense-Thirring}} = \frac{-6\,J}{a_{\mathrm{I}}^3(1-e_{\mathrm{I}}^2)^{3/2}} \cos I_{\mathrm{I}} \cong 32 \text{ milliarcsec/year} \tag{9}$$

and for LAGEOS II:

$$\dot{\omega}_{\mathrm{II}}^{\text{Lense-Thirring}} = \frac{-6\,J}{a_{\mathrm{II}}^3(1-e_{\mathrm{II}}^2)^{3/2}} \cos I_{\mathrm{II}} \cong -57 \text{ milliarcsec/year}\,. \tag{10}$$

Thus, for example, if one is able to determine the quantity $ea\omega$ with an accuracy, over several orbits, of about 2 cm (indeed, for LAGEOS II, the r.m.s. of the residuals of the best orbital fits is, over several orbits, of the order of a few cm), one can determine $a\omega$ with an accuracy of about 140 cm at the LAGEOS II altitude, corresponding to about 24 milliarcsec. Then, since the Lense-Thirring drag of the LAGEOS II argument of the perigee is -57 milliarcsec/year, one can achieve, in determining $a\dot{\omega}$ in one year period, an accuracy of the order of 40% of the Lense-Thirring effect on the LAGEOS II perigee.

In summary, since the Lense-Thirring drag is almost twice larger, in magnitude, on the perigee of LAGEOS II than on the perigee of LAGEOS and since the orbital eccentricity of LAGEOS II is much larger than the eccentricity of LAGEOS, in order to determine the «frame-dragging» effect, in addition to the nodes of LAGEOS and LAGEOS II, one can also use the LAGEOS II perigee. By using the perigee one introduces an observational error much larger than in case of the nodes. However, this «new» observable quantity, the perigee of LAGEOS II, allows to eliminate errors, due to the uncertainties in the gravity field, that are much larger than the observational errors in the LAGEOS II perigee. Let us investigate how the errors in the C_{2n0} propagate on the perigee rate of LAGEOS II and compare these perigee errors with the size of the Lense-Thirring effect on the perigee. The observed perigee precession is for LAGEOS II: $\dot{\omega}_{\mathrm{LAGEOS\,II}} \cong 160°/\mathrm{year}$, and the classical perigee precession is:

$$(11)\qquad \dot{\omega}^{\mathrm{Class}} = -\frac{3}{4}\,n\left(\frac{R_\oplus}{a}\right)^2 \frac{1-5\cos^2 I}{(1-e^2)^2}\,J_2 - $$

$$-[[15nR_\oplus^4(108+135e^2+208\cos(2I)+252e^2\cos(2I)+196\cos(4I)+$$

$$+189e^2\cos(4I))]/(1024a^4(1-e^2)^4)]\,J_4+\Sigma P_{2n}\times J_{2n}\,,$$

where the P_{2n} are the coefficients (in the equation for the perigee rate) of the nonnormalized even zonal harmonics $J_{2n} \equiv -\sqrt{4n+1}\,C_{2n0}$. Thus, for the perigee of LAGEOS II, one has (in units of $\dot{\omega}_{\mathrm{II}}^{\text{Lense-Thirring}}$):

	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to JGM3 estimated errors	$\delta\dot{\omega}_{\mathrm{II}}/\dot{\omega}_{\mathrm{II}}^{\mathrm{L\text{-}T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 1.1	~ 5.9
δC_{40}	~ 2.1	~ 5.3
δC_{60}	~ 0.41	~ 0.32
δC_{80}	~ 0.68	~ 0.8
$\delta C_{10,0}$	~ 0.22	~ 0.07

From these uncertainties in the perigee rate of LAGEOS II, similarly to what inferred for the nodal rates, it is manifest that the dominating error sources are due to the uncertainties in C_{20} and C_{40}.

Thus, summarizing, we have now the three unknowns δC_{20}, δC_{40} and Lense-Thirring effect, and the three observable quantities $\dot{\Omega}_{\mathrm{LAGEOS}}$, $\dot{\Omega}_{\mathrm{LAGEOS\,II}}$, and $\dot{\omega}_{\mathrm{LAGEOS\,II}}$.

The main unmodeled part of the LAGEOS I nodal rate, due to the uncertainties in the even zonal harmonics, to the errors in the value of the orbital parameters (mainly the inclination), and including the Lense-Thirring effect (to be determined), is:

$$(12)\qquad \delta\dot{\Omega}_I = (-9.3\cdot 10^{11})\times\delta C_{20} - (4.62\cdot 10^{11})\times\delta C_{40} + \Sigma N_{2n}\times\delta C_{2n0} + 6\times\delta I_I + 31\mu\,,$$

where $\delta\dot{\Omega}$ is in units of milliarcsec/year, and δI in milliarcsec. This formula shows the main error sources in the calculated nodal rate (apart from the errors due to tides and to nongravitational perturbations; see below). In this formula the first two contributions are due to the uncertainties δC_{20} and δC_{40}, we then have the error due to the uncertainties in the higher even zonal harmonics δC_{2n0} (with $2n \geq 6$), and the error due to the uncertainties in the determination of the inclination δI_I. In this formula we have also included the Lense-Thirring [2] parameter μ, by definition 1 in general relativity: $\mu^{\mathrm{GR}} \equiv 1$, that, if not incorporated in the modeling of the orbital perturbations, will affect the orbital residuals. One can write a similar expression for the node of LAGEOS II:

$$(13)\qquad \delta\dot{\Omega}_{\mathrm{II}} = (17.17\cdot 10^{11})\times\delta C_{20} +$$

$$+(1.68\cdot 10^{11})\times\delta C_{40} + \Sigma N''_{2n}\times\delta C_{2n0} + 5.3\times\delta I_{\mathrm{II}} + 31.5\mu$$

and for the perigee of LAGEOS II:

$$(14) \qquad \delta\dot{\omega}_{II} = (-11.87\cdot 10^{11}) \times \delta C_{20} - $$
$$-(11.79\cdot 10^{11}) \times \delta C_{40} + \Sigma P_{2n} \times \delta C_{2n0} - 16 \times \delta I_{II} - 57\mu \, .$$

Thus, using these three observable quantities, we can solve for μ and eliminate δC_{20} and δC_{40}:

$$(15) \qquad \delta\dot{\Omega}_I + k_1\,\delta\dot{\Omega}_{II} + k_2\,\delta\dot{\omega}_{II} = \mu(31 + 31.5k_1 - 57k_2) \text{ milliarcsec/year} +$$
$$+[\text{other errors sources } (\delta C_{60}, \delta C_{80}, \ldots, \delta I_I, \delta I_{II}, \ldots)],$$

where k_1 and k_2 are:

$$k_1 = (\dot{\Omega}_4^{I}\dot{\omega}_2^{II} - \dot{\Omega}_2^{I}\dot{\omega}_4^{II})/(\dot{\Omega}_2^{II}\dot{\omega}_4^{II} - \dot{\Omega}_4^{II}\dot{\omega}_2^{II}) \cong 0.295$$

and

$$(16) \qquad k_2 = (\dot{\Omega}_4^{II}\dot{\Omega}_2^{I} - \dot{\Omega}_2^{II}\dot{\Omega}_4^{I})/(\dot{\Omega}_2^{II}\dot{\omega}_4^{II} - \dot{\Omega}_4^{II}\dot{\omega}_2^{II}) \cong -0.35 \, ,$$

where $\dot{\Omega}_2$, $\dot{\Omega}_4$, $\dot{\omega}_2$, and $\dot{\omega}_4$, are the coefficients of C_{20} and C_{40} in the equations for nodal rates, $\dot{\Omega}$, and perigee rate, $\dot{\omega}$.

Finally, we need to consider the other error sources and the error budget corresponding to this new method to measure the Lense-Thirring effect. It is necessary and important to observe that by using this new method one eliminates not only the error in the static part of C_{20} and C_{40}, but also the errors arising from the unknown, mismodeled or unmodeled, temporal variations in C_{20} and C_{40}, including their tidal variations, and their secular and seasonal variations.

If we combine the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II according to formula (15), and by using the JGM-3 covariance matrix we have: Error $(\delta\dot{\Omega}_I + k_1\delta\dot{\Omega}_{II} + k_2\delta\dot{\omega}_{II})^{C_{2n0} \geq C_{60}} \cong 17\% \times (60.2 \text{ milliarcsec/year})$, that is $(\delta\mu)^{C_{2n0} \geq C_{60}} \cong 17\%\mu$.

In regard to tidal perturbations and other temporal variations in the Earth's gravity field, we have used and adapted to the present analysis previous results and studies of the LAGEOS III experiment [7, 13, 14]; of course tidal perturbations have different effects on the orbital elements of LAGEOS II and LAGEOS III. One of the main tidal errors is due to the 18.6 year tidal perturbation of the satellites orbits associated with the period of the Moon's node. A large part of the error associated with this 18.6 year tide is due to the uncertainty, δC_{20}, in its $l = 2, m = 0$ component, however any tidal error in C_{20} and C_{40}, and any error due to other unmodeled temporal variations in C_{20} and C_{40}, including their secular and seasonal variations, is eliminated using the combination (15). Thus, we obtain a preliminary upper limit for the error due to tidal perturbations and temporal variations in C_{2n0}: $\delta\mu^{\text{tides}} \lesssim 5\%\mu$.

Then, we have to consider the nongravitational perturbations, they are particularly effective on the perigee. The nongravitational perturbations include direct solar radiation pressure, Earth's albedo, Yarkovsky anisotropic thermal radiation, Rubincam effect (anisotropic re-radiation of Earth's infrared radiation absorbed by the LAGEOS retro-reflectors), particle drag and errors due to the estimated value of the satellite reflectivity and estimated 15-day along-track acceleration.

With regard to the mismodeling of the nodal rates due to nongravitational perturbations, previous extensive error analyses and results for the LAGEOS III experiment [7, 13, 14] have shown a total error of less than 3%. Then, we may substantially apply these results, for the nongravitational perturbations, to the nodal rates of LAGEOS and LAGEOS II. In regard to the effects of neutral and charged particle drag on the perigee, one has to consider that any drag-like force of the type $\vec{F} \sim v\vec{v}$ gives a null secular contribution to the perigee rate; this result also applies to any unmodeled, constant, 15-day acceleration. In regard to the radiation pressure, the unmodeled effects on the nodes of LAGEOS and LAGEOS III, and also LAGEOS II, have been calculated to be small. However, the effect of unmodeled radiation pressure may be large on the perigee. The radiation pressure effect is large on the perigee because of the special dependence of the perigee rate on the transversal and radial components of this perturbation. In this preliminary error analysis we have assumed an error of about 5% in the value of the reflectivity of LAGEOS II (after estimating it by best fits of the observations), and an error of about 0.1% in the value of the solar constant. Indeed, the radiation pressure effects on a satellite have a unique and characteristic signature, one can thus separate them from the other perturbations and one can accurately estimate and then adjust the reflectivity to best fit the observations. Furthermore, whereas the Lense-Thirring effect on the perigee rate is secular, the direct radiation pressure perturbations of the perigee are nonsecular, they are periodical, on the perigee, with known periods. Then, by choosing a long enough period of observation, one can average to a small number the direct radiation pressure perturbations. In addition, by identifying the characteristic frequencies of the radiation pressure perturbations on the perigee one can remove, over a long enough period of observation, the unmodeled radiation pressure effects on the perigee with these typical frequencies. Then, we have simulated the orbit of LAGEOS II corresponding to different cases, each with a different value of the reflectivity of the satellite according to the above-mentioned uncertainties. Then, by removing the main direct radiation pressure effects with periods of less than 3 years we have estimated a total error of the gravitomagnetic parameter μ of less than $20\% \times 1/3 \sim 7\%$; the factor 1/3 originates in formula (15), where the perigee rate contributes to μ by about 1/3 only.

Then, in conclusion, the effect of the nongravitational perturbations on the combination (15) should not exceed 10%; that is $\delta\mu^{\text{nongrav}} \lesssim 10\%\mu$.

The order of magnitude of the observational errors (one can obtain the accurate figure by performing the analysis with the real data) may be estimated by observing that the uncertainty in the determination of the orbit, indicated by the r.m.s. of the residuals of the orbital fits, is of the order of 3 cm, for a 15-day period; this corresponds to an uncertainty of the order of 0.5 milliarcsec in the nodal longitude, and of the order of $0.5/e \sim 35$ milliarcsec in the argument of perigee of LAGEOS II. Then, we may estimate the uncertainty due to observational errors in the combination (15) to be: $\delta\mu^{\text{obs}} \lesssim 10\%\mu$, over about 3 years.

Finally, one has to evaluate another type of error that we call here the «imprint» of the Lense-Thirring effect. The Earth's gravity field solution JGM-3 has been obtained by using the observations of several Earth's satellites and by using a set of models and parameters to describe the orbital perturbations, including the a priori, theoretical, general relativistic value of the Lense-Thirring effect. Thus, the gravitational field solution JGM-3, that we use to measure μ, contains some kind of «imprint» of the a priori hypothesized, theoretical value of the Lense-Thirring effect. In other words, even in the case that there is no Lense-Thirring effect in nature, the Earth's gravity field solution JGM-3 would contain in the even zonal harmonic coefficients an «imprint» of the hypothesized, theoretical value of the Lense-Thirring effect. This conceivable error source has been evaluated by performing several simulations testing the influence of the Lense-Thirring effect on the estimation of the Earth's gravity field. The result of these simulations has shown that, in the event of no Lense-Thirring drag in nature, with regard to the effects on the orbits of LAGEOS and LAGEOS II, most of the a priori value of the Lense-Thirring effect is absorbed in the first 3 even zonal harmonic coefficients, and mainly in C_{20} and C_{40}, a minor part is absorbed in the higher harmonic coefficients, and part of it is absorbed in the other parameters usually estimated and adjusted (initial conditions, polar motion, ...). Thus, using the new method described in this paper, by eliminating every error contained in the first two even zonal harmonics, C_{20} and C_{40}, there is a small influence on the combination (15), of nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II, due to the Lense-Thirring «imprint» in the harmonics with $2n \geq 6$. Indeed, even in the worst possible, unrealistic, case, when nothing is allowed to be estimated and adjusted, there is an error induced by this «imprint» effect, contained in the C_{2n0} with $2n \geq 6$, of only 10% of the total effect (15). However, this «imprint» effect is generally very small in the other performed tests, not exceeding a few percent of the Lense-Thirring combination (15). Then $\delta\mu^{\mathrm{LT\,imprint}} \lesssim 10\%\mu$.

In conclusion, by the use of the new method to measure the Lense-Thirring effect described in this paper, by these preliminary analyses, we have estimated the total error in the determination of the parameter μ to be

$$\delta\mu \lesssim 25\%\mu \,. \tag{17}$$

A comprehensive error analysis and error budget will be the subject of a following paper.

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