Res. Popul. Ecol. 39(2), 1997, pp. 139–148.

Researches on POPULATION ECOLOGY Original paper

Optimal Age-Dependent Sustainable Harvesting of Natural Resource Populations: Sustainability Value

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Abstract. We studied the optimal age-dependent harvesting of a natural resource population that achieves a maximum income under the constraint of sustainability, i.e. the reproductive adults numbers must exceed a given minimum. The resource species is assumed to be semelparous (a single reproduction over a life). The economic value and natural mortality coefficient can vary with age. The optimal age-dependent harvesting under the sustainability constraint is obtained using Pontryagin's maximum principle. The constraint of resource sustainability can be treated as an additional term measured in the same units as economic income. Specifically, three terms: (1) current harvesting value, (2) future harvesting value, and (3) sustainability value, are calculated for each age, and the resources should be harvested at the maximum rate when their current harvesting value is greater than the sum of future harvesting value and sustainability value, and should not be harvested otherwise. Numerical analyses of several cases demonstrated that the optimal harvesting schedule depends critically on the natural mortality coefficient and the functional form of the economic value of the resource.

Key words: age-dependent harvesting, maximum principle, semelparity, sustainability.

Introduction

A sustainable management of natural resources, such as commercial fishery resource or game mammals, is an important issue closely related to both conservation practice and population ecology in general. Traditionally, harvesting policy that achieves an optimal economic income has been studied (Clark 1985). Due to the growth of individuals and population dynamics, an optimal harvesting policy is obtained by considering the tradeoffs of the immediate income by a higher harvesting rate and the future expected income by a lower harvesting rate. Typically an inter-generation tradeoff exists between harvesting a recent resource generation and recruitment of the following generations. Such a tradeoff over different points of time can be dealt with using techniques of dynamic optimization, such as dynamic programming or Pontryagin's maximum principle. Reed (1974, 1979) studied optimal policy with fluctuating recruitment by using dynamic programming, and found that a constant escapement policy maximized the long term fishery yield. Clark and Kirkwood (1984) analyzed optimal policy using the maximum principle, considering both fluctuation and estimation errors in recruitment. Their numerical solution showed that escapement should increase with estimated recruitment.

On the other hand, within a generation, there is also a tradeoff between harvesting resources of different age classes. Since the abundance of resources as measured in number declines with age due to mortality, more resources of early stages are available for harvesting than of older stage resources. On the other hand, harvesting resources of early stages would reduce the abundance of older stages that are often more valuable. Clark (1990) analyzed the optimal age-dependent fishery problem so as to maximize the economic income within a generation using the maximum principle. However such a decision based on the economics of the current generation can be quite

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problematical, because nonsustainable resource use, exemplified by North Pacific Whale fishing early this century, would deprive the future generations of the opportunity of using it. We must not apply the economic choice theory to harvesting policy based only on intra-generation considerations if the decision has such a strong and one-sided effect on future generations.

It is often assumed that the value of future income is discounted by a negative exponential function of time to consider an alternative use of investment funds in other means (Clark 1976). If the growth rate of the resource population is smaller than the discounting rate, then catching all the resources immediately becomes the recommended policy. This also occurs when the economic value of the resource population is likely to decline in the future. To prevent such situations, we need to consider sustainability of the resource population as a separate constraint in management problems, rather than an outcome of economic decision making of the current generation.

With consideration to long term population dynamics, several authors have studied age-dependent harvesting policies. Beddington and Taylor (1973) analyzed age-dependent harvesting strategy with iteroparous species using a multi-cohort model. Reed (1980) analyzed a multi-cohort model including density dependent survival by using nonlinear programing. Getz and Haight (1989) developed Reed's study, who had also analyzed the optimal harvesting policy of an age-structured iteroparous species population. Those models are useful in numerically calculating harvesting policy for several age classes. Nevertheless, these are neither adequate to analyze more detailed management policy, nor for providing a general theoretical perspective on the relationship of sustainability and economic values.

In this paper, we examine how the optimal harvest policy should be affected by a sustainability constraint. To clarify the concept, we study a simplest case in which the resource species is semelparous (single reproduction) with perfectly synchronized life cycle (like annual animals), in which the harvesting intensity at each age can be controlled easily. Alternatively the resource population may include individuals of different ages, but using different gears or locations, the age-specific harvesting policy can be realized relatively easily. The economic value (price or size) of an individual and the natural mortality rate are given as functions of age. The number of spawners at the terminal age is constrained to exceed a minimum level to maintain the next generation. The study showed that the optimal harvesting schedule is strongly affected by age-dependence of mortality and economic value.

Optimal harvesting under a sustainability constraint

Pontryagin's maximum principle

We consider a semelparous resource species which reproduces only at the end of its lifetime. Harvesting is assumed to be able to distinguish between each age cohort of the resource taken, an assumption most applicable to annual fishes, without a generation overlap. However, if characteristics of the resource differ with age (e.g. size, behavior or habitat), the model can be applied to resources with overlapping generations. A further assumption is that catch intensity can be precisely controlled throughout the life cycle of the resource species. For example, in aquaculture situations, such an assumption can be satisfied. Fisheries of Pacific salmon in North America or Ayu in Japan may satisfy the above situation.

Both the economic value (e.g. price) and natural mortality coefficient of an individual resource depend upon its age, t, denoted by $V(t)$ and $M(t)$, respectively. Given these functions, the harvesting policy can be chosen, expressed in terms of a harvesting mortality coefficient of the resource at each age t, denoted by $F(t)$. There is a maximum value, F_{MAX}, and the harvesting mortality satisfies: $0 \leqq F(t) \leqq F_{MAX}$.

The amount of resource at age t is denoted by $N(t)$. If the initial recruitment is N_0, $N(t)$ is represented by

$$N(t) = N_0 e^{-\int_0^t (F(s)+M(s))ds}. \tag{1}$$

The total economic yield of the harvesting over the resource life cycle is

$$\Phi(T) = \int_0^T V(s)F(s)N(s)ds, \tag{2}$$

where T is the terminal age of the individual. In the present model, for simplicity, the cost of the harvesting is not considered. Immediately before terminal age T, the spawners reproduce the next generation. The optimal harvesting schedule is that which maximizes Eq. (2).

On the other hand, to maintain the resource population, sufficient spawners must be left to produce a recruitment more than or equal to the initial population size N_0. The number of these spawners must be larger than $N_T{}^*$, which is determined by the initial population size N_0 and the spawner-recruitment relationship:

$$N(T) = N_0 e^{-\int_0^t (F(s)+M(s))ds} \geq N_T{}^*. \tag{3}$$

The optimal harvesting schedule $F(t)$ is to maximize Eq. (2) under the constraint (3). Eq. (3) may be called the *sustainability constraint*.

This maximization problem can be solved by using Pontryagin's maximum principle (Pontryagin et al. 1962). Pontryagin's maximum principle has been applied to the study of lifetime strategies (King and Roughgarden 1982a, b; Chiariello and Roughgarden 1984; Iwasa and Roughgarden 1984). Optimal energy allocation between growth and reproduction of fish species have been well studied using this technique (Kitahara et al. 1987; Hiyama et al. 1988; Hiyama and Kitahara 1993). The optimal age-dependent harvesting problem can also be solved by using a similar approach.

Hamiltonian of this problem is

$$\begin{aligned} H &= \dot{\Phi} + \lambda(t)\dot{N} \\ &= [\{V(t)-\lambda(t)\}F(t)-\lambda(t)M(t)]N(t), \end{aligned} \tag{4}$$

in which $\lambda(t)$ is a costate variable. From the maximum principle, the optimal harvesting $F^*(t)$ maximizes Eq. (4) for each value of t. If $V(t)$ is smaller than $\lambda(t)$, $F^*(t)=0$. Hence when $V(t)$ is greater than $\lambda(t)$, $F^*(t)$ is the maximum value F_{MAX}. However, if $V(t)$ is equal to $\lambda(t)$, $F^*(t)$ can not be determined uniquely and a singular control must be considered in which the coefficient of $F(t)$ vanishes over some time interval (but singular control is in fact non-existent in the present model as discussed below). At this point, the optimal harvesting schedule is

$$\begin{aligned} F^*(t) &= 0 && \text{if} && V(t)<\lambda(t), \\ &F_{MAX} && \text{if} && V(t)>\lambda(t), \\ &\text{and in between} && \text{only if} && V(t)=\lambda(t). \end{aligned} \tag{5}$$

The dynamics of $\lambda(t)$ are

$$\begin{aligned} \frac{d\lambda}{dt} &= -\frac{\partial H}{\partial N} \\ &= -\{V(t)-\lambda(t)\}F(t)+\lambda(t)M(t). \end{aligned} \tag{6}$$

Integration of Eq. (6) from t to T with the terminal condition $\lambda(T)=\lambda_T$ gives

$$\lambda(t)=\int_t^T V(s)F(s)e^{-\int_t^s (F(s')+M(s'))ds'}ds+\lambda_T e^{-\int_t^T (F(s)+M(s))ds}. \tag{7}$$

If there is no constraint on spawner number, the extreme point at T is free and λ_T becomes 0. In such a case, the second term of Eq. (7) vanishes, and $\lambda(t)$ is the same as the first term of Eq. (7). This term is an integral of the product of the current harvesting value $V(t)$ and harvesting coefficient $F(t)$ that is discounted by harvesting and natural mortality. This term is similar to Fisher's reproductive value of an organism at age t, in which the maternity of an individual is integrated with a discounting factor using the population growth rate and individual mortality. The reproductive value expresses the value of an individual, in relation to its contribution to future generations. Thus, the first term of Eq. (7) can be considered to express the value of an individual, which is an expected future income within the resource life cycle. Hereafter, $V(t)$ and the first term of $\lambda(t)$ are referred to as the 'current harvesting value' and the 'future harvesting value' of an individual resource at time t, respectively.

When the spawner number is not constrained ($\lambda_T=0$), the harvesting coefficient $F^*(t)$ can be determined from the current and future harvesting values. According to Eq. (5), harvesting should be stopped ($F(t)^*=0$) when the current harvesting value is smaller than the future harvesting value ($V(t)<\lambda(t)$). On the other hand, harvesting activity should be at a maximum ($F(t)^*=F_{MAX}$) if the current harvesting value exceeds the future harvesting value ($V(t)>\lambda(t)$).

The spawner number is partly constrained by the inequality, the number of spawners being any value greater than or equal to N_T^* ($N(T)\geq N_T^*$). When the optimal path is above the boundary ($N(T)>N_T^*$), λ_T must be equal to 0 as it is not constrained. If instead the equality of the boundary condition is satisfied in the optimal trajectory ($N(T)=N_T^*$), λ_T must be positive and the value is determined in the processes determining the optimal harvesting trajectory. Hence either of the two cases, ($N(T)>N_T^*$ and $\lambda_T=0$) or ($N(T)=N_T^*$ and $\lambda_T>0$), can hold.

When λ_T is not equal to 0, according to Eq. (7), $\lambda(t)$ is represented by the sum of the future harvesting value and λ_T with a certain discount coefficient. It seems, therefore, that the future harvesting value is supplemented by a value related to the spawner constraint. According to Eqs.(5) and (7), when $t=T$, the harvesting coefficient is determined from a comparison between the current harvesting value $V(t)$ and λ_T, suggesting that λ_T can be regarded as the economic value of a spawner left at the terminal age. On the other hand, when $t<T$, λ_T is discounted by the sum of natural and harvesting mortalities from t to T. This suggests that a younger resource has a smaller potential contribution as a spawner, because of the greater likelihood of its death (natural- or harvesting-mortalities induced) before reaching reproductive age. Thus, the second term of Eq. (7) can be regarded as the expected contribution of a resource to the economic value of a spawner. Hereafter, the term is referred to as the 'sustainability value' indicating contribution to spawner value.

The costate variable $\lambda(t)$ can now be considered to represent the expected future value of a resource, being the sum of the future harvesting value and the sustainability value. According to Eq. (7), the harvesting coefficient is determined by comparison between the current harvesting value and the expected future value. That is, the harvesting should be active, only if

(Current Harvesting Value) >

(Future Harvesting Value) + (Sustainability Value). (8)

When the condition (8) is not satisfied, the harvesting should be stopped. If a large number of spawners must be left, the economic value of a spawner (λ_T) becomes high and harvesting activity should be suppressed.

With regard to the possibility of singular control, if $V(t)$

coincides with $\lambda(t)$, the solution $F^*(t)$ cannot be determined uniquely. At this point, the existence of singular control in which $V(t)$ is equal to $\lambda(t)$ over a certain period, must be examined. According to the Appendix, the control variable $F(t)$ is not included in the singular solution. Thus, we can not purposely trace the singular control, which implies that the latter is not generally included in the optimal control solution.

Numerical analysis

If functions $V(t)$ and $M(t)$ are given, the harvesting policy maximizing total yield can be obtained by numerical analysis. In this, the dynamics of variables ($N(t)$ and $\lambda(t)$) are traced based on Euler's method, allowing determination of the optimal harvesting coefficients at each individual age.

The procedure is started from the initial resource age 0, at which the value of $N(0)$ is already determined. On the other hand, the value of $\lambda(0)$ is chosen as an arbitrary positive number, which will be determined by trial and error as explained below: When $\lambda(t)$ is given, according to Eq. (5), $F(t)$ can be determined by comparison between $V(t)$ and $\lambda(t)$. Next, $N(t+\Delta t)$ and $\lambda(t+\Delta t)$ can be derived approximately from

$$N(t+\Delta t)=N(T)-\Delta t N(t)(F(t)+M(t)), \tag{9a}$$

$$\lambda(t+\Delta t)=\lambda(t)+\Delta t\frac{d\lambda}{dt}(t), \tag{9b}$$

respectively. By iteration of this calculation, $N(t)$, $\lambda(t)$ and $F(t)$ can be determined from the initial age $t=0$ to the terminal age $t=T$.

Since the extremum point at age T is free, λ_T must be equal to 0, and, based on this procedure, the optimal policy can be determined in two steps. In the first step, the optimal trajectory without the sustainability constraint ($N(T)>N_T{}^*$) is derived. When the above procedure reaches terminal age T, $\lambda(T)$ is compared with 0. If $\lambda(T)$ differs from 0, the value of $\lambda(0)$ is changed and the procedure repeated. This procedure is repeated using different $\lambda(0)$ until λ_T nearly equals 0. When λ_T is sufficiently close to 0, the resource population size $N(T)$ is examined. Two cases are possible:

Case 1: If the population size is greater than $N_T{}^*$, the derived $F(t)$ is the optimal harvesting policy $F^*(t)$ maximizing total yield, in which the sustainability constraint has no effect.

Case 2: If the derived terminal population size conflicts with the sustainability condition ($N(T)<N_T{}^*$), the optimal policy must be examined further (second step).

In the second step, at terminal age T, the population size $N(T)$ is compared with the constraint $N_T{}^*$, rather than $\lambda(T)$ being compared with 0. The procedure is repeated until $N(T)$ nearly equals to $N_T{}^*$, by which the derived $F(t)$ is the optimal harvesting policy $F^*(t)$ maximizing the total yield under the sustainability (or spawner) constraint.

The first step considers the case where the optimal policy, ignoring the sustainability constraint, satisfies the sustainability requirement. Intuitively, if the spawner constraint is absent, it seems to be better to catch almost all the resource. Nevertheless, a certain number of spawners is often retained even in the freely-derived optimal policy. When an excessive harvesting in the early stages reduces the number of valuable old resource, we should refrain from harvesting of the former. In addition to this, when an upper limit of harvesting rate constrains harvesting of the latter, some number of spawners are necessarily left at terminal age in the optimal policy. If the recommended number of spawners is greater than the lower condition $N_T{}^*$, such a freely determined optimal orbit coincides with the optimal policy under the sustainability constraint. If instead, the sustainability condition is limiting in the optimal trajectory, it must cross the constraint boundary; thus the extreme point at T is set on the boundary condition (i.e. $N_T{}^*$), by which λ_T becomes greater than 0.

The optimal harvesting activity pattern is highly dependent upon functional forms of the current harvesting value $V(t)$ and the natural mortality coefficient $M(t)$. To illustrate its behavior we here show several simple cases in Figures 1–4. The optimal harvesting policies are illustrated in Figs. 1c, 2c, 3c, and 4c. In these figures, the optimal policies with sustainability constraint are represented by solid lines and those without sustainability constraints are represented by dotted lines.

Figure 1 shows cases with age-independent value and mortality of a resource. In this case, when the sustainability constraint is absent, it is the best policy to harvest resource throughout the season. However, if we include the constraint, the resource should be harvested in the early stages. This is because that when harvesting activity must be suppressed for sustainability, we should leave neither variable nor abundant old resource. Figure 2 illustrates a case, in which the natural mortality coefficient is constant and the value of a resource increases linearly with age. Independently of the sustainability coefficient, the resource should not be harvested until it reaches a certain value. Thereafter, maximum catch rates can apply until terminal age T. In this case, harvesting the old, valuable resource can be more advantageous, and harvesting younger resource is not favored. Such a suppression of harvesting automatically satisfies sustainability even if the constraint is absent. Nevertheless, supplementary analyses showed that if the natural mortality coefficient is higher with the same resource value function, the optimal policy is harvesting at an intermediate stage. This is because, in such a case, the valuable late stage resource is reduced due to high mortality.

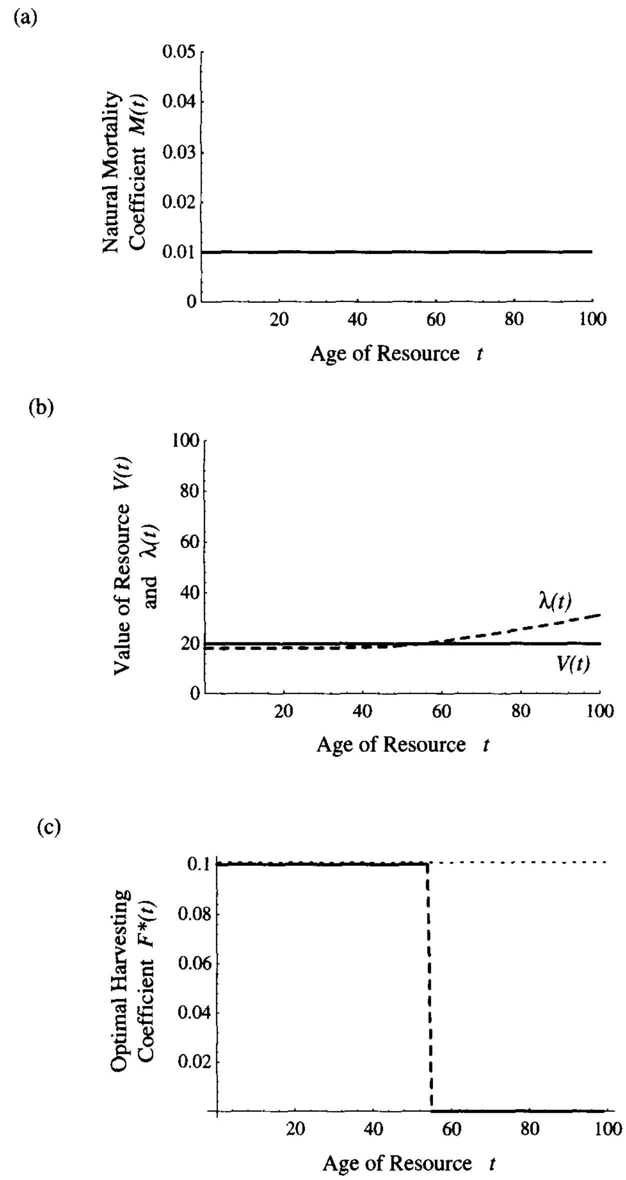

Fig. 1. A numerical solution of the optimal harvesting policy. (a) Natural mortality coefficient per age $M(t)$, (b) value of a resource $V(t)$ (solid line) and expected future value $\lambda(t)$ (broken curve), and (c) optimal change of harvesting coefficient with age. The solid line represents the optimal policy with a sustainability constraint, the dotted line that without the constraint. Age t is relative to resource longevity. Maximum harvesting coefficient is 0.1. The initial recruitment is 10,000, with 10 spawners required to be left at terminal age T. If changes in natural mortality and resource value are given, the optimal harvesting schedule is derived. When the current value of an individual resource is greater than the expected future value, harvesting should become active. In this case, the harvesting should be directed only at early age stages.

Since natural mortality is generally severe for larvae or early stage individuals, a variable mortality coefficient that is higher in the early stages was considered. In this situation, several harvesting patterns can be found, depending on the functional forms of $V(t)$ and $M(t)$. In Figs. 3 and 4, the shapes of the functions are similar to each other, although the minimums of the natural mortality coefficients differ. In both case, when there is no sustainability constraint, the harvesting season consists of two separate periods. This can be explained by considering the benefits of these periods separately. In the early

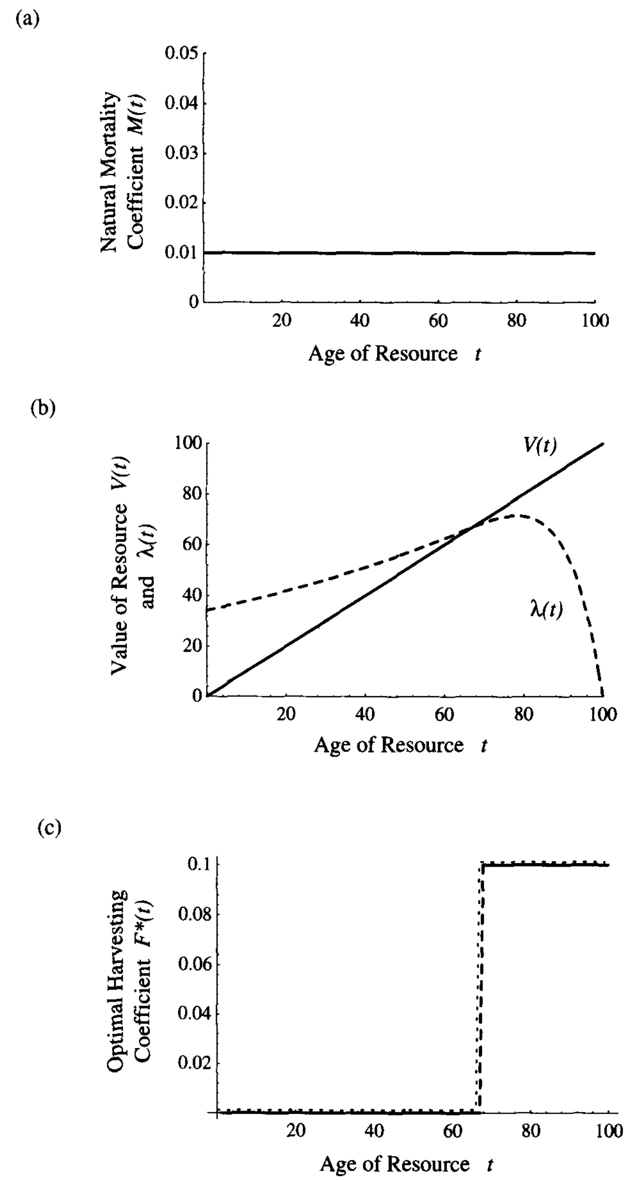

Fig. 2. A numerical solution of the optimal harvesting policy. (a) Natural mortality coefficient per age $M(t)$, (b) value of a resource $V(t)$ (solid line) and expected future value $\lambda(t)$ (broken line), and (c) optimal change of harvesting coefficient with age. The solid line represents the optimal policy with a sustainability constraint, the dotted line that without the constraint. Parameters as for Fig. 1. In this case, harvesting should be directed only at old age stages.

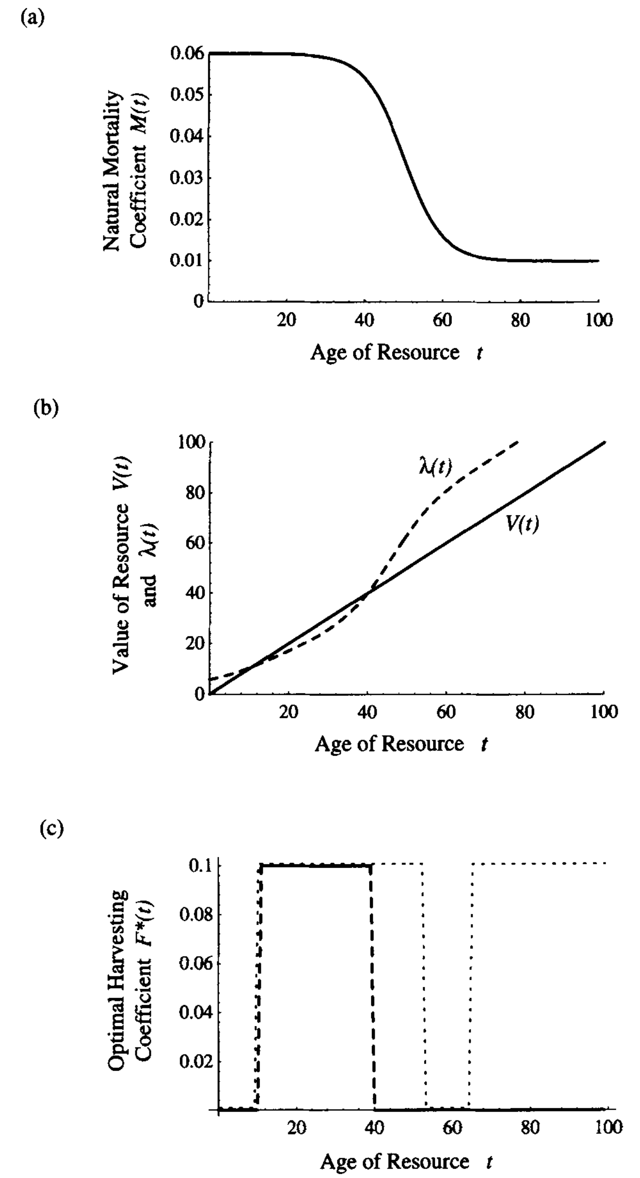

Fig. 3. A numerical solution of the optimal harvesting policy. (a) Natural mortality coefficient per age $M(t)$, (b) value of a resource $V(t)$ (solid line) and expected future value $\lambda(t)$ (broken line), and (c) optimal change of harvesting coefficient with age. The solid line represents the optimal policy with a sustainability constraint, the dotted line that without the constraint. Parameters as for Fig. 1. In this case, the harvesting should be directed at intermediate age stages.

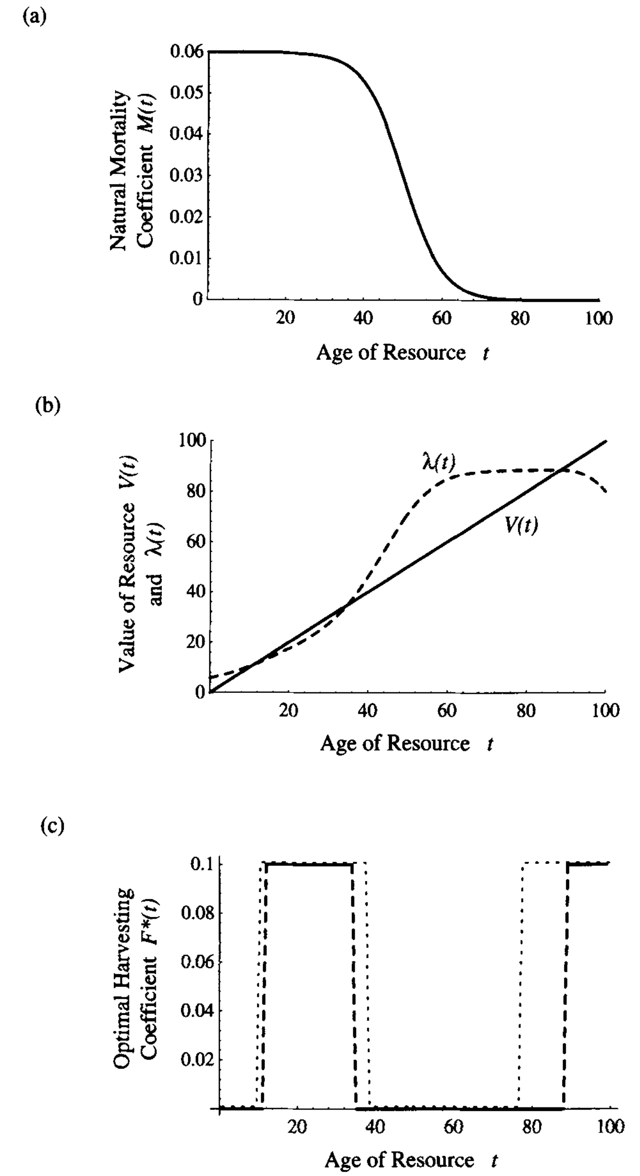

Fig. 4. A numerical solution of the optimal harvesting policy. (a) Natural mortality coefficient per age $M(t)$, (b) value of a resource $V(t)$ (solid line) and expected future value $\lambda(t)$ (broken line), and (c) optimal change of harvesting coefficient with age. The solid line represents the optimal policy with a sustainability constraint, the dotted line that without the constraint. Parameters as for Fig. 1. In this case, optimal harvesting includes two separate age periods.

period, it is of benefit to harvest resources before they die due to high mortality. Contrarily, in the later period of low mortality, it is advantageous to wait for growth of the surviving resources. On the other hand, if a sustainability constraint is introduced, harvesting is suppressed and harvesting period is shortened. The optimal harvesting schedule accompanying the sustainability constraint varies between Fig. 3 and 4, one of which represents harvesting at an intermediate stage, the other being sequences at both intermediate and late stages. This is because relatively higher mortality in the later period causes a reduction of the resource population, of which harvesting in that period tends not to satisfy the sustainability condition.

In Figs. 1b, 2b, 3b and 4b, the expected future value $\lambda(t)$ is shown together with the current harvesting value $V(t)$, showing that only when the former is less than the latter, should harvesting be active. These figures show that $\lambda(t)$ changes in a complex manner.

Application to real fishery data

We also applied the present analysis to real fishery data. The data analyzed are from the brine boats fishery of Gulf of Carpentaria prawn, cited in Clark and Kirkwood (1979). The real fishery has an accompanying cost for fishing activities. Although the analysis of the model presented neglects this fishery cost, we included it in the numerical analysis and examined its effects. The result is illustrated in Fig. 5, where line (i) represents the optimal fishery with neither the sustainability constraint nor fishery cost considered, line (ii) is optimal with the sustainability constraint but without fishery cost, and line (iii) is optimal with the fishery cost but without the sustainability constraint. For the sustainability constraint, it is assumed that 1/50th of the initial population size must be left at the end of fishery season.

The results indicate that in the real brine boats fishery of Gulf of Carpentaria prawn, if there are neither fishery costs nor sustainability constraints, the fishery should be active over all the season. Once the sustainability constraint is included for maintenance of the resource population, the optimal fishery season is restricted to the relatively early period of the season. Fishery costs have a similar effect to the sustainability constraint, which also causes a restriction of the fishery season. Nevertheless, in the presented sample case, the inclusion of the sustainability constraint more severely restricts the fishery season than that of fishery cost. Namely, in this particular case the optimal solution with only fishery cost considered does not leave a resource population sufficient to realize sustainable fishery. If we are to manage the resource population by introducing costs for fishery we need to choose such cost levels with care.

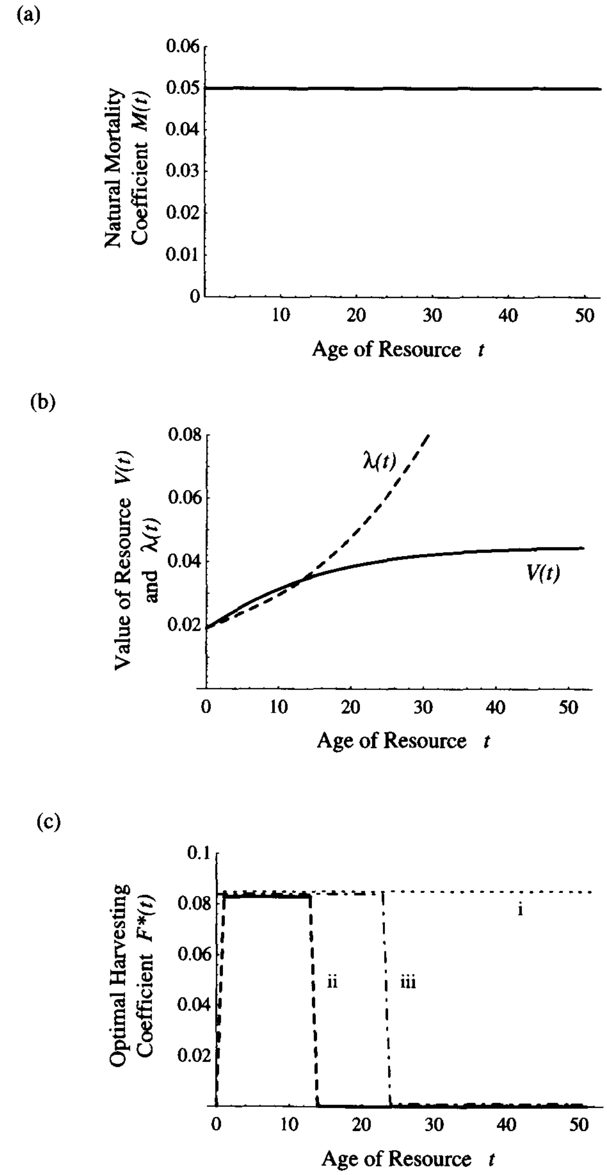

Fig. 5. Application of analysis on real fishery data. The data are cited from Clark and Kirkwood (1979). Unit of time step is weekly, and a year includes 52 weeks ($T=52$). Parameters are $F_{MAX}=8.28\times10^{-4}$ (the number of brine boat is assumed to be 120), $M(t)=0.05$ (constant), $N_0=3.7\times10^8$. Functional form of $V(t)$ is $0.045\{1-\exp[-0.08(t+17.3)]\}^3$ ($), which is determined by Bertalanffy growth function. Three types of optimal solution are illustrated: (i) solution with neither sustainability constraint nor fishery cost, (ii) solution with sustainability constraint ($N(T)=N_0/50$) but without fishery cost, and (iii) solution without sustainability constraint but with fishery cost that is 5.07×10^5 ($) per unit harvesting rate.

Discussion

In the present paper, we focused on a tradeoff between early and late harvesting within a generation, where harvesting resources at early stages would reduce the abundance of late and old stages that are often more valuable than the young stages. Here, we considered a semelparous resource population, and assumed that the harvesting intensity can be flexibly chosen as a function of age. This is the case if the resource is perfectly synchronized in life cycle, as is the case for annual fish. Even if the resource population includes individuals of different ages with overlapping generations, the harvesting intensity can be controlled by using different harvesting locations or harvesting methods if the morphology, behavior or habitat of resource population varies with age. In particular, in aquaculture situations, the harvest of each stage can be controlled quite precisely.

This paper outlines a method of deriving numerical solutions for optimal harvesting schedules. The analysis of several simple examples demonstrated that a solution maximizing total yield is critically dependent on the functions of natural mortality and value of an individual (Figs. 1–4). One conclusion of the analysis is that optimal harvesting may comprise several distinct harvesting periods. For example, Fig. 4 illustrates a case in which optimal policy is to harvest at two separate age cohorts of the resource population, but not to exploit cohorts of intermediate ages between them. If we restricted the search for the optimal policy into simpler classes, such as 'start harvesting at a certain age' or 'stop harvesting at a certain age', optimal harvesting would not be realized.

Figures 1c, 2c, 3c and 4c illustrated the effects of the sustainability constraint on harvesting policy. These showed that optimal harvesting policies with a sustainability constraint are often very different from those without the constraint, clearly demonstrating the importance of a sustainability constraint. Such changes of policy introducing the constraint cause a relaxation of harvesting, by which the number of spawners is increased. According to these results, if we aim for a sustainable resource use, a policy without the sustainability constraint is quite inadequate, and may cause severe overcatch and extinction of the resource population.

Based on the theory of optimal harvesting under a sustainability constraint for the spawner numbers needed to sustain the resource population, we can discuss the economical use of resources and the preservation of wildlife within a single general framework. Several previous studies of optimal harvesting which maximized the long term yield used the number of spawners as the policy strategy in the maximization (Reed 1974, 1979; Clark and Kirkwood 1984). However treating the minimum number of spawners as a constraint, set separately from the optimality criterion as a measure of economic use, is conceptually quite important in identifying two distinct considerations: sustainability and economical resource use.

According to the analysis in the present paper, harvest activity is determined by whether or not the current harvesting value is greater than the expected future value. The expected future value is the sum of future harvesting value and the sustainability value. As assumed in many previous treatments of optimal harvesting (Clark 1976, 1990), if there is no spawner constraint (or sustainability constraint), the sustainability value term does not exist. The sustainability value is measured by the same units as the other two terms (current and future harvesting values) and economic income. If the resource population is exploited so much that the criterion of leaving a minimum number of spawners is at risk, then we must constrain resource utilization by posing an additional control even if its direct effect is economically costly. The risk of violating the sustainability constraint at such a stage would cause a need for posing more stringent control in the future. Thus, the risk can in principle be measured in the same terms as economic income, because it is a reduction of income in the future due to posing a more stringent control in order to fulfill the sustainability constraint. This is precisely the meaning of 'sustainability value' as derived in the present paper.

An illustrating example of the model applied to real resource management shows that harvesting cost has a similar effect as the sustainability constraint, both of which restrict the harvesting period (Fig. 5). Thus, the resource population may be conserved by introducing a tax on harvesting activity. Nevertheless, it is not easy to determine the correct level of such a cost which will cause an adequate amount of fishery activity realizing the sustainability of the resource population (see line (ii) in Fig. 5). The concepts of a sustainability constraint and the sustainability value which we introduced in this paper suggest a method that combines conservation of resource population and economical use of the resource in a single framework.

Here, we discuss a potential application of such an economic interpretation to real fishery management policies. In welfare economics the possibility of a well planned tax system to realize economic welfare has been studied (Pigou 1932). The present model proposes a method for evaluating such a tax for bio-resource management.

The model implies that when a spawner can be evaluated as λ_T economic value (which is determined through the optimization process), fishers adopt the optimal harvesting schedule so as to maximize their income even if they are not aware of sustainability. (It is notable that the value of a spawner λ_T is different from the marketing value $V(T)$). Such a situation can be realized by the government paying λ_T bonus to fishers for each spawner. When we shift the standard of the evaluation of the bonus, this can then be rearranged to a penalty for overcatch, setting a critical spawner level above which λ_T bonus is paid for each spawner, but below which λ_T penalty is imposed for each spawner overcaught. Now, we consider the case in which such a penalty being levied on a fishers' association. In this situation, a personal harvesting operation running outside of the optimal schedule causes two economic costs for the association, one of which is a reduction of harvest in future yield, and another being the penalty for fewer spawners paid to the government. Expected values of these costs are exactly the 'future harvesting value' and the 'sustainability value', respectively. Thus, the association should request the offender to pay these costs for both yield and sustainability. Several types of penalty can be considered for illegal harvesting operations, although application of a 'sustainability value' proposes a consistent policy accompanying the optimal harvesting policy.

Applications of the costate value $\lambda(t)$ to resource or environmental management have been studied in economics, in which the costate value $\lambda(t)$ is referred to as the 'shadow price' or 'shadow value'. The shadow price appears in any economical problem with some accompanying constraints. Miyagawa (1965) discussed the shadow price using a linear programing technique. The shadow price was also discussed in relation to the maximum principle (for example, Kitabatake 1995). There are theoretical studies for renewable bio-resources, in which the shadow price can be applied to management as taxes on utilization or development of that resource (Kitabatake 1989; Clark 1990; Ueda et al. 1991). Kitabatake (1995) demonstrated a consideration of the shadow price for a non-renewable resource with respect to a constraint for remaining resource stock. This basic idea is common to the approach developed in the present paper for the case of semelparous bio-resources. Never-theless, the analysis presented can be developed to the management of iteroparous species. In addition, bio-resources reduce in abundance due to natural mortality. Including such an aspect, the present model evaluates the shadow price (sustainability value) in a plausible formulation.

Although the present model proposes important view points for sustainable harvesting policy, it neglects several factors important in actual fisheries. For example, price may in general be a decreasing function of supply. By assuming economic values independent of catch, we in effect consider a very large market in which the price is determined. However the situation would be different if the harvesting site we are modelling is of a not very small fraction of the whole market. The cost of harvesting, and the cost of changing the harvesting level are also neglected. In addition, the maximum harvesting coefficient fixed in the model, in reality, may vary with resource age and/or seasonal conditions. The model can be expanded from various view points, in particular, from that of the reproductive behavior of the resource. Although a semelparous (single big reproduction) organism was assumed here, the model can be expanded for iteroparous (repeated reproduction) species. These would probably make analytically explicit solutions difficult to obtain, but can be included easily in numerical analysis of the optimization.

Acknowledgments: This work was partly supported by a Grant-in-aid from the Japan Ministry of Education, Science and Culture (#07760179). We thank Drs. Hiroyuki Matsuda, Yutaka Natsukari and Kenichi Tatsukawa, and members of the division of Population Dynamics of Marine Organisms, Ocean Research Institute, University of Tokyo for their comments and encouragement.

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Received 1 January 1997; Accepted 10 November 1997

Appendix

According to Eq. (4), a singular orbit is

$$\lambda(t)=V(t). \tag{A1}$$

Differentiating Eq. (A1),

$$\frac{d\lambda}{dt}=\frac{dV}{dt}. \tag{A2}$$

is derived. Substituting Eq. (6) into Eq. (A2), the dynamics of $V(t)$ on a singular orbit is

$$\frac{dV}{dt}=-\{V(t)-\lambda(t)\}F(t)+\lambda(t)M(t). \tag{A3}$$

Substituting Eq. (A1) into Eq. (A3), results in

$$\frac{dV}{dt}=V(t)M(t). \tag{A4}$$

The integration of Eq. (A4),

$$V(t)=Ce^{\int M(t)dt}, \tag{A5}$$

does not generally hold for some intervals. Hence it can be concluded that singular control is not part of the optimal solution in the present model.