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A REMARK ON THE UNCONDITIONAL STRUCTURE OF $L(E, F)$

BY
CARSTEN SCHÜTT†

ABSTRACT

We present a sequence of symmetric Banach spaces E_n, $n \in N$, with $d(E_n, l_n^\infty)$, $n \in N$, unbounded and $\mathrm{ubc}(L(E_n^*, E_n))$, $n \in N$, uniformly bounded.

0. Introduction

In this note we consider the unconditional structure of the Banach space of operators $L(E, F)$ of two Banach spaces E and F with unconditional bases $\{e_i\}_{i=1}^n$ and $\{f_j\}_{j=1}^m$, respectively. In [5] it was shown that the unconditional basis constant $\mathrm{ubc}(\{e_i^* \otimes f_j\}_{i,j=1}^{n,m})$ is, up to a constant, the same as $\mathrm{lust}(L(E, F))$ and $\mathrm{gl}(L(E, F))$. The only known cases for uniformly bounded $\mathrm{ubc}(L(E, F))$ were described by the inequality [5]

$$\mathrm{ubc}(L(E, F)) \leqq \min\{d(E^*, l_n^\infty), d(F, l_m^\infty)\}$$

while assuming that $\mathrm{ubc}(\{e_i\}_{i=1}^n) = \mathrm{ubc}(\{f_j\}_{j=1}^m) = 1$. On the other hand, using results of [1] and [5] one gets that

$$\mathrm{ubc}(L(E, F)) \geqq C(\log(n+m))^{-1/2} \min\{d(E^*, l_n^\infty)^{1/2}, d(F, l_m^\infty)^{1/2}\}.$$

The factor $(\log(n+m))^{-1/2}$ enters because in [1] Gaussian and not Bernoulli random variables are considered. By a result of Lewis [2] it was suggested that the factor may be dropped, namely, Lewis proved that

$$\mathrm{ubc}(\underbrace{E \otimes_\varepsilon E \otimes_\varepsilon \cdots \otimes_\varepsilon E}_{k \text{ factors}})$$

tends with the number of factors k to infinity if and only if $d(E, l_n^\infty) > 1$. We give a counterexample.

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1. Preliminaries

We say that a Banach space E has a symmetric basis $\{e_i\}_{i=1}^n$ if it is normalized and we have, for all $a \in \mathbf{R}^n$, all permutations π of the set $\{1,\cdots,n\}$, and all sequences of signs $\varepsilon = (\varepsilon_1,\cdots,\varepsilon_n)$

$$\left\|\sum_{i=1}^n a_i e_i\right\| = \left\|\sum_{i=1}^n \varepsilon_i a_{\pi(i)} e_i\right\|.$$

We say a Banach space has a C-unconditional basis if for all $a \in \mathbf{R}^n$ and all $\varepsilon = (\varepsilon_1,\cdots,\varepsilon_n)$

$$\left\|\sum_{i=1}^n a_i e_i\right\| \leqq C\left\|\sum_{i=1}^n \varepsilon_i a_i e_i\right\|.$$

We put $\mathrm{ubc}(\{e_i\}_{i=1}^n) = \inf C$ and

$$\mathrm{ubc}(E) = \inf\{\mathrm{ubc}(\{e_i\}_{i=1}^n) \mid \{e_i\}_{i=1}^n \text{ is a basis of } E\}.$$

The Gordon–Lewis constant $\mathrm{gl}(E)$ is defined by

$$\mathrm{gl}(E) = \sup\{\gamma_1(A)/\pi_1(A) \mid A \in L(E,l^2)\}$$

where $\gamma_1(A)$ denotes the 1-factorizing norm and $\pi_1(A)$ the 1-absolutely summing norm of A [5]. The Banach–Mazur distance of two Banach spaces E and F is given by

$$d(E,F) = \inf\{\|J\|\,\|J^{-1}\| \mid J \in L(E,F),\ J \text{ is isomorphism}\}.$$

By $\mathrm{card}\, M$ or $|M|$ we denote the cardinality of a set M. By $[r]$ we denote the smallest natural number greater than the real number r. By $x^* \otimes y$ we denote the operator mapping x onto $\langle x^*, x\rangle y$.

2. An estimate for $\mathrm{ubc}(L(E,F))$

The estimate is an immediate consequence of two known results.

PROPOSITION 1. *Suppose that $\{e_i\}_{i=1}^n$ and $\{f_j\}_{j=1}^m$ are 1-unconditional bases of the Banach spaces E and F, respectively. Then*

$$\mathrm{gl}(L(E,F)) \geqq C(\log(n+m))^{-1/2}\min\{d(E^*, l_n^\infty)^{1/2}, d(F, l_n^\infty)^{1/2}\}$$

where C is an absolute constant.

Please note that lemma 5 in [5] says that under the same hypothesis of Proposition 1 we have

$$\mathrm{ubc}(L(E,F)) \leqq \min\{d(E^*, l_n^\infty), d(F, l_m^\infty)\}.$$

PROOF. First we prove that for some $C \in \mathbf{R}$

$$\min_{\pm}\left\|\sum_{i,j=1}^{n,m} \pm e_i^* \otimes f_j\right\| \tag{1}$$
$$\leqq C(\log(n+m))^{1/2} \max\left\{\left\|\sum_{i=1}^{n} e_i^*\right\|^{1/2}\left\|\sum_{j=1}^{m} f_j\right\|, \left\|\sum_{i=1}^{n} e_i^*\right\|\left\|\sum_{j=1}^{m} f_j\right\|^{1/2}\right\}.$$

Indeed, we have for $\varepsilon = (\varepsilon_{ij})_{i,j=1}^{n,m}$, $\varepsilon_{ij} = \pm 1$, [1]

$$\min_{\pm}\left\|\sum_{i,j=1}^{n,m} \pm e_i^* \otimes f_j\right\| \leqq 2^{-nm}\sum_{\varepsilon}\left\|\sum_{i,j=1}^{n,m} \varepsilon_{ij} e_i^* \otimes f_j\right\|$$
$$\leqq \sqrt{\pi/2}\int_{\Omega}\left\|\sum_{i,j=1}^{n,m} g_{ij}(\omega) e_i^* \otimes f_j\right\| d\omega$$

where g_{ij}, $1 \leqq i \leqq n$, $1 \leqq j \leqq m$, denote independent Gaussian random variables. By a result of Chevet [1] the last expression is less than a constant times

$$\sup_{\|x\|=1}\left(\sum_{i=1}^{n}|\langle e_i^*, x\rangle|^2\right)^{1/2}\sup_{1\leqq k\leqq m}\int_{\Omega}\left\|\sum_{j=1}^{k} g_j f_j\right\| d\omega$$
$$+\sup_{\|y^*\|=1}\left(\sum_{j=1}^{m}|\langle f_j, y^*\rangle|^2\right)^{1/2}\sup_{1\leqq k\leqq n}\int_{\Omega}\left\|\sum_{i=1}^{k} g_i e_i^*\right\| d\omega$$

where g_i, $1 \leqq i \leqq n$, and g_j, $1 \leqq j \leqq m$, are independent Gaussian random variables. Using the general fact

$$\int_{\Omega}\left\|\sum_{j=1}^{k} g_j(\omega) f_j\right\| d\omega \leqq C\sqrt{\log k}\, 2^{-k}\sum_{\varepsilon}\left\|\sum_{i=1}^{k}\varepsilon_j f_j\right\|$$

and the same inequality for $\{e_i^*\}_{i=1}^n$ we get for the last expression a constant times

$$\sup_{\|x\|=1}\left(\sum_{i=1}^{n}|\langle e_i^*, x\rangle|^2\right)^{1/2}\sup_{1\leqq k\leqq m}\sqrt{\log k}\left\|\sum_{j=1}^{k} f_j\right\|$$
$$+\sup_{\|y^*\|=1}\left(\sum_{j=1}^{m}|\langle f_j, y^*\rangle|^2\right)^{1/2}\sup_{1\leqq k\leqq n}\sqrt{\log k}\left\|\sum_{i=1}^{k} e_i^*\right\|.$$

Since $|\langle e_i^*, x\rangle| \leqq 1$ and $|\langle f_j, y^*\rangle| \leqq 1$ for all $i = 1,\cdots,n$ and $j = 1,\cdots,m$ we get that

$$\frac{1}{C}\min_{\pm}\left\|\sum_{i,j=1}^{n,m} \pm e_i^* \otimes f_j\right\|$$

$$\leqq \sup_{\|x\|=1}\left(\sum_{i=1}^{n} |\langle e_i^*, x\rangle|\right)^{1/2} \sqrt{\log m}\left\|\sum_{j=1}^{m} f_j\right\|$$

$$+ \sup_{\|y^*\|=1}\left(\sum_{j=1}^{m} |\langle f_j, y^*\rangle|\right)^{1/2} \sqrt{\log n}\left\|\sum_{i=1}^{n} e_i^*\right\|$$

$$\leqq \sqrt{\log m}\left\|\sum_{i=1}^{n} e_i^*\right\|^{1/2}\left\|\sum_{j=1}^{m} f_j\right\| + \sqrt{\log n}\left\|\sum_{j=1}^{m} f_j\right\|^{1/2}\left\|\sum_{i=1}^{n} e_i^*\right\|.$$

From this (1) follows immediately. By [5] we have

$$\mathrm{gl}(L(E,F)) \geqq \frac{1}{4}\,\mathrm{ubc}(\{e_i^* \otimes f_j\}_{i,j=1}^{n,m})$$

$$\geqq \frac{1}{4}\left\|\sum_{i,j=1}^{n,m} e_i^* \otimes f_j\right\|\left(\min_{\pm}\left\|\sum_{i,j=1}^{n,m} \pm e_i^* \otimes f_j\right\|\right)^{-1}.$$

By (1) we get for some $C' > 0$

$$\mathrm{gl}(L(E,F)) \geqq C'(\log(n+m))^{-1/2} \min\left\{\left\|\sum_{i=1}^{n} e_i^*\right\|^{1/2}, \left\|\sum_{j=1}^{m} f_j\right\|^{1/2}\right\}.$$

Using now the obvious inequality $d(E^*, l_n^\infty) \leqq \|\Sigma_{i=1}^n e_i^*\|$ and the same inequality for F we finish the proof. □

3. The example

We give the definition of the considered space. E_n is the space R^n equipped with a norm defined for the vectors $\Sigma_{i=1}^k e_i$, where e_i, $i = 1, \cdots, n$ are the natural unit vectors, as

$$k_1 = 1, \quad k_{j+1} = 2^{k_j}, \qquad j = 1, 2, \cdots,$$

(2)
$$\left\|\sum_{i=1}^{k} e_i\right\| = \varphi(k) = j \qquad \text{for } k_j \leqq k < k_{j+1}.$$

For the dual basis $\{e_i^*\}_{i=1}^n$ and all permutations π of $\{1, \cdots, n\}$ we put

$$\left\|\sum_{i=1}^{k} e_{\pi(i)}^*\right\| = k\left\|\sum_{i=1}^{k} e_i\right\|^{-1}.$$

Now we take the convex hull of

$$(3)\quad M = \left\{\left\|\sum_{i=1}^{k} e_i^*\right\|^{-1} \sum_{i=1}^{k} \pm e_{\pi(i)}^* \,\middle|\, k = 1, \cdots, n,\ \pi \text{ is permutation of } \{1, \cdots, n\}\right\}$$

as the dual unit ball. In particular, M is the set of extreme points of the dual unit ball. We introduce the function $r\colon \{8^8, 8^8+1, \cdots\} \to \mathbf{N}$

$$r(k) = [\log_2\log_2\log_2\log_2 k] \tag{4}$$

where the brackets mean to take the smallest natural number greater than or equal to $\log_2\log_2\log_2\log_2 k$. We observe the property

$$\left\| \sum_{i=1}^{k} e_i \right\| \leqq C \left\| \sum_{i=1}^{r(k)} e_i \right\| \qquad \text{for } k = 8^8, 8^8+1, \ldots \tag{5}$$

and for some absolute constant C.

By corollary 3 of [4] we have

$$\left\| \sum_{i=1}^{n} e_i \right\|^{1/2} \leqq \sqrt{2}\, d(E_n, l_n^\infty)$$

and therefore $d(E_n, l_n^\infty)$ tends with n to infinity. Nevertheless, we have the following theorem.

THEOREM 2. *There is an absolute constant C such that*

$$\operatorname{ubc}(L(E_n^*, E_n)) \leqq C.$$

For the proof we need two lemmas.

LEMMA 3. *Let φ and r be as defined by* (2) *and* (4) *and let $(a_{ij})_{i,j=1}^{k,l}$ be a real-valued matrix with $|a_{ij}| \leqq 1$ for all $i = 1, \cdots, k$ and $j = 1, \cdots, l$. Then there are absolute constants C^* and C so that for all $l, k \in N$ with $C^* \leqq \log_2\log_2 k \leqq l \leqq k$ and*

$$lk = \varphi(k)\varphi(l) \sum_{i,j=1}^{k,l} |a_{ij}| \tag{6}$$

we have subsets $r(K)$ and $r(L)$ of $\{1, \cdots, k\}$ and $\{1, \cdots, l\}$ of cardinality $r(k)$ and $r(l)$ such that

$$\sum_{i,j=1}^{k,l} |a_{ij}| \leqq C \frac{lk}{r(l)r(k)} \left| \sum_{\substack{i \in r(K) \\ j \in r(L)}} a_{ij} \right| \tag{7}$$

LEMMA 4. *Let φ and r be as defined by* (2) *and* (4) *and let $(a_{ij})_{i,j=1}^{k,l}$ be a real-valued matrix. Suppose $1 \leqq l \leqq \log_2\log_2 k$ and*

$$1 = \frac{\varphi(k)\varphi(l)}{kl} \sum_{i,j=1}^{k,l} |a_{ij}| \geqq \frac{\varphi(l)}{l} \sum_{j=1}^{l} |a_{ij}| \qquad \textit{for all } i = 1, \cdots, k. \tag{8}$$

Then there is a subset $r(K)$ of $\{1,\cdots,k\}$ of cardinality $r(k)$ and a sequence $(\varepsilon_j)_{j=1}^l$ of signs such that

$$\sum_{i,j=1}^{k,l} |a_{ij}| \leqq C\frac{k}{r(k)}\left|\sum_{\substack{j=1\\ i\in r(K)}}^{l} \varepsilon_j a_{ij}\right| \tag{9}$$

where C is an absolute constant.

Proof of Theorem 2. We have to show that there is a constant $C \geqq 1$ so that we have for any matrix $A = (a_{ij})_{i,j=1}^n$

$$\max_{\pm}\left\|\sum_{i,j=1}^{n} \pm a_{ij}e_i\otimes e_j\right\| \leqq C\min_{\pm}\left\|\sum_{i,j=1}^{n} \pm a_{ij}e_i\otimes e_j\right\|.$$

Obviously this is equivalent to showing for all matrices A

$$\left\|\sum_{i,j=1}^{n} |a_{ij}|\, e_i\otimes e_j\right\| \leqq C\left\|\sum_{i,j=1}^{n} a_{ij}e_i\otimes e_j\right\|.$$

Without restriction of generality we may assume that $\|\sum_{i,j=1}^n |a_{ij}|\, e_i\otimes e_j\| = 1$. Since the extreme points of the dual unit ball are of the form (3) we get for some k, l, $1 \leqq k, l \leqq n$, and some subsets K, L of $\{1,\cdots,n\}$ with $|K| = k$, $|L| = l$

$$1 = \left\|\sum_{i,j=1}^{n} |a_{ij}|\, e_i\otimes e_j\right\| = \left(\left\|\sum_{i=1}^{k} e_i^*\right\|\left\|\sum_{i=1}^{l} e_i^*\right\|\right)^{-1}\sum_{\substack{i\in K\\ j\in L}} |a_{ij}|. \tag{10}$$

Obviously we may assume that $C^* \leqq l \leqq k$ for some $C^* \in \mathbf{R}$. Now we consider two cases. First, suppose $\log_2\log_2 k \leqq l$. Then, because of (10) the assumptions of Lemma 3 are fulfilled. Thus we get by (10) and Lemma 3

$$\left\|\sum_{i,j=1}^{n} |a_{ij}|\, e_i\otimes e_j\right\| = \frac{\varphi(k)\varphi(l)}{kl}\sum_{\substack{i\in K\\ j\in L}} |a_{ij}|$$

$$\leqq C\frac{\varphi(k)\varphi(l)}{r(k)r(l)}\left|\sum_{\substack{i\in r(K)\\ j\in r(L)}} a_{ij}\right|.$$

Because of (5) the last expression is less than

$$C'\frac{\varphi(r(k))\varphi(r(l))}{r(k)r(l)}\left|\sum_{\substack{i\in r(K)\\ j\in r(L)}} a_{ij}\right| \leqq C'\left\|\sum_{i,j=1}^{n} a_{ij}e_i\otimes e_j\right\|.$$

Second, we have the case $1 \leqq l \leqq \log_2\log_2 k$. The condition (8) is fulfilled because of (10). Therefore we may apply Lemma 4.

The computation is the same as in the first case. □

The following lemma is a consequence of a solution to Zarankiewicz's problem [3]. For completeness we give a short proof.

LEMMA 5. *Let φ and r be as defined by* (2) *and* (4). *Suppose that* $1 \leqq \log_2\log_2 k \leqq l \leqq k$ *and* $(a_{ij})_{i,j=1}^{k,l}$ *is a* $0,1$-*matrix with more than* $C(\varphi(k)\varphi(l))^{-2}kl$ *ones. Then there is a constant* $C^* = C^*(C)$ *so that for all* $k, l \in N$ *with* $C^* \leqq k, l$ *there is a submatrix of size* $r(k)$ *by* $r(l)$ *consisting entirely of ones.*

PROOF. We assume that $(a_{ij})_{i,j=1}^{k,l}$ has no desired submatrix and construct a contradiction. We define

$$n_i = \operatorname{card}\{j \mid a_{ij} = 1\}.$$

Obviously, we can choose in the ith row $\binom{n_i}{r(k)}$ different subsets containing exactly $r(k)$ ones. Thus, for the whole matrix this makes $\Sigma_{i=1}^{l}\binom{n_i}{r(k)}$. Clearly, since we assume that there is no submatrix of size $r(k)$ by $r(l)$ containing entirely ones we have

$$r(l)\binom{k}{r(k)} \geqq \sum_{i=1}^{l}\binom{n_i}{r(k)}. \tag{11}$$

On the other hand, by convexity and monotonicity of the function $g(t) = \binom{t}{a}$ and by

$$\sum_{i=1}^{l} n_i \geqq Ckl(\varphi(k)\varphi(l))^{-2} \geqq Ckl(\varphi(k))^{-4}$$

we get

$$\sum_{i=1}^{l}\binom{n_i}{r(k)} \geqq l\binom{Ck\varphi(k)^{-4}}{r(k)}. \tag{12}$$

By (11), (12) and the inequality

$$\frac{(m-n)^n}{n!} \leqq \binom{m}{n} \leqq \frac{m^n}{n!}$$

we get

$$\frac{l}{r(k)} \leqq \frac{l}{r(l)} \leqq \left(\frac{k}{Ck\varphi(k)^{-4} - r(k)}\right)^{r(k)} \leqq (2C\varphi(k)^4)^{r(k)}$$

provided k is large enough so that $Ck\varphi(k)^{-4} \geqq 2r(k)$. Thus

$$\log_2\log_2 k \leqq l \leqq r(k)(2C\varphi(k)^4)^{r(k)}$$

or

$$\log_2\log_2\log_2 k \leqq \log_2 r(k) + r(k)\log_2(2C\varphi(k)^4).$$

Considering the definition of r and φ, (2) and (4), we conclude that this is impossible. □

PROOF OF LEMMA 3. We introduce the following sets

$$N_m = \{(i,j) \mid 2^{-m} \geqq |a_{ij}| > 2^{-m-1}\}$$

for m with $0 \leqq m \leqq \log_2 4\varphi(k)\varphi(l) = \gamma$. Thus we get by (6)

$$\sum_{m \leqq \gamma} \sum_{(i,j)\in N_m} |a_{ij}| \geqq \frac{kl}{\varphi(k)\varphi(l)} - \frac{kl}{2\varphi(k)\varphi(l)} \geqq \frac{1}{2}\frac{kl}{\varphi(k)\varphi(l)}.$$

At least for one m_0 the sum $\Sigma_{(i,j)\in N_{m_0}} |a_{ij}|$ is greater than or equal to the average:

$$\sum_{(i,j)\in N_{m_0}} |a_{ij}| \geqq (\log_2 8\varphi(k)\varphi(l))^{-1}\frac{kl}{2\varphi(k)\varphi(l)} \geqq 2C\frac{kl}{(\varphi(k)\varphi(l))^2}$$

for some $C > 0$. In particular, since $|a_{ij}| \leqq 1$ for all $i = 1, \cdots, k$ and $j = 1, \cdots, l$, we may assume without restriction of generality that

$$\operatorname{card}\{(i,j) \in N_{m_0} \mid a_{ij} > 0\} \geqq C\frac{kl}{(\varphi(k)\varphi(l))^2}.$$

Now we have by Lemma 5 that there is a constant C^* so that for all $k, l \in \mathbf{N}$ with $C^* \leqq k, l$ there is a submatrix of $(a_{ij})_{i,j=1}^{k,l}$ of size $r(k)$ by $r(l)$ with $2^{-m_0} \geqq a_{ij} > 2^{-m_0-1}$.

Thus, denoting the index sets of this submatrix by $r(K)$ and $r(L)$ we get

$$4\sum_{r(K)\times r(L)} a_{ij} \geqq \frac{r(k)r(l)}{\varphi(k)\varphi(l)} = \frac{r(k)r(l)}{kl}\frac{kl}{\varphi(k)\varphi(l)} = \frac{r(k)r(l)}{kl}\sum_{i,j=1}^{k,l}|a_{ij}|.$$ □

PROOF OF LEMMA 4. By (8) we get

$$\frac{k}{\varphi(k)}\sum_{j=1}^{l}|a_{ij}| \leqq \sum_{i,j=1}^{k,l}|a_{ij}| \qquad \text{for all } i = 1, \cdots, k. \tag{13}$$

Therefore, for more than $\frac{1}{8}k/\varphi(k)$ numbers i, $i = 1, \cdots, k$, we have

$$\sum_{j=1}^{l}|a_{ij}| \geqq \frac{1}{4k}\sum_{i,j=1}^{k,l}|a_{ij}|. \tag{14}$$

If not, we have for more than $k - \frac{1}{8}k/\varphi(k)$ numbers i, $i = 1, \cdots, k$

$$\sum_{j=1}^{l}|a_{ij}| \leqq \frac{1}{4k}\sum_{i,j=1}^{k,l}|a_{ij}|$$

and for the other numbers i, $i = 1, \cdots, k$, we have (13). Therefore we get a contradiction:

$$\begin{aligned}\sum_{i,j=1}^{k,l} |a_{ij}| &\leqq \left(k - \frac{1}{8}\frac{k}{\varphi(k)}\right)\frac{1}{4k}\sum_{i,j=1}^{k,l}|a_{ij}| + \frac{1}{8}\frac{k}{\varphi(k)}\frac{\varphi(k)}{k}\sum_{i,j=1}^{k,l}|a_{ij}| \\ &\leqq \left(\frac{1}{4} - \frac{1}{32}\frac{1}{\varphi(k)}\right)\sum_{i,j=1}^{k,l}|a_{ij}| + \frac{1}{8}\sum_{i,j=1}^{k,l}|a_{ij}| \\ &\leqq \frac{1}{2}\sum_{i,j=1}^{k,l}|a_{ij}|.\end{aligned}$$

Among $\frac{1}{8}k/\varphi(k)$ rows we find more than $r(k)$ rows where corresponding coordinates have the same signs. Indeed, since $l \leqq \log_2 \log_2 k$ we have at most $\log_2 k$ rows that have at least in one coordinate a different sign. Thus we have at least $\frac{1}{8}k/(\log_2 k)\varphi(k)$ rows of the same signs. This number is eventually greater than $r(k)$. Therefore, choosing $r(k)$ rows of equal signs with the property (14) we get for a proper sequence of signs $(\varepsilon_j)_{j=1}^{l}$

$$\sum_{i,j=1}^{k,l}|a_{ij}| \leqq 4k\frac{1}{r(k)}\sum_{\substack{j=1\\ i\in r(K)}}^{l}|a_{ij}| = 4\frac{k}{r(k)}\left|\sum_{\substack{j=1\\ i\in r(K)}}^{l}\varepsilon_j a_{ij}\right|$$

where $r(K)$ denotes the index set of the submatrix. □

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Department of Mathematics
The Hebrew University of Jerusalem
Jerusalem, Israel
and
Department of Mathematics
Johannes Kepler Universität
Linz, Austria