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Reversed preservation properties of some negative aging conceptions and stochastic orders[1]

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Abstract

Reversed preservation properties of some negative aging conceptions are developed for the parallel and series system which are composed of independent and identical elements. If the system is of NWU(2) (IMRL, NWUC) properties then the elements is also of NWU(2) (IMRL, NWUC) properties. Reversed preservation properties of the right spread order and the total time on test transform order under the taking of maxima and minima are investigated respectively, applications in moments of NBUE ordered populations is presented as well.

Key Words IMRL; NWUC; NWU(2); Parallel system; RS order; Series system; TTT transform order

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1 Introduction and Preliminaries

Nonparametric aging classes of life distributions have been found to be quite useful in maintenance policy and system reliability analysis. Several extensions of IFR and NBU, for example, DMRL (*decreasing mean residual life*), NBU(2) (*new better than used in second stochastic dominance*) and NBUC (*new better than used in convex ordering*) etc., have been proposed and surveyed in recent decades. Many authors have paid their attention to investigate behavior of aging properties in coherent structure, parallel (series) system, k-out of-n system, convolution, mixture and renewal process. For more details, readers can see *Barlow and Proschan* (1981), *Langberg et al* (1980), *Hendi et al* (1993), *Chen* (1994), *Cai and Wu* (1997), *Belzunce et al* (1999), *Li et al* (2000), *Li and Kochar* (2001), *Franco et al* (2001), *Belzunce et al* (2001), *Li and Zuo* (2002), *Franco et al* (2003) etc.

We firstly give an overview of some related criteria of stochastic comparison and the concerned aging conceptions.

Assume X and Y be two non-negative random variables, representing equipment lives with distributions F and G, denote their survival functions by $\bar{F} = 1 - F$ and $\bar{G} = 1 - G$, set F^{-1} and G^{-1} as their right continuous inverses, i.e., $F^{-1}(p) \equiv \inf\{x : F(x) \geq p\}$, $G^{-1}(p) \equiv \inf\{x : G(x) \geq p\}$. Let $X_t = X - t | X > t$ be the residual life at age $t > 0$ of the random life X, denote its distribution function and survival function by F_t and $\bar{F}_t$.

Definition 1.1 (a) Y is said to be larger than X in the increasing convex order (denoted by $X \leq_{icx} Y$) if, for all $t \geq 0$,

$$\int_t^{\infty} \bar{F}(x)dx \leq \int_t^{\infty} \bar{G}(x)dx.$$

(b) Y is said to be larger than X in the increasing concave order (denoted by $X \leq_{icv} Y$) if, for all $t \geq 0$,

$$\int_0^t \bar{F}(x)dx \leq \int_0^t \bar{G}(x)dx.$$

(c) Y is said to be larger than X in the right spread order (denoted by $X \leq_{RS} Y$) if, for all $0 < p < 1$,

$$\int_{F^{-1}(p)}^{\infty} \bar{F}(x)dx \leq \int_{G^{-1}(p)}^{\infty} \bar{G}(x)dx.$$

(d) Y is said to be larger than X in the total time on test transform order (denoted by $X \leq_{ttt} Y$) if, for all $0 < p < 1$,

$$\int_0^{F^{-1}(p)} \bar{F}(x)dx \leq \int_0^{G^{-1}(p)} \bar{G}(x)dx.$$

For a comprehensive discussion on the above stochastic orders please see *Shaked and Shanthikumar* (1994), *Shaked and Shanthikumar* (1998), *Fernandez-Ponce, Kochar and Muñoz-Pérez* (1998) and *Kochar, Li and Shaked* (2002).

Definition 1.2 (a) X is IFR (DFR) if X_t is decreasing (increasing) in $t \geq 0$ in stochastic order.

(b) X is DMRL (IMRL) if EX_t is decreasing (increasing) in $t \geq 0$.

(c) X is NBU (NWU) if $X_t \leq_{st} (\geq_{st}) X$ for all $t \geq 0$.

(d) X is NBU(2) (NWU(2)) if $X_t \leq_{icv} (\geq_{icv}) X$ for all $t \geq 0$.

(e) X is NBUC (NWUC) if $X_t \leq_{icx} (\geq_{icx}) X$ for all $t \geq 0$.

(f) X is NBUE (NWUE) if $EX_t \leq (\geq) EX$ for all $t \geq 0$.

For more details about NBU(2), DMRL and NBUC aging properties see *Deshpande et al* (1986) and *Cao and Wang* (1991, 1992). The following chain of implications can be easily established,

$$\text{IFR} \Longrightarrow \text{DMRL} \Longrightarrow \text{NBUC (NBU(2))} \Longrightarrow \text{NBUE}.$$

It can be easily verified that, for any integer $n \geq 1$,

$$\min\{X_1, \ldots, X_n\} \text{ is IFR (DFR, NBU, NWU)} \Longrightarrow X \text{ is also IFR (DFR, NBU, NWU)}.$$

The following two partial orderings of random lives, which will be involved in sequel, are often used to measure the degree of IFRA and NBUE.

Definition 1.3 (a) Y is said to be larger than X in the star-shaped order (denoted by $X \leq_* Y$) if $G^{-1}F(x)$ is star-shaped with respect to $x \geq 0$.

(b) Y is said to be larger than X in the NBUE order (denoted by $X \leq_{NBUE} Y$) if, for all $1 > p > 0$,

$$\frac{\int_{F^{-1}(p)}^{\infty} \bar{F}(x)dx}{EX} \leq \frac{\int_{G^{-1}(p)}^{\infty} \bar{G}(x)dx}{EY}.$$

It is shown in *Kochar and Wiens* (1987) that

$$X \leq_* Y \Longrightarrow X \leq_{NBUE} Y.$$

For more on these two orders please refer to *Barlow and Proschan* (1981), *Kochar and Wiens* (1987).

In recent decades, many authors have devoted themselves to investigating preservation properties of some positive aging conceptions which can be regarded as extensions of IFR and NBU. For examples, *Klefsjö* (1985) showed that a parallel system of i.i.d. IFRA(IFR) units is IFRA(IFR); *Abouammoh and El-Neweihi* (1986) showed that a parallel system of i.i.d. DMRL (NBUE) units is also DMRL (NBUE); *Hendi et al* (1993) proved that a parallel system of i.i.d. NBUC units is also NBUC; *Franco et al* (2001) proved that a series system of independent NBU(2) units is also NBU(2), it was found there furthermore that IFR(2) is preserved also by the formation of parallel systems of i.i.d. units; Recently, *Belzunce et al* (2003) have shown that the IFR(2) class is equivalent to the IFR class for continuous distributions, the discrete case is trivial. *Li and Kochar* (2001) got simultaneously the preservation property of NBU(2) class. *Cai and Wu* (1997), *Li et al* (2000) and *Pellerey and Petakos* (2002) obtained the above preservation property of NBUC under the formation of parallel systems of independent units.

In this note, we will make a discussion on the reversed preservation property of some negative aging conceptions and some stochastic orders. In section 2, it is proved that, if a parallel (series) system of i.i.d. units is NWU(2) (NWUC, IMRL), then its units are also NWU(2) (NWUC, IMRL). In section 3, we firstly investigate some moments inequalities of two NBUE ordered populations, afterward, the reversed preservation properties of the right spread order and the total time on test transform order under the parallel system and the series system are surveyed respectively, some applications are developed as well.

Throughout this note, we discuss nonnegative variables with common left end point 0 of their supports, the term *increasing* is used for *monotone nondecreasing*, and expectations are always assumed to be finite when used.

2 Reversed preservation properties of some negative aging classes

Before stating our main conclusions, we firstly introduce the following two lemmas which will be frequently used in sequel.

Lemma 2.1 (*Pellerey and Petakos*, 2002) For any positive integer n and $t \geq 0$, it holds that

$$(\max\{X_1, \ldots, X_n\})_t \leq_{st} \max\{(X_1)_t, \ldots, (X_n)_t\}, \tag{1}$$

where $X_1, \ldots, X_n$ are i.i.d. non-negative random variables.

Lemma 2.2 (*Barlow and Proschan*, 1981) Assume that $W(x)$ is a *Lebesgue-Stieltjes* measure, not necessarily positive.

(a) If $h(x)$ is nonnegative and increasing, and

$$\int_t^\infty dW(x) \geq 0, \qquad \text{for all } t \geq 0,$$

then $\int_0^\infty h(x)dW(x) \geq 0$.

(b) If $h(x)$ is nonnegative and decreasing and

$$\int_0^t dW(x) \geq 0, \qquad \text{for all } t \geq 0,$$

then $\int_0^\infty h(x)dW(x) \geq 0$.

Our first main result presents the reversed preservation property of NWU(2) class of life distributions.

Theorem 2.1 Assume that $X_1, \cdots, X_n$ are i.i.d. copies of X. For any fixed integer $n \geq 1$, if $\max\{X_1, \ldots, X_n\}$ is NWU(2), then X is also NWU(2).

Proof $\max\{X_1, \ldots, X_n\}$ is NWU(2), it holds that, for any $t \geq 0$,

$$(\max\{X_1, \ldots, X_n\})_t \geq_{icv} \max\{X_1, \ldots, X_n\}.$$

By (1), we have, for any $t \geq 0$,

$$\max\{(X_1)_t, \ldots, (X_n)_t\} \geq_{icv} \max\{X_1, \ldots, X_n\}.$$

That is, for any $t \geq 0$ and $x \geq 0$,

$$\int_0^x [1 - F_t^n(y)]\, dy \geq \int_0^x [1 - F^n(y)]\, dy;$$

Equivalently,

$$\begin{aligned} & \int_0^x [F^n(y) - F_t^n(y)]\, dy \\ = & \int_0^x [F(y) - F_t(y)] \left[F^{n-1}(y) + F^{n-2}(y)F_t(y) + \ldots + F(y)F_t^{n-2}(y) + F_t^{n-1}(y)\right] dy \\ \geq & \ 0. \end{aligned}$$

Notice that the function

$$\left[F^{n-1}(x) + F^{n-2}(x)F_t(x) + \cdots + F(x)F_t^{n-2}(x) + F_t^{n-1}(x)\right]^{-1}$$

is nonnegative and decreasing, it follows from Lemma 2.2 (b) that, for all $x \geq 0$ and $t \geq 0$,

$$\int_0^x \left[\bar{F}_t(x) - \bar{F}(x)\right] dx = \int_0^x [F(y) - F_t(y)]\, dy \geq 0.$$

Thus, it holds that $X_t \geq_{icv} X$, which asserting that X is also of NWU(2) property.

Theorem 2.2 in the following provides similar properties of IMRL and NWUC, it can be proved in a similar manner.

Theorem 2.2 Assume that $X_1, \cdots, X_n$ are i.i.d. copies of X. For any integer $n \geq 1$,

(i) if $\min\{X_1, \ldots, X_n\}$ is NWUC, then X is also NWUC;

(ii) if $\min\{X_1, \ldots, X_n\}$ is IMRL (DMRL), then X is also IMRL (DMRL).

Proof (i) $\min\{X_1, \ldots, X_n\}$ is NWUC, it holds that, for any $t \geq 0$,

$$(\min\{X_1, \ldots, X_n\})_t \geq_{icx} \min\{X_1, \ldots, X_n\}.$$

By the fact that

$$(\min\{X_1, \ldots, X_n\})_t \stackrel{st}{=} \min\{(X_1)_t, \ldots, (X_n)_t\} \tag{2}$$

we have, for any $t \geq 0$,

$$\min\{(X_1)_t, \ldots, (X_n)_t\} \geq_{icx} \min\{X_1, \ldots, X_n\}.$$

That is, for any $t \geq 0$ and $x \geq 0$,

$$\int_x^\infty \bar{F}_t^n(y)dy \geq \int_x^\infty \bar{F}^n(y)dy;$$

Equivalently,

$$\begin{aligned} & \int_x^\infty \left[\bar{F}_t^n(y) - \bar{F}^n(y)\right] dy \\ = & \int_x^\infty \left[\bar{F}_t(y) - \bar{F}(y)\right] \left[\bar{F}^{n-1}(y) + \bar{F}^{n-2}(y)\bar{F}_t(y) + \ldots + \bar{F}(y)\bar{F}_t^{n-2}(y) + \bar{F}_t^{n-1}(y)\right] dy \\ \geq & \ 0. \end{aligned}$$

Notice that the function

$$\left[\bar{F}^{n-1}(x) + \bar{F}^{n-2}(x)\bar{F}_t(x) + \cdots + \bar{F}(x)\bar{F}_t^{n-2}(x) + \bar{F}_t^{n-1}(x)\right]^{-1}$$

is nonnegative and increasing, it follows from Lemma 2.2 (a) that, for all $x \geq 0$ and $t \geq 0$,

$$\int_x^\infty \left[\bar{F}_t(y) - \bar{F}(y)\right] dy \geq 0.$$

Thus, $X_t \geq_{icx} X$, and hence X is also of NWUC property.

(ii) According to *Cao and Wang* (1992), X is IMRL(DMRL) if and only if, for all $t \geq s \geq 0$, $X_t \geq_{icx} (\leq_{icx}) X_s$, the desired result can be proved by the fact (2) and Lemma 2.2 (a) in a similar manner.

3 Reversed preservation properties of the RS order and the TTT transform order

Suppose $X_1, \dots, X_n$ and $Y_1, \dots, Y_n$ are i.i.d. copies of X and Y, respectively. Let

$$V_n = \max\{X_1, \dots, X_n\} - \min\{X_1, \dots, X_n\},$$

$$W_n = \max\{Y_1, \dots, Y_n\} - \min\{Y_1, \dots, Y_n\},$$

Bartoszewicz (1998) obtain the following inequalities under the assumption that $X_i \le_* Y_i$, $i = 1, \dots, n$, and EX, EY be finite,

$$\frac{E[\max\{X_1, \dots, X_n\}]}{EX} \le \frac{E[\max\{Y_1, \dots, Y_n\}]}{EY}, \tag{3}$$

$$\frac{E[\min\{X_1, \dots, X_n\}]}{EX} \ge \frac{E[\min\{Y_1, \dots, Y_n\}]}{EY}, \tag{4}$$

$$\frac{EV_n}{EX} \le \frac{EW_n}{EY}. \tag{5}$$

We will derive in this section these inequalities under a milder assumption that $X_i \le_{NBUE} Y_i$, $i = 1, \dots, n$. Furthermore, it holds also that

$$\frac{Var[\max\{X_1, \dots, X_n\}]}{E^2X} \le \frac{Var[\max\{Y_1, \dots, Y_n\}]}{E^2Y}. \tag{6}$$

Now, we give proofs for our main results.

Theorem 3.1 Let $X_i \le_{NBUE} Y_i$, $i = 1, \dots, n$. Then, inequalities (3), (4), (5) and (6) hold.

Proof It is easy to verify that

$$X \le_{NBUE} Y \Longleftrightarrow \frac{X}{EX} \le_{RS} \frac{Y}{EY} \Longleftrightarrow \frac{Y}{EY} \le_{ttt} \frac{X}{EX}.$$

According to Theorem 5.1 of *Kochar, Li and Shaked* (2002), the right spread order is preserved under the maxima, and the total time on test transform order is preserved under the minima. That is,

$$(X_i \le_{RS} Y_i \quad i = 1, \dots, n) \Longrightarrow \max\{X_1, \dots, X_n\} \le_{RS} \max\{Y_1, \dots, Y_n\}, \tag{7}$$

and

$$(X_i \leq_{ttt} Y_i \quad i = 1, \ldots, n) \Longrightarrow \min\{X_1, \ldots, X_n\} \leq_{ttt} \min\{Y_1, \ldots, Y_n\}. \tag{8}$$

By (7) and (8), we have

$$\frac{\max\{X_1, \ldots, X_n\}}{EX} \leq_{RS} \frac{\max\{Y_1, \ldots, Y_n\}}{EY}$$

and

$$\frac{\min\{Y_1, \ldots, Y_n\}}{EY} \leq_{ttt} \frac{\min\{X_1, \ldots, X_n\}}{EX}.$$

Notice the fact that $X \leq_{RS} Y$ implies both $EX \leq EY$ and $VarX \leq VarY$, inequalities (3) and (6) follows directly, and the inequality (4) follows also from the fact that $X \geq_{ttt} Y$ implies $EX \geq EY$.

Inequality (5) follows immediately from (3) and (4).

Theorem 3.2 Let $X_i \leq_{RS} Y_i$, $i = 1, \ldots, n$. Then, for all $0 < p < 1$,

$$\int_{F^{-1}(p)}^{+\infty} \left[\bar{F}_{n:n}(x) - \bar{F}_{1:n}(x)\right] dx \leq \int_{G^{-1}(p)}^{+\infty} \left[\bar{G}_{n:n}(x) - \bar{G}_{1:n}(x)\right] dx. \tag{9}$$

Proof The survival functions of the maxima of n i.i.d. copies of X and Y are, respectively,

$$\bar{F}_{n:n}(x) = 1 - F^n(x),$$

$$\bar{G}_{n:n}(x) = 1 - G^n(x).$$

The survival functions of the minima of n i.i.d. copies of X and Y are, respectively,

$$\bar{F}_{1:n}(x) = (1 - F(x))^n,$$

$$\bar{G}_{1:n}(x) = (1 - G(x))^n.$$

The right spread order $X \leq_{RS} Y$ asserts that

$$\int_t^{+\infty} \bar{F}(x) d(G^{-1}F(x) - x) \geq 0, \quad t \geq 0.$$

Since the function $1 + F(x) + \ldots + F^{n-1}(x) - (1 - F(x))^{n-1}$ is increasing and positive for all $x \geq 0$, it follows from Lemma 2.2 (a), for all $t \geq 0$,

$$\begin{aligned}
& \int_t^{+\infty} \left[\bar{F}_{n:n}(x) - \bar{F}_{1:n}(x)\right] d(G^{-1}F(x) - x) \\
= & \int_t^{+\infty} \bar{F}(x) \left[1 + F(x) + \ldots + F^{n-1}(x) - (1 - F(x))^{n-1}\right] d(G^{-1}F(x) - x) \\
\geq & \ 0.
\end{aligned}$$

That is to say,

$$\int_{F^{-1}(p)}^{+\infty}\left[\bar{F}_{n:n}(x)-\bar{F}_{1:n}(x)\right]dx\le\int_{G^{-1}(p)}^{+\infty}\left[\bar{G}_{n:n}(x)-\bar{G}_{1:n}(x)\right]dx,$$

for all $0<p<1$.

Remark If $X\le_{NBUE}Y$, then $X/EX\le_{RS}Y/EY$, putting $p\to 0$ in corresponding (9) will give rise to (5).

Now, let us turn to the reversed preservation properties of these two stochastic orderings.

Theorem 3.3 For any integer $n>0$,

(i) If $\min\{X_1,\ldots,X_n\}\le_{RS}\min\{Y_1,\ldots,Y_n\}$, then $X\le_{RS}Y$.

(ii) If $\max\{X_1,\ldots,X_n\}\le_{ttt}\max\{Y_1,\ldots,Y_n\}$, then $X\le_{ttt}Y$.

Proof (i) $\min\{X_1,\ldots,X_n\}\le_{RS}\min\{Y_1,\ldots,Y_n\}$ implies that, for all $0<p<1$,

$$\int_{F_{1:n}^{-1}(p)}^{\infty}\bar{F}_{1:n}(x)dx\le\int_{G_{1:n}^{-1}(p)}^{\infty}\bar{G}_{1:n}(x)dx.$$

This is equivalent to, for all $t\ge 0$,

$$\int_t^{\infty}\bar{F}_{1:n}(x)d\left(G_{1:n}^{-1}F_{1:n}(x)-x\right)\ge 0.$$

Notice the fact that

$$\bar{F}_{1:n}(x)=\bar{F}^n(x)$$

and

$$G_{1:n}^{-1}F_{1:n}(x)=G^{-1}F(x),$$

we have, for all $t\ge 0$,

$$\int_t^{\infty}\bar{F}^n(x)d\left(G^{-1}F(x)-x\right)\ge 0.$$

By the increasingness of $\left(\bar{F}^{n-1}(x)\right)^{-1}$ and Lemma 2.2 (a), it holds that, for all $t\ge 0$,

$$\int_t^{\infty}\bar{F}(x)d\left(G^{-1}F(x)-x\right)\ge 0,$$

which is equivalent to, for all $0<p<1$,

$$\int_{F^{-1}(p)}^{\infty}\bar{F}(x)dx\le\int_{G^{-1}(p)}^{\infty}\bar{G}(x)dx.$$

Therefore, $X \leq_{RS} Y$.

(ii) $\max\{X_1, \ldots, X_n\} \leq_{ttt} \max\{Y_1, \ldots, Y_n\}$ states that, for all $0 < p < 1$,

$$\int_0^{F_{n:n}^{-1}(p)} \bar{F}_{n:n}(x)dx \leq \int_0^{G_{n:n}^{-1}(p)} \bar{G}_{n:n}(x)dx.$$

This is equivalent to, for all $t \geq 0$,

$$\int_0^t \bar{F}_{n:n}(x)d\left(G_{n:n}^{-1}F_{n:n}(x) - x\right) \geq 0.$$

Since

$$\bar{F}_{n:n}(x) = 1 - F^n(x)$$

and

$$G_{n:n}^{-1}F_{n:n}(x) = G^{-1}F(x),$$

we have, for all $t \geq 0$,

$$\int_0^t (1 - F^n(x))\, d\left(G^{-1}F(x) - x\right) \geq 0.$$

By the decreasingness of

$$\left(F(x) + \ldots + F^{n-1}(x)\right)^{-1}$$

and Lemma 2.2 (b), it follows that, for all $t \geq 0$,

$$\int_0^t \bar{F}(x)d\left(G^{-1}F(x) - x\right) \geq 0,$$

which is equivalent to, for all $0 < p < 1$,

$$\int_0^{F^{-1}(p)} \bar{F}(x)dx \leq \int_0^{G^{-1}(p)} \bar{G}(x)dx.$$

So, $X \leq_{ttt} Y$.

As an application, the following Corollary 3.4 presents conclusions about the NBUE ordering.

Corollary 3.4 For any positive integer n,

(i) if

$$\frac{\min\{X_1, \ldots, X_n\}}{EX} \leq_{RS} \frac{\min\{Y_1, \ldots, Y_n\}}{EY},$$

then $X \leq_{nbue} Y$.

(ii) if

$$\frac{\max\{X_1,\ldots,X_n\}}{EX} \leq_{ttt} \frac{\max\{Y_1,\ldots,Y_n\}}{EY},$$

then $X \geq_{nbue} Y$.

Proof Notice that

$$\frac{X}{EX} \leq_{RS} \frac{Y}{EY} \Longleftrightarrow X \leq_{nbue} Y \Longleftrightarrow \frac{X}{EX} \geq_{ttt} \frac{Y}{EY},$$

the proof can be easily followed from Theorem 3.3.

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