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Gauge Fields and the Algebra of Polarizations.

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Summary. — Rigorous c-number solutions of the source-free (but nonlinear, because of the curvature tensor) field equations of a gauge field B_μ for the local group G are found. The polarization matrices are forced thereby to satisfy a Lie algebra. If this is Abelian, one gets the usual gauge-field theory, with any Lie group G desired. In the non-Abelian case, some curious new features emerge. The polarizations turn out to be the generators of the little group of a timelike momentum, and G is fixed as a group containing the homogeneous Lorentz group L. If one uses these well-determined polarizations to build the interaction-picture quantized gauge field, and requires G to be « internal » (*i.e.* to commute with the « external » Poincaré group), then a continuous infinity of independent polarization states are required, even though as a group-theoretical object B_μ belongs to the mass > 0, spin-1 representation space. Interpreting B_μ as W_μ, the intermediate boson, one gets an effective current-current interaction invariant against this internal Lorentz group which, since $SU_3 \not\supset L$, breaks SU_3 in a specific way.

1. – Introduction.

It is well understood [1] that gauge fields, introduced to secure invariance against local transformations of some « internal » Lie group G, are nothing but what has been extensively studied in non-Riemannian geometry under the name of « linear connection ». Thus we know that any gauge-field equations must be written in terms of the curvature tensor, the only tensor formable from the connection. This curvature tensor shows a « self-coupling » nonline-

[1] For example C. N. YANG and R. L. MILLS: *Phys. Rev.*, **96**, 191 (1954); R. UTIYAMA: *Phys. Rev.*, **101**, 1597 (1956); J. SAKURAI: *Ann. of Phys.*, **11**, 1 (1960).

arity (²) of a very special type. We study here what limitations on gauge-field theory are imposed by taking this nonlinear structure seriously.

In the usual treatement the gauge field in the Interaction Picture (I.P.) satisfies the Klein-Gordon equation but the polarization vectors (³) $e_\mu(k)_a{}^b$ (space-time vectors that is, matrices of course on the internal space) are undetermined by the field equations. We find however that if we wish to get rigorous c-number transverse plane wave solutions $B_\mu(x) = e_\mu(k)_a{}^b \varphi(k\cdot x)$ to the source-free (*i.e.* only self-coupled) gauge-field equations, we must force the polarization matrices (for any k) to form a Lie algebra $\mathscr{L}$ (⁴). In the limit $f \to 0$ as the coupling to the fermion fields vanishes, these rigorous solutions ($\varphi(k\cdot x)$ = elliptic functions) go into solutions of the Klein-Gordon equation ($\varphi(k\cdot x) \to$ linear combination of $\exp[\pm ik\cdot x]$, $k^2 =$ const) but now with well-determined polarization matrices. When we build the quantized I.P. gauge field, we accept the polarizations as determined this way.

There are two cases. Case I): $\mathscr{L}$ is Abelian; Case II): $\mathscr{L}$ is non-Abelian. Case I) yields the usual type of theory with invariance under any internal Lie group G desired, and a finite number of polarizations $e_\mu^{(i)}(k)_a{}^b$, $i = 1, \ldots, n$, with the usual particle interpretation of these states of the quantized field. Thus Case I) seems to be the proper framework for a theory of vector fields interacting with the full symmetry G with fermion fields, *e.g.* Yang-Mills fields or Sakurai's vector mesons which mediate the strong interactions, or possibly, with $G = SU_3$, the vector-meson resonances.

Case II) yields some curious and, as far as we know, new concepts. Lorentz invariance of B_μ *fixes* G as $L \equiv$ homogeneous Lorentz group, and $\mathscr{L}$ as the SU_2 subalgebra (the $e_\mu(k)$ in fact generate the little group of the timelike momentum k, namely $SO_3 \simeq SU_2$). There is a (continuous) infinity of polarization states, so that it is not clear what the particle interpretation is, nor how these states would be recognized experimentally. Finally, this effective interaction, being L-invariant, breaks SU_3. This suggests that Case II) could not apply to a hypothetical carrier of the strong interaction, but might describe W_μ, the intermediate boson.

Interpreting B as W, then, weakly interacting particles would have to fall into internal L-multiplets labeled by (j, k), j and k half-integers. In another

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(²) Except for the case dim $G = 1$ (photon).

(³) Notation: $a, b, c, \ldots = 1, \ldots, M$ are internal indices, and will usually be suppressed. The foolproof matrix convention on these (or any other) indices is that adjacent index pairs are contracted. Thus $\Gamma_\mu \psi$ means $(\Gamma_\mu \psi)_a = \Gamma_{\mu a}{}^b \psi_b$, $(kA)_\mu = k_\nu A^\nu{}_\mu$, etc. In general we use the notation of Jauch and Rohrlich in their book (*Theory of Photons and Electrons* (Cambridge, Mass., 1955)) but with $t \equiv x^4$. $\bar\psi^a \equiv \psi_a{}^* A$, where A is the Dirac operator $= i\gamma^4$ in the standard representation. Thus $\bar\psi$ belongs to the G-representation adjoint to that of ψ: if $\psi_a \to D(g)_a{}^b \psi_b$ under G, then $\bar\psi^a \to \bar\psi^b D^{-1}(g)_b{}^a$.

(⁴) $\mathscr{L}$ is a subalgebra of the Lie algebra of G, see Sect. 3.

paper [5] we have investigated the properties of the resulting weak-interaction theory using only the special multiplets $(j, 0)$.

2. – Notation and background.

The Lagrangian for a massive [6] gauge field $W_\mu(x)_a^{\ b}$ coupled to a set of fermion fields $\psi_a(x)$ is

$$\mathscr{L} = -\operatorname{Tr}\bar{\psi}(\gamma^\mu\tilde{\nabla}_\mu + M)\psi - \frac{1}{4}\operatorname{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^2}{2}\operatorname{Tr}W_\mu W^\mu; \tag{2.1}$$

$\tilde{\nabla}_\mu$ is the covariant derivative: $\tilde{\nabla}_\mu\psi = \partial_\mu\psi - \tilde{\Gamma}_\mu\psi$ [3], where

$$\tilde{\Gamma}_\mu = P_L\Gamma_\mu\,, \qquad P_L = (1 + i\gamma_5)/2\,, \qquad \Gamma_\mu \equiv 2ifW_\mu\,. \tag{2.2}$$

$W_{\mu a}^{\ \ b}$ has the form $W_\mu^{\ A}T_{Aa}^{\ \ b}$, where the T_A, $A = 1, \dots, N$, are matrices generating the internal Lie group G on the internal space spanned by the ψ_a;

$$-2ifF_{\mu\nu} \equiv R_{\mu\nu}\,, \qquad R_{\mu\nu} \equiv -\partial_\mu\Gamma_\nu + \partial_\nu\Gamma_\mu - [\Gamma_\mu, \Gamma_\nu]\,. \tag{2.3}$$

$R_{\mu\nu}(\equiv R_{\mu\nu a}^{\ \ \ \ b})$ is the curvature tensor of the (reduced) linear connection Γ_μ. Internal indices $a, b, \dots$ will generally be suppressed; the trace in (2.1) saturates these indices.

If we write

$$\mathscr{L} = \mathscr{L}_\psi + \mathscr{L}_w + \mathscr{L}_1\,, \qquad \mathscr{L}_1 \equiv fW_\mu^{\ A}j^\mu_{\ A}\,, \tag{2.4}$$

where

$$j^\mu_{\ A} \equiv i\operatorname{Tr}\bar{\psi}\gamma^\mu(1 + i\gamma_5)T_A\psi\,, \tag{2.5}$$

then $j^\mu_{\ A}$ is the well-normalized source (fermion) current density, whose vector and axial parts satisfy the G-current algebra [7].

We have allowed the total linear connection $\tilde{\Gamma}_\mu$ to act on both the internal and Dirac spinor indices. Then the separable form (2.2) was chosen, with P_L the left-handed projection in Dirac spinor space and the *reduced* connection Γ_μ

(5) $\Delta T = \frac{1}{2}$ *rule and the Cabibbo angle*, to be published.

(6) It must be emphasized that the introduction of the mass term in (2.1) violates the whole spirit of linear connection theory, and cannot survive in a correct formulation. We include it as representing some sort of phenomenology, since otherwise the effective current-current four-fermion interaction (Sect. 5) behaves wrong at small momentum transfers.

(7) Thus the $Q_A(t) \equiv \frac{1}{2}\int d^3x j^4_{\ A}$ satisfy the G-Lie algebra with the same structure constants as the T_A.

having only internal indices, to give the interaction the known $V-A$ form (before renormalization).

3. – The polarization algebra.

The field equations for W_μ are

$$\nabla_\mu F^{\mu\nu} - \mu^2 W^\nu = -fj^\nu\,, \qquad j^{\nu}{}_a{}^b \equiv i\bar{\psi}^b \gamma^\nu (1 + i\gamma_5)\psi_a \tag{3.1}$$

where ∇_μ is formed with Γ_μ.

For the source-free gauge field (set $j^\nu = 0$) we get the nonlinear field equations

$$\Box W^\nu + 4f^2\big[W_\mu, [W^\mu, W^\nu]\big] - \mu^2 W^\nu + 2if[W_\mu, \partial^\nu W^\mu] = 0 \tag{3.2}$$

since $0 = \nabla_\mu W^\mu \equiv \partial_\mu W^\mu - 2if[W_\mu, W^\mu] = \partial_\mu W^\mu$ follows from these field equations.

If we look for c-number plane wave solutions of (3.2), $W^{\nu}{}_a{}^b = e^\nu(k)_a{}^b \varphi(k\cdot x)$, $k_\nu e^\nu(k)_a{}^b = 0$, the last term in (3.2) vanishes. Then in order that the double commutator be proportional to $e^{\nu}{}_a{}^b$, we demand that the polarization matrices $e^\nu \equiv e^\nu(k)$ form a Lie algebra ([8]):

$$[e_\mu, e_\nu] = ic_{\mu\nu\lambda\xi} k^\xi e^\lambda \tag{3.3}$$

The structure constants $c_{\mu\nu\lambda}(k)$ must be skew symmetric in μ, ν, and if we require that they be *linear* in k, as in (3.3), then $c_{\mu\nu\lambda\xi}$ must be totally antisymmetric in μ, ν, λ to guarantee $k\cdot e = 0$.

Further, in order that (3.3) be self-consistent ($\equiv$ hold for any k), we must impose a transformaton law

$$e_\mu(k\Lambda) = D(\Lambda, k)^{-1} e_\nu(k) D(\Lambda, k) \Lambda^\nu{}_\mu \tag{3.4}$$

for some matrices $D(\Lambda, k)$ acting on the internal space ([9]).

([8]) This idea is due to G. GOEDECKE.

([9]) Actually the transformation law (3.4) is sufficient, not necessary. And in fact, in the Abelian case I, the separable solutions $e_\mu^{(\lambda B)}(k)_a{}^b = e_\mu^{(\lambda)}(k) e^{(B)A} T_{Aa}{}^b$ transform according to (suppress a, b) $e_\mu^{(\lambda B)}(k\Lambda) = \sum_{\lambda'} e_\nu^{(\lambda' B)}(k) d_{\lambda'\lambda}(\Lambda, k)\Lambda^\nu{}_\mu$, where $d_{\lambda'\lambda}(\Lambda, k)$ is the rotation induced on the space-time polarizations by Λ, and $D(\Lambda, k) = \mathbf{1}$, all Λ. This is more general than (3.4). However, (3.5) still holds for all k and λB because one actually has the stronger $[e_\mu^{(\lambda B)}(k), e_\nu^{(\lambda' B)}(k)] = 0$, $\lambda, \lambda' = 1, 2, 3$.

If the $c_{\mu\nu\lambda\xi}=0$ we get the Abelian case:

$$[e_\mu(k)\,,\,e_\nu(k)]=0\,, \qquad \text{Case I)}. \tag{3.5}$$

Otherwise the $c_{\mu\nu\lambda\xi}$ are not all zero, and it can be shown [10] that they are proportional to the totally antisymmetric quantity $\varepsilon_{\mu\nu\lambda\xi}$. Hence by normalizing the e_μ appropriately we get

$$[e_\mu(k), e_\nu(k)]=i\varepsilon_{\mu\nu\lambda\xi}e^\lambda(k)k^\xi\,, \qquad \text{Case II)}. \tag{3.6}$$

The linking of space-time and internal indices through (3.6) is the basic reason why the internal symmetry G is usually broken by Case II).

A *complete* set of solutions [9] of I) (where we define $e_\mu(k)\equiv e_\mu(k)^A T_A$) can be taken in the separable form $e_\mu^{(\lambda B)}(k)^A=e_\mu^{(\lambda)}(k)e^{(B)A}$, where $e_\mu^{(\lambda)}(k)$, $\lambda=1,2,3$, are orthonormal spacelike and $k\cdot e^{(\lambda)}(k)=0$, and $e^{(B)A}$, $B=1,\ldots,N'\leqslant N$, is any orthonormal set in the adjoint representation space of G. (The theory will be G-invariant if and only if $N'=N$.)

By « complete » we mean a sufficiently large set of polarizations that the two-point function $\langle W_\mu(x)\,W_\nu(y)\rangle_0$ of the I.P. gauge field be proportional, in momentum space, to $(g_{\mu\nu}+k_\mu k_\nu/\mu^2)$.

As for Case II), we recognize (3.6) as none other than the commutation relations for the generators $w_\mu(k)$ of the little group of k! Hence $e_\mu(k)=w_\mu(k)$ $=\frac{1}{2}\varepsilon_{\mu\nu\lambda\xi}k^\nu M^{\lambda\xi}$ is a solution for any finite-dimensional representation $M^{\lambda\xi}$ of the L-Lie algebra. Thus in Case II) G is forced to be the homogeneous Lorentz group, or a group containing it. In the transformation law (3.4), $D(\Lambda,k)$ becomes $S(\Lambda)\equiv\exp[i/2]\omega^{\mu\nu}M_{\mu\nu}$, where $\omega^{\mu\nu}$ are the parameters of Λ.

Now given one set of matrices $M_{\mu\nu}$, then of course $M^{(S)}{}_{\mu\nu}\equiv S^{-1}M_{\mu\nu}S$, for any nonsingular matrix S, gives another, usually different, solution of II); and whether these represent « independent polarizations » can only be decided with reference to the two-point function (or the propagator). We shall find that completeness, in the sense of above, demands a continuous infinity of « independent » polarizations.

Getting back to the problem posed at the beginning of this Section, we find from (3.2) and (3.5), (3.6) that the phase function $\varphi(k\cdot x)$ satisfies

$$(\Box-\mu^2)\varphi=0\Rightarrow\varphi=\exp[\pm ik\cdot x]\,,\qquad k^2+\mu^2=0; \qquad \text{Case I)}, \tag{3.7}$$

$$\varphi''+\varphi-8f^2\varphi^3=0\,,\; k^2+\mu^2=0; \qquad \text{Case II)}, \tag{3.8}$$

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[10] Write (3.3) for the $e_\lambda(k\Lambda)$ and use (3.4). The $D(\Lambda,k)$ cancel, and one is left with $c_{\mu\nu\lambda\xi}\Lambda^\mu{}_\varrho\Lambda^\nu{}_\tau\Lambda^\lambda{}_\zeta\Lambda^\xi{}_\omega=c_{\varrho\tau\zeta\omega}$. Thus $c_{\mu\nu\lambda\xi}=c\varepsilon_{\mu\nu\lambda\xi}$, $c=$ complex number and $\neq 0$ since $c_{\mu\nu\lambda\xi}\neq 0$, *q.e.d.*

where $\varphi'(z) \equiv d\varphi/dz$. (3.8) is the anharmonic oscillator equation and can be solved exactly in terms of elliptic functions. As $f \to 0$, $\varphi \to \exp[\pm ik\cdot x]$.

We remark that if there is no mass term, however, since the double commutator term in (3.2) is proportional to k^2, just like the term $\Box W^\nu$, φ satisfies the equation $\varphi'' - 8f^2\varphi^3 = 0$ in Case II) where $k^2 =$ any constant. *I.e.* in the case $\mu^2 = 0$ in the Lagrangian, the wave functions need not be massless.

4. – The quantum field and its propagator.

We concentrate on the new Case II). The quantized gauge field in the I.P. is then ([11])

$$(4.1) \qquad W_\mu(x) = (2\pi)^{-\frac{3}{2}}\mu^{-1}N\int d^4k\theta(k)\delta(k^2+\mu^2)\sum_i e_\mu^{(i)}(k)\cdot \\ \cdot\{a^{(i)}(k)\exp[ik\cdot x] + a^{(i)}(k)^*\exp[-ik\cdot x]\},$$

where the polarizations $e_\mu^{(i)}$ satisfy the algebra (3.6) and i runs over a complete set, to be determined. N normalizes the sum over i. Since $e_\nu^{(i)} = w_\nu^{(i)}$, little group generators, $\mu^{-1}e_\nu^{(i)}$ are dimensionless.

One then finds

$$(4.2) \qquad \langle W_\mu(x)\, W_\nu(y)\rangle_0 = (2\pi)^{-3}\int d^4k\theta(k)\delta(k^2+\mu^2)\exp[ik\cdot(x-y)]A_{\mu\nu}(k),$$

where

$$(4.3) \qquad A^{\mu\nu}(k)(=A_{\mu\nu}(k)_a{}^b{}_c{}^d) \equiv \mu^{-2}N^2\sum_i e_\mu^{(i)}(k)\otimes e_\nu^{(i)}(k)\,.$$

Now since a and b are internal indices, they should not be rotated by (external) Poincaré transformations. So we impose the conventional relativistic transformation law

$$(4.4) \qquad U(L)\,W^\mu(x)_a{}^b\,U(L)^{-1} = \Lambda^\mu{}_\nu W^\nu(L^{-1}x)_a{}^b\,, \qquad L^{-1}x \equiv \Lambda^{-1}(x-a)\,,$$

(note the unaffected internal indices). Specializing to a pure Lorentz transformation ($a = 0$), this gives via (4.2)

$$(4.5) \qquad A_{\mu\nu}(k\Lambda) = A_{\lambda\xi}(k)\Lambda^\lambda{}_\mu\Lambda^\xi{}_\nu\,.$$

([11]) We have chosen to make $W_\mu(x)^A$ (obtained by factoring out the generators $T_A = M_{\lambda\xi}$) a self-adjoint field. By using the polarizations corresponding to the inequivalent L-representations (j, k) and (k, j) in the annihilation and creation parts respectively, a more general theory might be obtained.

But from the definition (4.3) and the law (3.4) with $D(\Lambda, k) = S(\Lambda)$ we get

$$A_{\mu\nu}(k\Lambda) = S(\Lambda)^{-1} \otimes S(\Lambda)^{-1} A_{\lambda\xi}(k) S(\Lambda) \otimes S(\Lambda) \Lambda^{\lambda}{}_{\mu} \Lambda^{\xi}{}_{\nu} \,. \tag{4.6}$$

Thus for each index pair $\lambda\xi$, $A_{\lambda\xi}(k)$ must be invariant under conjugation by $S(\Lambda)$.

This can never be realized by a finite number of polarizations. If we take an infinite number, however, one for each Lorentz rotation Λ, and define

$$e_{\mu}^{(\Lambda)} \equiv S(\Lambda)^{-1} e_{\mu}^{(0)} S(\Lambda) \,, \tag{4.7}$$

where $e_{\mu}^{(0)}$ is defined with some standard matrices $M_{\lambda}^{(0)}$, and

$$A_{\mu\nu}(k) = \mu^{-2} N^2 \int_L \mathrm{d}\Lambda_1 e_{\mu}^{(\Lambda_1)}(k) \otimes e_{\nu}^{(\Lambda_1)}(k) \,, \tag{4.8}$$

where the integration goes over the whole group manifold L and $\mathrm{d}\Lambda_1$ is the invariant group volume element, then $A_{\lambda\xi}(k)$ is indeed invariant under conjugation by any $S(\Lambda)$, since these matrices simply get « absorbed ».

To evaluate (4.8), note that $A_{\mu\nu}(k)$ must have the form

$$A_{\mu\nu}(k) = \mu^{-2}(-k^2 g_{\mu\nu} + k_{\mu} k_{\nu})(a/2) M_{\lambda\xi} \otimes M^{\lambda\xi} \,, \tag{4.9}$$

where a is a pure number, and $M_{\mu\nu} = S(\Lambda)^{-1} M_{\mu\nu}^{(0)} S(\Lambda)$ for any $\Lambda \in L$. This follows from $k^{\mu} A_{\mu\nu} = k^{\nu} A_{\mu\nu} = 0$, $[S(\Lambda) \otimes S(\Lambda), A_{\mu}{}^{\mu}] = 0$, and $A_{\mu\nu}$ quadratic homogeneous in k. Hence to evaluate $A_{\mu\nu}$ it is enough to evaluate $A_{\mu}{}^{\mu}$. Putting in the explicit expressions for $e_{\mu}^{(\Lambda_1)}(k)$, the integrals can be done ([12]) and we get finally $a = \frac{1}{6}$. Equation (4.1) becomes now

$$W_{\mu}(x) = (2\pi)^{-\frac{3}{2}} \mu^{-1} N \int \mathrm{d}^4 k \theta(k) \delta(k^2 + \mu^2) \int \mathrm{d}\Lambda e_{\mu}^{(\Lambda)}(k) \cdot \\ \cdot \{a^{(\Lambda)}(k) \exp[ik\cdot x] + a^{(\Lambda)}(k)^* \exp[-ik\cdot x]\} \,, \tag{4.10}$$

$$e^{(\Lambda)}{}_{\mu}(k) = (\tfrac{1}{2}) \varepsilon_{\mu\nu\lambda\xi} k^{\nu} M^{(\Lambda)\lambda\xi} \,, \qquad M^{(\Lambda)\lambda\xi} \equiv S(\Lambda)^{-1} M^{(0)\lambda\xi} S(\Lambda) \,.$$

([12]) These are evaluated by considering $k_4' = (k\Lambda_1)_4$ a complex variable and rotating the path to the imaginary axis. The justification is that otherwise we get a noninvariant result, nonsensical because we know that $A_{\mu\nu}$ commutes with $S(\Lambda) \otimes S(\Lambda)$ and is thus $\propto M_{\mu\nu} \otimes M^{\mu\nu}$. This « paradox » is not so surprising when we consider that these integrals are divergent, because L is noncompact ($N^{-1} = \infty$), and a limiting procedure must be used.

Then the transformation law (4.4) implies

$$U(\Lambda)\, a^{(\Lambda_1)}(k)\, U(\Lambda)^{-1} = a^{(\Lambda^{-1}\Lambda_1)}(k\Lambda)\,, \tag{4.11}$$

as is easily worked out by changing dummy variables a few times, using (3.4) and the invariance of the volume elements d^4k and $d\Lambda$. The commutation relations are

$$[a^{(\Lambda)}(k),\, a^{(\Lambda')}(k')^*] = 2\omega_k\, \delta(\boldsymbol{k}-\boldsymbol{k}')\, \delta(\Lambda-\Lambda')\,, \tag{4.12}$$

and these are invariant under (4.11).

The quantum field thus shows several new features relative to the usual case I). There is a continuous infinity of « polarization states » correlated 1-1 with Lorentz rotations. These do not maintain their identity under Lorentz transformations (cf. (4.11)). Moreover, the interpretation of the « one-gauge-particle state » $a^{(\Lambda)}(k)^*|0\rangle$ is not clear. Group-theoretically speaking, $W_\mu(x)_a{}^b$ given by (4.10) belongs to the mass $= \mu$, spin $= 1$ representation of the Poincaré group for each a, b, as one sees from (4.4) together with $\partial_\mu W^\mu = 0$. It differs thus *dynamically* from the usual such gauge vector (Case I)), *i.e.* in its structure as an operator in state-vector Hilbert space. The infinite number of operators $a^{(\Lambda)}(k)$, linearly independent by (4.12), is not the same thing as the finite number of operators in the Case I) theory.

5. – The effective weak interaction.

The propagator turns out, after some labor, to be [13]

$$i\langle T W_\mu(x)\, W_\nu(y)\rangle_0 = \frac{1}{6}\,(J_i \otimes J^i - K_i \otimes K^i)\cdot \cdot (2\pi)^{-4}\!\int d^4k \left(g_{\mu\nu} + \frac{k_\mu k_\nu}{\mu^2}\right) \frac{\exp[ik\cdot(x-y)]}{k^2+\mu^2-i\delta}\,, \tag{5.1}$$

where $J_i = J^i \equiv M_{jk}$ (ijk cyclic permutation of 123),

$$K_i = K^i \equiv M_{i4}\,, \qquad i, j, k = 1, 2, 3.$$

Hence taking $O(f^2)$ Møller scattering type graph, we find that in the small momentum transfer limit $|k_\nu|^2 \ll \mu^2$, $\nu = 1, \ldots, 4$, the theory leads to the ef-

[13] Here, just as in ordinary vector meson theory, the vanishing of certain singular integrals in the complex k^4-plane allow the replacement $(k^4)_{\text{mass shell}} \equiv \omega_k \to k^4$ in the factor $(-k^2 g_{\mu\nu} + k_\mu k_\nu)_{\text{m.s.}}$ in the propagator. We chose to make this replacement in the term $k_\mu k_\nu$ but kept $-k^2 = \mu^2$ in the $-k^2 g_{\mu\nu}$ term.

fective current-current weak interaction

$$\mathscr{H}_w = \frac{G}{\sqrt{2}}(\boldsymbol{J}_\mu \cdot \boldsymbol{J}^\mu - \boldsymbol{K}_\mu \cdot \boldsymbol{K}^\mu)\,, \qquad \frac{G}{\sqrt{2}} = -\frac{f^2}{3\mu^2}\,. \tag{5.2}$$

Here the currents $\boldsymbol{J}$ and $\boldsymbol{K}$ are defined by the general formula (2.5), taking $T_A = M_{23}, M_{31}, M_{12}$ and M_{14}, M_{24}, M_{34} respectively.

To make contact with physics, it remains the assign to weakly interacting particles to L-multiplets (j, k) (« weak multiplets ») and more (since $SU_3 \not\supset L = SO_{3,1}$ and thus SU_3 is broken) to assign the actual matrices $\boldsymbol{M}_{\mu\nu}$ with whose bases the hadrons are identified. This is explored in another paper (ref. (6)), so we shall conclude here by just making a few general remarks.

$\mathscr{H}_w$ is « rotationally invariant », meaning that it commutes with the $\boldsymbol{J}$ (actually it commutes with the $\boldsymbol{K}$ also). Hence neutral currents are necessarily implied. In ref. (6) it is shown how this allows a derivation of the $\Delta T = \frac{1}{2}$ rule, with the correct small admixture of $\Delta T = \frac{3}{2}$ for the nonleptonic decays. However, these neutral currents also lead to some of the usual difficulties.

Note that the currents $\boldsymbol{J}$ and $\boldsymbol{K}$ have the same commutation relations as vector and axial parts. However $\mathscr{H}_w$ has no cross terms in these, *i.e.* we cannot describe $\mathscr{H}_w$ by a total current with V and A parts. This was a direct consequence of having the Lorentz group as internal group. What significance this may have, we do not know.

RIASSUNTO (*)

Si trovano rigorose soluzioni in numeri c e delle equazioni di campo libere da sorgenti (ma non lineari, a causa del tensore di curvatura) del campo di gauge B_μ per il gruppo locale G. Si costringono così le matrici di polarizzazione a soddisfare a un'algebra di Lie. Se questa è abeliana, allora si ha la normale teoria dei campi di gauge, con un qualsivoglia gruppo di Lie G. Nel caso non abeliano, emergono alcune nuove caratteristiche abbastanza insolite. Le polarizzazioni risultano essere i generatori del piccolo gruppo di una quantità di moto temporale, e G è fissato come un gruppo contenente il gruppo omogeneo di Lorentz L. Se si usano queste polarizzazioni ben determinate per costruire il campo di gauge quantizzato del modello di interazione e si richiede che G sia « interno » (cioè che commuti col gruppo di Poincaré « esterno »), allora si richiede un'infinità continua di stati di polarizzazione indipendenti, anche se B_μ della teoria dei gruppi appartiene allo spazio delle rappresentazioni con massa > 0 e spin 1. Considerando B_μ come W_μ, il bosone intermedio, si ha un effettiva interazione corrente-corrente invariante rispetto a questo gruppo interno di Lorentz che, poiché $SU_3 \not\supset L$, rompe la simmetria di SU_3 in modo specifico.

(*) *Traduzione a cura della Redazione.*

Калиброванные поля и алгебра поляризаций.

Резюме (*). — Получены строгие с-численные решения уравнений поля без источников (но нелинейные из-за тензора кривизны) для калибровочного поля B_μ для локальной группы G. При этом требуется, чтобы матрицы поляризации удовлетворяли алгебре Ли. Если это абелев случай, то получается обычная калибровочная теория поля, с любой желаемой G группой Ли. В неабелевом случае появляются некоторые любопытные новые особенности. Оказывается, что поляризации представляют генераторы маленькой группы времени-подобного импульса, и G определяется, как группа, содержащая однородную группу Лорентца L. Если использовать эти хорошо определенные поляризации для построения картины взаимодействия квантованного калибровочного поля, и требовать, чтобы G являлась « внутренней » ([illegible] заменить с « внешней » группой Пуанкаре), то требуется непрерывная бесконечность независимых состояний поляризации, даже если теоретико-групповой объект B_μ принадлежит пространству представлений с массой > 0 и спином 1. Интерпретируя B_μ, как W_μ, промежуточный бозон, можно получить эффективное ток-токовое взаимодействие, инвариантное относительно этой внутренней группы Лорентца, которое нарушает SU_3 определенным образом, так как $SU_3 \not\supset L$.

(*) *Переведено редакцией.*