

About Charged Tachyons.

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1. — In recent times many authors ⁽¹⁾ expressed the opinion that *tachyons* (faster-than-light particles) are probably neutral. We think that often this opinion was based essentially on considerations which may be depicted as follows.

Let us consider the complex plane ⁽²⁾ of the (complex) *proper masses* m_0 of « elementary » particles, as in Fig. 1. According to the « switching principle » ⁽³⁾, we may associate antiparticles ⁽⁴⁾ to the (negative) rest masses, symmetrical to the particle (positive) discrete spectrum. The point 0 corresponds to photons, and usually it is argued that they are neutral also because « coincide » just with their own antiparticles. Lastly, the tachyon proper masses fall on the imaginary axis. As the distinction between « tachyons » and « antitachyons » does *not* rest ⁽⁵⁾ on the sign of their proper mass $m_0 = \pm i\mu$ (μ real), but obviously ⁽⁵⁾ on the sign of their relativistic energy

$$E = \pm \sqrt{\mathbf{p}^2 - \mu^2} \quad [\mu < |\mathbf{p}| < \infty],$$

it can be heuristically thought that, in the m_0 complex plane, the correspondence *particle-antiparticle* is produced by the symmetry with respect to the

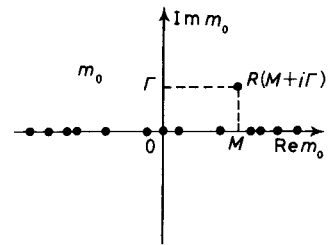


Fig. 1. — Complex plane of the proper masses m_0 . On the real axis we may represent the (discrete) spectrum of *bradyons* ^(b)—common particles, with $v < c$ and $p^2 > 0$ —(positive semiaxis) and of their antiparticles (negative semiaxis). The point $m_0 = 0$ corresponds to *luxons*—*e.g.* photons, with $v = c$ and $p^2 = 0$ —which have the conventional proper mass zero. *Tachyons*—with $v > c$ and $p^2 < 0$ —shall fall on the imaginary axis (see also ref. ⁽⁴⁾).

⁽¹⁾ a) J. DHAR and E. C. G. SUDARSHAN: *Phys. Rev.*, **174**, 1808 (1968), and references therein; b) M. BALDO and E. RECAMI: *Lett. Nuovo Cimento*, **2**, 643 (1969); c) T. ALVAGER: private communication.

⁽²⁾ a) E. RECAMI: *Giornale di Fisica*, **10**, 195 (1969); b) *Rendic. Accad. Naz. Lincei* (to appear).

⁽³⁾ a) See *e.g.*: E. C. G. SUDARSHAN: Report NYO-3399-191/SU-1206-191 (Syracuse, Dec. 1968);

b) V. S. OLKHOVSKY and E. RECAMI: *Nuovo Cimento*, **63 A**, 814 (1969).

⁽⁴⁾ See also: R. P. FEYNMAN: *Phys. Rev.*, **76**, 749 (1949).

⁽⁵⁾ a) M. BALDO and E. RECAMI: *Lett. Nuovo Cimento*, **2**, 643 (1969); b) O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. Journ. Phys.*, **30**, 718 (1962). See also ref. ⁽⁴⁾.

⁽⁶⁾ W. KROLIKOWSKY: Report P-1060/VII/PH (Warszawa, 1969).

imaginary axis. Thus, analogously to *luxons*, also tachyons «should coincide» with their antiparticles and «should» be neutral.

In this letter, we want to emphasize that this conclusion is by no means a necessary one.

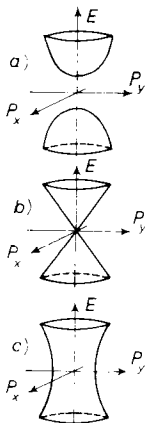


Fig. 2. — The hypersurfaces $E^2 - p^2 = m_0^2$: a) for bradyons (type I particles); b) for luxons (type II particles); c) for tachyons (type III particles). An axis (p_z) is, of course, not represented. Here we use natural units.

In fact, the *indistinguishability* between particles and antiparticles requires neutrality only for *bradyons* (= common particles), for which the *matter-antimatter* character is an absolute one⁽²⁾, *i.e.* independent of the (inertial) observers^(*). Namely, if *antiparticles* appear as the (orthodox) reinterpretations^(3b) of «negative-energy particles travelling backwards in time»^(**), such an indistinguishability is possible only when the electric charge is lacking (besides other conditions).

The luxon case is a limit one, and the absence of electric charge on photons (*e.g.*) can be also *justified* by remembering that they are just the quanta of the electromagnetic field!

For tachyons, the *coincidence* of «particles» and «antiparticles» means merely that their material/antimaterial character is *relative* to the observer^(5b). As results from Fig. 2, for particles of type III a Lorentz transformation can bring (with continuity) a point of the upper sheet onto the lower sheet^(2a-5b). To clarify this point, we must recall that it is always (possible and) better to construct theories by considering process *amplitudes* rather than states⁽⁷⁾. Which is particularly true for theories including tachyons, as a particle that appears as an *entering* tachyon to an observer may appear as an *escaping* antitachyon to another observer, and *vice versa*^(1b).

In a certain sense, this «switching principle» for tachyons plays the same role as the «crossing principle» for bradyons, the main difference being that the *crossing* for the reactions (*e.g.*)

$$(1) \quad \begin{cases} A) & a - b \rightarrow c \div d, \\ B) & a \div \bar{c} \rightarrow \bar{b} \div d, \end{cases}$$

where the bar means «antiparticle», requires that an analytic function $\mathcal{A}(s, t)$ exists, which gives the amplitudes for both reactions $A), B)$ in correspondence to their respective allowed domains D_A, D_B of the invariants^(***) s, t . In the case of tachyons, on the contrary, D_A and D_B must coincide, as s and t are just Lorentz-invariant variables, and reactions $A), B)$ may be the (different) descriptions of the same physical process according to two *inertial observers*. Thus, in general, for a theory regarding tachyons to be said «Lorentz-invariant», we might require that it *conserves* the reaction amplitudes, in the above-mentioned sense⁽⁷⁾. Besides, as usual, according to the

(*) As usual, physical observers are considered to be of type I. With respect to (unphysical) superluminal inertial frames, tachyons would behave as bradyons, and *vice versa*.

(**) In ref. (2b) the synthetic name *buffons* has been proposed for the «negative-energy particles travelling backwards in time».

(***) The variables s and t , as is well known, are the «Mandelstam invariants».

(7) M. GLÜCK: *Nuovo Cimento*, **62** A, 791 (1969).

relativity principle, a physical law valid in a reference frame, must be valid also in another one. For instance, the electric charge Q must be conserved in every process as seen by any observer, but—strictly speaking—the relativity principle could be asked not to imply that Q be also a Lorentz invariant (*i.e.* that it maintain the same value under a Lorentz boost). A close inspection of the real meaning of « coincidence » between tachyons and antitachyons brings one—in fact—to such an expectation.

Therefore, the fact that the number of tachyons (and antitachyons) is not Lorentz-invariant (^{1-3,5}) does not necessarily impose that faster-than-light particles be neutral. To repeat ourselves (and to speak roughly), within a physical theory, the *propositions* that *must* remain true under Lorentz boosts could be just nothing else and no more than the physical laws of that theory (essentially the conservation laws, and the *relations* expressing process amplitudes (⁷), in the sense before clarified) (*). The « paradox » (arising in *standard* electrodynamics) relative to a charged *radiating* tachyon, which can appear to a certain set of observers as bearing relativistic energy *zero* (with divergent velocity),—and which therefore seemed not to be able to radiate (**) according to that set of reference frames—has been destroyed too by RHEE (⁸). To conclude our comments about tachyonic electric charge, we ought to show that it would be possible to define a charge operator Q —for a set of interacting particles, including tachyons (***)—which commutes with the total Hamiltonian, but no longer with the generic Lorentz boost. Let us confine ourselves, for simplicity, to a set of scalar (***) tachyons.

However, on the one side, the usual expressions for the Lorentz-boost operators cannot be applied directly to tachyons systems, either because they *should not* conserve in this case the number of « particles », or because (as previously outlined) we must deal with *amplitudes* rather than with tachyonic states. On the other side, if we assume tachyons to have electric charge $\pm q$, the operator Q is nothing but the operator « number of tachyons minus number of antitachyons » as introduced in ref. (1)—apart from a multiplicative constant.

Thus, let us turn our attention to the subject of the number of tachyons, that is not Lorentz-invariant as initial tachyons in one process may result as final antitachyons to different observers (^{1,7}). To illustrate this known fact, let us report some examples, out from existing literature. A first example, following FEINBERG, is depicted in Fig. 3 (see its caption). Other situations are discussed by SUDARSHAN and coworkers in ref. (^{1,13}).

(*) One has no reasons to require that also the *description* of processes be invariant (ref. (^{5b})). In particular, two different inertial observers might judge the total electric charge involved in a physical reaction as having two different values (this may be just the case for charged tachyons): the main point is that *every* observer must see the total electric charge to be conserved in that reaction.

(**) In ref. (⁸) it has been shown (within usual electrodynamics) that also a zero-energy charged tachyon may radiate, subtracting energy to the applied field.

(⁸) J. W. RHEE: Technical report 70-025 (College Park, Aug. 1969).

(***) Differently from what asserted in ref. (^{5b}), a tachyon cannot decay in a set of particles and/or antiparticles and/or photons, for trivial kinematical reasons. Further (different) mistakes are contained in ref. (⁸): see ref. (^{5b,10}).

(⁹) R. FOX, C. G. KUPER and S. G. LIPSON: *Proc. Roy. Soc.*, A 316, 515 (1970).

(¹⁰) G. ECKER: *Quantum field theory with spacelike momentum spectrum*, p. 23, Report I.T.P. (Wien, 1969); S. A. BLUMAN and M. A. RUDERMAN: *Noncausality and instability in ultradense matter*, preprint (1970).

(***) Spin-zero tachyons may correspond both to unitary (ref. (¹¹)) and to nonunitary (ref. (¹²)) representations of the Lorentz group.

(¹¹) P. WINTERNITZ: Rutherford Lab. preprint RPP/T/3 (Aug. 1969).

(¹²) K. B. WOLF: *Nucl. Phys.*, 11 B, 159 (1969).

(¹³) M. E. ARONS and E. C. G. SUDARSHAN: *Phys. Rev.*, 173, 1622 (1968).

To formalize the (scalar)-tachyon system behaviour under Lorentz boosts, we might preliminarily (*) proceed as follows. Let us consider a reaction process, with respect to an inertial reference frame, and the creation operators a, b of its initial (2) and final (3) usual scattering states, respectively:

$$(2) \quad a^{\text{in}}(\mathbf{k}), \quad b^{\text{in}}(\mathbf{k}),$$

$$(3) \quad a^{\text{out}}(\mathbf{k}), \quad b^{\text{out}}(\mathbf{k}),$$

and the corresponding annihilation operators a^\dagger, b^\dagger . The operators a, b refer to tachyons and antitachyons respectively; we emphasize that we are always dealing with *positive*

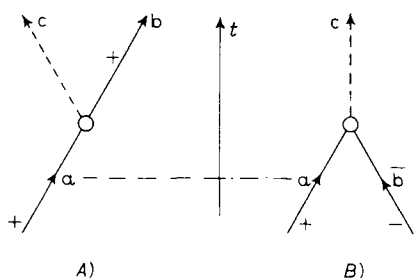


Fig. 3. - In *A*) we depict (following FEINBERG) a tachyon a which emits a photon c , transforming into another tachyon b with different (greater) velocity. It shall be possible to find other inertial frames, according to which the same process appears as in *B*), where the *final* tachyon b now is interpreted as an *initial* antitachyon \bar{b} . The arrow t indicates symbolically the time-flow direction. The electric-charge signs are explicitly shown: the total electric charge is conserved during the reaction in every frame of reference, but changes its value under the Lorentz boost (*i.e.* it is *not* a Lorentz invariant, for tachyons). See the text.

energies. Besides, we may formally consider the vector-space $H = \mathcal{H} \oplus \bar{\mathcal{H}}$ direct product of the vector spaces spanned respectively by the states (**):

$$(4) \quad \mathcal{H} : \begin{cases} a^{\text{in}}(\mathbf{k})|0\rangle, a^{\text{in}}(\mathbf{k}) a^{\text{in}}(\mathbf{k}')|0\rangle, \dots \\ b^{\text{in}}(\mathbf{k})|0\rangle, b^{\text{in}}(\mathbf{k}) b^{\text{in}}(\mathbf{k}')|0\rangle, \dots \\ a^{\text{in}}(\mathbf{k}) b^{\text{in}}(\mathbf{k}')|0\rangle, \dots \end{cases}$$

$$(5) \quad \bar{\mathcal{H}} : \begin{cases} \langle 0|a^{\dagger\text{out}}(\mathbf{k}), \langle 0|a^{\dagger\text{out}}(\mathbf{k}) a^{\dagger\text{out}}(\mathbf{k}'), \dots \\ \langle 0|b^{\dagger\text{out}}(\mathbf{k}), \langle 0|b^{\dagger\text{out}}(\mathbf{k}) b^{\dagger\text{out}}(\mathbf{k}'), \dots \\ \langle 0|a^{\dagger\text{out}}(\mathbf{k}) b^{\dagger\text{out}}(\mathbf{k}') \dots \end{cases}$$

Under a Lorentz boost L (which *a priori* acts onto H), we can formally assume

$$(6) \quad L a^{\text{in}}(\mathbf{k})|0\rangle = a^{\text{in}}(\mathbf{k}')|0\rangle \cdot \Theta(k'_0) + \langle 0|b^{\dagger\text{out}}(\mathbf{k}') \cdot \Theta(-k'_0),$$

$$(7) \quad L b^{\text{in}}(\mathbf{k})|0\rangle = b^{\text{in}}(\mathbf{k}')|0\rangle \cdot \Theta(k'_0) + \langle 0|a^{\dagger\text{out}}(\mathbf{k}') \cdot \Theta(-k'_0),$$

(*) A more simple and more «consistent» formalism is being developed elsewhere; see ref. (14).

(14) A. AGODI; private communication (Catania, 1970).

(**) Notice that *a priori* symbols $| \rangle$ and $\langle |$ have here a different meaning than in Dirac formalism.

where $k' \equiv Lk$; and, more in general ($k'_u \equiv Lk_u$ [$u \equiv j, h$]),

$$(8) \quad L \left[\prod_{j=1}^i a^{\text{in}}(k_j) \prod_{h=i+1}^n b^{\text{in}}(k_h) |0\rangle \right] = \prod_{j=1}^i a^{\text{in}}(k'_j) \prod_{h=i+1}^n b^{\text{in}}(k'_h) |0\rangle \prod_{i=1}^n \Theta(k'_{0i}) +$$

$$+ \sum_{p=1}^i \sum_{q=1}^{n-i} \sum_{j_1 \neq j_2 \neq \dots \neq j_p} \sum_{h_1 \neq h_2 \neq \dots \neq h_q} \left\{ \prod_{j=1}^i a^{\text{in}}(k'_j) \prod_{h=i+1}^n b^{\text{in}}(k'_h) |0\rangle \right\} \otimes \left\{ \langle 0 | \prod_{r=1}^p b^{\text{tout}}(k'_{0r}) \prod_{s=1}^q a^{\text{tout}}(k'_{0s}) \right\} \cdot$$

$$\cdot \prod_{\substack{j=1 \\ j \neq j_1, \dots, j_p}}^i \Theta(k'_{0j}) \cdot \prod_{\substack{h=i+1 \\ h \neq h_1, \dots, h_q}}^n \Theta(k'_{0h}) \cdot \prod_{r=1}^p \Theta(-k'_{0r}) \cdot \prod_{s=1}^q \Theta(-k'_{0s}).$$

The above formulae get their actual meaning—for what has been previously emphasized—only when entering in « scalar products », which we define as follows. For simplicity, we confine ourselves to the two particular cases corresponding to Fig. 3 and formula (1) (interpreted for tachyons):

$$(9) \quad \langle 0 | c^{\text{tout}}(k_1) a^{\text{tout}}(k_2) a^{\text{in}}(k_3) |0\rangle \stackrel{L}{\rightarrow} \langle 0 | c^{\text{tout}}(k_1) a^{\text{tout}}(k_2) L^\dagger L a^{\text{in}}(k_3) |0\rangle =$$

$$= [b^{\text{in}}(k'_2) |0\rangle \otimes \langle 0 | c^{\text{tout}}(k'_1)] \cdot a^{\text{in}}(k'_3) |0\rangle \stackrel{\text{def}}{=} \langle 0 | c^{\text{tout}}(k'_1) b^{\text{in}}(k'_2) a^{\text{in}}(k'_3) |0\rangle,$$

where the operator c refers to the photon entering in Fig. 3;

$$(10) \quad \langle 0 | a^{\text{tout}}(k_1) a^{\text{tout}}(k_2) a^{\text{in}}(k_3) a^{\text{in}}(k_4) |0\rangle \stackrel{L}{\rightarrow} \langle 0 | a^{\text{tout}}(k_1) a^{\text{tout}}(k_2) L^\dagger L a^{\text{in}}(k_3) a^{\text{in}}(k_4) |0\rangle =$$

$$= [b^{\text{in}}(k'_1) |0\rangle \otimes \langle 0 | a^{\text{tout}}(k'_2)] \cdot [a^{\text{in}}(k'_3) |0\rangle \otimes \langle 0 | b^{\text{tout}}(k'_4)] \stackrel{\text{def}}{=} \langle 0 | a^{\text{tout}}(k'_2) b^{\text{tout}}(k'_4) b^{\text{in}}(k'_1) a^{\text{in}}(k'_3) |0\rangle.$$

If we want an invariant theory (for tachyons), as before outlined, we wish to assume the equality of every scattering amplitude and of the boosted one (in particular of the amplitudes appearing in formulae (9) and (10)). It follows that, in the H -space, boost operators are *unitary* (as any Lorentz transformation), even if they may connect « process descriptions » with different numbers of initial and final particles.

That equality of the amplitudes could be expressed also in the S -matrix formalism, provided that we adopt (in the H -space) the natural, formal convention

$$(11) \quad S^{\text{out}} \langle k_1, \dots, k_m | \leftrightarrow {}^{\text{in}} \langle k_1, \dots, k_m | S.$$

For example, formula (9) becomes (*)

$$(9 \text{ bis}) \quad \langle 0 | c^{\text{tin}}(k_1) a^{\text{tin}}(k_2) S a^{\text{in}}(k_3) |0\rangle = \langle 0 | c^{\text{tin}}(k_1) a^{\text{tin}}(k_2) L^\dagger S L a^{\text{in}}(k_3) |0\rangle =$$

$$= [\langle 0 | c^{\text{tin}}(k'_1) \otimes b^{\text{out}}(k'_2) |0\rangle] S \cdot a^{\text{in}}(k'_3) |0\rangle \equiv \langle 0 | c^{\text{tin}}(k'_2) S b^{\text{in}}(k'_1) a^{\text{in}}(k'_3) |0\rangle.$$

2. — Recently RHEE (8) pointed out the possible importance of (charged) tachyons in astrophysics, because of their behaviour in (interstellar) electromagnetic fields, different from bradyons. If it would be again possible to apply some standard-electro-

(*) In this case we could write $L a^{\text{in}}(k) L^\dagger \equiv a^{\text{in}}(k') \cdot \Theta(k'_0) + b^{\text{tout}}(k') \cdot \Theta(-k'_0)$ and simplify the whole formalism.

dynamics formulae to tachyons, we could here recall the relation for the power lost, through *bremstrahlung* radiation, by tachyons in a random (pure) electric field E , in the form ($\beta \equiv v/c$)

$$(12) \quad P \propto (\gamma r_t)^2 \left(1 - \frac{\beta^2}{3}\right) q,$$

where

$$\gamma \equiv \frac{1}{\sqrt{\beta^2 - 1}}, \quad r_t \equiv \frac{q^2}{\mu c^2}, \quad q \equiv \frac{E^2}{8\pi},$$

the quantity q being the (probable) electric charge of tachyons.

We want to point out that from eq. (12) it follows that tachyons with $v > c\sqrt{3}$ will absorb radiation, decelerating; whilst tachyons having $c < v < c\sqrt{3}$ will emit radiation, accelerating. The interesting point is that—in both cases—tachyons should tend to the *asymptotical* velocity $v_\infty = c\sqrt{3}$.

Therefore, we could imagine that a (galactic) H_{II} region (*) sufficiently thick might work as a « *monokinetic* » lens for tachyons. Assuming that (probable) tachyons, coming from a H_{II} region, all possess just the velocity $c\sqrt{3}$, from their «time of flight» between two detectors one might get the proper-mass distribution of the considered tachyons—as their energy loss in the first detector ought reasonably to depend only on their own energies.

Besides, we cannot go on without mentioning that in ref. (13) tachyons have been shown to suffer a gravitational *repulsion* from usual matter. SALTZMAN and SALTZMAN started from papers (16) which exhibited « Schwarzschild » singular surfaces *co-ordinate-independent* (i.e. *physically* meaningful), in contrast with a general belief.

3. — Lastly, to touch again the question of the detection of tachyons, one of the most immediate « ways » was suggested since 1968, particularly in ref. (14) (**). The theoretical problems had been already discussed by other authors (15).

Briefly, if the differential scattering amplitude between two common bradyons has a « resonance » peak for a fixed value of the momentum transfer—independent of the entering total energy—we could argue evidence for a tachyon. Indications in this direction have been forwarded very recently in ref. (17).

Even if the above-mentioned method has its value substantially within « peripheral-type models » (20) (in which so-called « virtual » particles are considered to be commonly exchanged), we wish to contribute signaling, e.g., the experimental results reported in ref. (18).

(*) i.e. a region of ionized hydrogen (protons).

(13) I. FERRETTI and M. VERDE: *Atti Acc. Scienze Torino* (1966), p. 318; M. VERDE: unpublished; F. T. HADJIOANNOU: *Nuovo Cimento* **44 A**, 185 (1966).

(16) F. SALTZMAN and G. SALTZMAN: *Lett. Nuovo Cimento*, **1**, 859 (1969), and ref. (14) therein.

(**) See also ref. (8*).

(17) A. M. GLEESON, M. G. GUNDZIG, E. C. G. SUDARSHAN and A. PAGNAMENTA: Report AEC-8/CPT-47 (Apr. 1970).

(18) AACHEN-BERLIN-BONN-CERN-CRACOW-HEIDELBERG-WARSAW COLLABORATION: *Nucl. Phys.*, **13 B**, 571 (1969); A. BIALAS, A. ESKREYS, W. KITTEL, S. POKORSKI, J. K. TUOMINIEMI and L. VAN HOVE: *Nucl. Phys.*, **11 B**, 479 (1969); A. BIALAS, G. BASSOMPIERRE, A. ESKREYS, Y. GOLDSCHMIDT-CLERMONT, A. GRANT, V. P. HENRI, B. JONGEJANS, D. LINGLIN, F. MULLER, J. M. PERREAU, H. PIOTROWSKA, J. K. TUOMINIEMI, G. WOLF, W. DE BAERE, J. DEBAISIEUX, E. DE WOLF, P. DUFOUR, F. GRARD, P. HERQUEL, L. PAPE, P. PEETERS and F. VERBEURE: *Nucl. Phys.*, **16 B**, 178 (1970).

Finally, without remembering the « compulsory principle » by GELL-MANN, we would like to conclude the present letter with the words (^{1a}): « If tachyons do exist, we ought to find them. If we do not find them, we ought to be able to find out why they could not exist. »

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