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Károlyházy's Quantum Space-Time Generates Neutron Star Density in Vacuum.

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Summary. — By simple arguments, we have shown that Károlyházy's model overestimates the quantum uncertainty of the space-time geometry and leads to absurd physical consequences. The given model can thus not account for gradual violation of quantum coherence and cannot predict tiny experimental effects either.

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In a pioneering paper [1], it was suggested that the quantum mechanics of macroscopic objects ought to be modified due to a certain eventual unsharpness of space-time geometry. Later on, the possibility of experimental verification of the model, too, has been developed [2, 3]. The idea went as follows.

By combining Heisenberg's uncertainty principle with gravitation, the following relation has been obtained for the minimum uncertainty Δs of a single (timelike) geodesic:

$$\Delta s^2 = \alpha^{4/3} s^{2/3}\,, \tag{1}$$

where s is the length of the geodesic and α is the Planck length (cf. eq. (3.1) of ref. [1]). Then this uncertainty is believed to be a universal lower bound, and so must appear in the space-time in an objective way. This way. This was done via random «gravitational waves».

The present authors [4] re-analysed the concept leading to eq. (1). A result is that in ref. [1] and [2] the value M of the mass realizing the least uncertainty along the given geodesic takes irrealistically high values $\sim (\hbar/c)\alpha^{-4/3} s^{1/3}$. For example, a geodesic of length $s = 1$ lightsecond would require a mass $M \sim 10^{10}$ g to be «realized». In other words, the optimum mass of a clock to measure a period of 1s would weight ten thousand metric tons.

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This result does not directly invalidate the concept of ref. [1] and [2]. Namely, the argumentation needs only the *existence* of a certain lower absolute bound for the uncertainty; it does not involve *real* clocks directly. However, the high-mass problem is intimately connected with another problem as will immediately be seen.

The original paper [1] as well as the further ones [2, 3] propose that the space-time uncertainties be represented by *random* gravitational waves. These gravitational waves γ satisfy the linearized vacuum Einstein equations:

$$\Box\gamma = 0 \tag{2}$$

—see eq. (3.2) of ref. [1]. Adopting all the time the conservative notations of ref. [1], the gravitational wave $\gamma(x, y, z, t)$ is expanded as a superposition of plane waves:

$$\gamma = \sum_k c_k \cos(k_x x)\cos(k_y y)\cos(k_z z)\cos(kct) + \dots . \tag{3}$$

The random coefficients c_k are uncorrelated. Their average is zero, while the spreads are given by

$$L^3 \bar{c}_k^2 = \alpha^{4/3} k^{-5/3} , \tag{4}$$

where L is the normalization volume (cf. eqs. (3.4) and (3.5) of ref. [1]). The above equation is the only one which is conform to the uncertainty relation (1).

According to the intentions implicit in ref. [1] and [2], the space-time geometries defined by eqs. (2) and (3) must be approximate solutions of the Einstein equations. However, it turns out that they will not. Though they satisfy, by construction, the *linearized* vacuum Einstein equations (2), the conditions for the linear approximation will seriously fail. We are going to test two rather trivial conditions. The first will hold but the second will not.

Let us calculate the mean-squared deviation of the metric tensor from its Minkowski value. Squaring both sides of the eq. (3) and taking stochastic averages of the coefficients c_k, one obtains

$$\bar{\gamma}^2 \sum_k \bar{c}_k^2 \sim \alpha^{4/3} L^{-3} \sum_k k^{-5/3} \sim (\alpha k_{\text{max}})^{4/3} . \tag{5}$$

One needs a finite cut-off on k otherwise the amplitude of the random waves would diverge. Károlyházy suggests $k_{\text{max}} = 10^{13}\ \text{cm}^{-1}$ and this assures that γ is much smaller than unity. This was the first condition for applying the linear form (2) of the Einstein equations.

As for the second condition, let us first invoke the expansion of the scalar curvature R up to the second order in γ (cf. ref. [5]):

$$R = \frac{1}{2}\Box\gamma_{ii} - \frac{1}{2}\gamma_{ij}\Box\gamma_{ij} + \frac{1}{4}\gamma_{ij,k}\gamma_{ij,k} + \frac{1}{4}(\gamma_{ij,k} - \gamma_{ik,j})(\gamma_{ij,k} - \gamma_{ik,j}) + \dots . \tag{6}$$

Now, by substituting the waves (3) into this equation, the first-order term indeed vanishes. The magnitude of the average of the remaining terms can be estimated by invoking eq. (4); one obtains

$$\bar{R} \sim \alpha^{4/3} k_{\text{max}}^{10/3} . \tag{7}$$

This curvature is extremely high. Using the previous cut-off we are led to

$\overline{R} \sim 1\ \mathrm{cm}^{-2}$. So the corresponding fluctuating metric is not at all the «extremely small smearing» [1] of the flat space-time, as thought before.

According to the exact Einstein equation $R = (8\pi\alpha^2/\hbar c)T$. Hence the curvature (7) would assume an average energy density in the order of

$$\overline{T} \sim \hbar c \alpha^{-2/3} k_{\max}^{10/3} . \tag{8}$$

Observe the dramatic change: in the energy density the Planck length appears with *inverse* (two-thirds) power. Therefore the interplay of two small length scales may result in anything. The original cut-off $k_{\max} = 10^{13}\ \mathrm{cm}^{-1}$ would yield

$$\frac{\overline{T}}{c^2} \sim 10^{26}\ \mathrm{g/cm}^3 , \tag{9}$$

i.e. 11 orders of magnitude *above neutron star density.*

In ref. [1] the details of the cut-off were thought of no importance. We have, however, pointed out that the original cut-off would imply absurd results for cosmological mass density. Since the cut-off $k_{\max}$ is the only free parameter in the model, one may hope to save the theory by choosing a lower value for it. Unfortunately, the choice $k_{\max} = 10^5\ \mathrm{cm}^{-1}$ familiar from, *e.g.*, the model of Ghirardi *et al.* [6], yields still-water density. Further decrease of $k_{\max}$ is needed. Then, however, there would be only macroscopic wavelengths $1/k$ and the gravitational fluctuations (3) would not play a role in the quantum-classical transition anymore. The trace (9) in itself could be removed by means of an incredibly high cosmological constant Λ, but in the Robertson-Walker–universe geometries two non-trivial components of the Einstein equations survive, and one cannot remove the problem from both.

Obviously, the Károlyházy model [1] has shown to overestimate something in the assumed quantum smearing of the space-time. The spectrum (4) of gravitational fluctuations is certainly wrong whatever cut-off is chosen. The proposals outlined in ref. [2, 3] derive extremely fine effects to be observed experimentally. In the light of the cosmological absurdity of the model we wonder if such tiny effects would have to be taken seriously.

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