Fission Fragment Angular Distribution in Proton-Induced Fission of ^{209}Bi.

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We have measured the angular distribution of fission fragments at ten proton energies between 30.6 and 44.5 MeV.

The experiment has been done with the external proton beam of the AVF cyclotron of the Milan University (1). The proton beam is collimated and focused on the target at the center of a 60 cm diameter scattering chamber in a spot of less than 7 mm diameter. The proton current on the target, varying from about 80 to 300 nA, is collected in a Faraday cup and fed to an integrator. The beam energy is deduced from the position of the stripping target in the cyclotron. This method, usually employed for simplicity, has been checked with the cross-over technique. The absolute value is given within $\pm$ 200 keV and the relative value within $\pm$ 100 keV.

The fission fragments are detected with solid-state counters Ortec 7901 of 100 mm^2 sensitive area and 60 μm depth.

Two detectors are used: one is set at 90° with respect to the incident beam and subtends a solid angle of $1.5\cdot10^{-2}$ steradians with an angular resolution of about $\pm 4°$. During the measurement this detector is used as a reference monitor. The second detector can be rotated around the axis of the scattering chamber from 15° to 170° in the laboratory system: the solid angle is $4.9\cdot10^{-3}$ steradians and the angular resolution $\pm 2.3°$.

The ^{209}Bi targets (supplied by CBNM Euratom-Geel), 295.6 μg/cm^2 thick, are evaporated on 22 μg/cm^2 Vyns backings. The uniformity and thickness are given within 5%.

The electronic system is conventional; the pulses from the two detectors are amplified and fed into a 2×512 channel analyser. The fission fragment energy spectra are directly integrated with counting scales.

The laboratory system counting rates and angles are converted to the c.m. system assuming fragment masses corresponding to symmetric fission. The Q-value is evaluated from $Q=\langle E_k\rangle-(E_{\text{inc}})_{\text{c.m.}}$ and the average fragment kinetic energy $\langle E_k\rangle$ is estimated with the formula $\langle E_k\rangle=(0.0171\,Z^2/A^{\frac{1}{3}}+22.2)$ MeV, where Z and A are the atomic and mass numbers of the compound nucleus (2). The calculated value, equal to

(1) M. CASTIGLIONI, G. DUTTO, M. FOIS, F. RESMINI, G. STRINI, C SUCCI and G TAGLIAFERRI: *Nuovo Cimento*, **41** B, 244 (1966).

(2) N. E. VIOLA jr.: *Nucl. Data.*, **1**, 391 (1966).

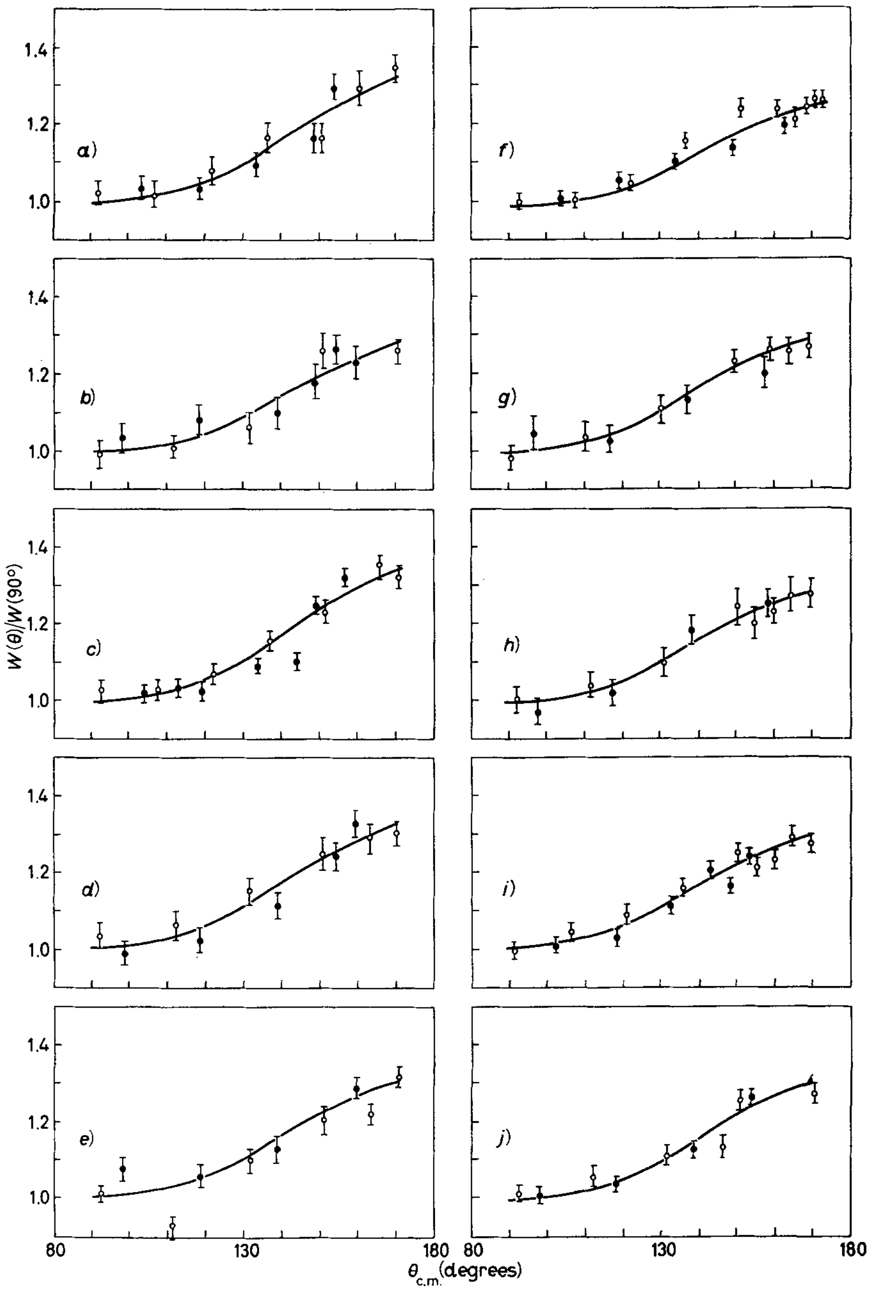

Fig. 1. – Center-of-mass angular distributions of fission fragments in proton-induced fission of ^{209}Bi for energies between 30.6 and 44.5 MeV. The solid lines are the least-squares fit to the experimental results (see text). Only the statistical errors are shown. *a*) $E_p = 30.6$ MeV. *b*) $E_p = 32.7$ MeV, *c*) $E_p = 34.6$ MeV, *d*) $E_p = 35.5$ MeV, *e*) $E_p = 36.8$ MeV, *f*) $E_p = 38.1$ MeV, *g*) $E_p = 39.5$ MeV, *h*) $E_p = 40.8$ MeV, *i*) $E_p = 42.05$ MeV, *j*) $E_p = 44.5$ MeV; ○ backward angle, ● forward angle.

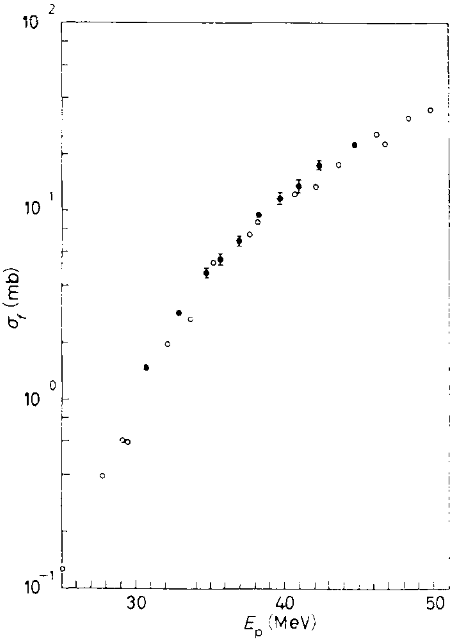

Fig. 2. – Proton-induced fission cross-sections of ^{209}Bi as a function of proton energy: ○ KODHAI-JOOPARI, ● our data.

149.3 MeV, is in rather good agreement with the measured one of 147 MeV obtained calibrating the detector energy scale with a ^{252}Cf source (3).

In Fig. 1 the measured anisotropy $W_{exp}(\theta) = \sigma(\theta)/\sigma(90)$ is shown as a function of $\theta_{c.m.}$. Due to the symmetry of the angular distributions with respect to 90°, the data at forward angles are reported at the corresponding backward angles. The given errors are purely statistical. In Fig. 2 are reported, as a function of the proton energy, the cross-sections evaluated integrating the best-fit theoretical anisotropies $W(\theta)$ (see the following). The values measured by JOOPARI (4), with the mica technique, are also shown. The agreement between the experimental results is quite good, while other values (5), obtained with radiochemical techniques, are smaller. In the theory of nuclear fission, first developed by BOHR (6), the fissioning nucleus, at saddle point, is characterized by the quantum numbers I (total angular momentum), M (projection of I on a quantization axis assumed coincident with the direction of the incident beam) and K (projection of I on the nuclear symmetry axis). For a Gaussian K distribution $f(K) \propto \exp[-K^2/2K_0^2]$, the exact expression of the angular distribution is given (7) by the following formula:

$$(1)\qquad W(\theta) \propto \sum_{I=0}^{\infty} \sum_{M=-S_{max}}^{+S_{max}} \left[\sum_{L=0}^{\infty} \sum_{S=|I_0-i|}^{I_0+1} \sum_{\mu=-I_0}^{I_0} \left(\frac{(2L+1)\, T_L \,|\langle SLM0|IM\rangle|^2 \, |\langle I_0 i \mu (M-\mu)|SM\rangle|^2}{\sum_{L=0}^{\infty} (2L+1)\, T_L} \right) \right] \cdot$$
$$\cdot \sum_{K=-I}^{I} \left[(2I+1)\, |d^I_{MK}(\theta)|^2 \exp\left[\frac{-K^2}{2K_0}\right] \Big/ \sum_{K=-I}^{I} \exp\left[\frac{-K^2}{2K_0^2}\right] \right],$$

where I_0, i, S are target, projectile and channel spins respectively; $d^I_{MK}(\theta)$ is the symmetrical top wave function (8), T_L are transmission coefficients.

The analysis of the experimental $W_{exp}(\theta)$ permits to evaluate the K_0^2-values; however,

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(3) H. W. SCHMITT, W. M. GIBSON, J. H. NEILER, F. J. WALTER and T. D. THOMAS: *Physics and Chemistry of Fission*, vol. **1** (Vienna, 1965), p. 331.

(4) A. KHODAI-JOOPARI: Thesis URCL 16489 (1966), unpublished.

(5) T. T. SUGIHARA, J. ROESMER and J. W. MEADOWS jr.: *Phys. Rev.*, **121**, 1179 (1961).

(6) A. BOHR: *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva*, 1955, vol. **2** (New York, 1956), p. 151.

(7) J. R. HUIZENGA, A. N. BEHKAMI and L. G. MORETTO: *Phys. Rev.*, **177**, 1826 (1969).

(8) J. A. WHEELER: *Channel analysis of fission*, in *Fast-Neutron Physics*, vol. **2**, edited by J. B. MARION and J. A. FOWLER (New York, 1963), p. 2051.

given the complexity of formula (1), that, when the number of contributing L-values is large, requires very long computing time, approximate expressions are usually introduced. An expansion of formula (1) in powers of $\beta = 1/2K_0^2$ is possible when the anisotropy is small. The relevant parameter is

$$C = \tfrac{1}{4}\beta L_M(L_M + 2)\,,$$

where $L_M = \sqrt{2\langle L^2\rangle}$ and $\langle L^2\rangle$ is given by

$$\langle L^2\rangle = \frac{\sum_L (2L+1)\,T_L\,L^2}{\sum_L (2L+1)\,T_L}\,. \tag{2}$$

In the considered cases where $C \sim 0.3$ and the transmission coefficients (calculated with the optical-model parameters reported in Table I) depend very little on the spin-orbit interaction, $W(\theta)$ is well approximated by the following expression [9]:

$$\begin{aligned}4\pi W(\theta) \propto \Big[\sum_L (2L+1)\,T_L\Big]^{-1}\cdot\sum_L (2L+1)\,T_L\Big\{1 + \frac{1}{6}\beta L(L+1)\,[3\cos^2\theta - 1] + \\ + \frac{1}{2160}\beta^2\big[69L^2(L+1)^2 + 80L(L+1)(I_0(I_0+1) + i(i+1)) - 189L(L+1)\big] - \\ - \frac{1}{72}\beta^2\cos^2\theta\big[15L^2(L+1)^2 + 8L(L+1)(I_0(I_0+1) + i(i+1)) - 36L(L+1)\big] + \\ + \frac{\beta^2}{16}\cos^4\theta\,[3L^2(L+1)^2 - 6L(L+1)]\Big\}\,.\end{aligned} \tag{3}$$

TABLE I. – *Optical-model parameters for* $p + {}^{209}Bi$. The optical-model potential used in the calculations of the transmission coefficient is $u(r) = V_c(r) + (V + cE)(1 + e^{x})^{-1} + i(W + c'E)(1 + e^{x'})^{-1} + \lambda_\pi^2 V_s(1/r)(d/dr)(1 + e^{x})^{-1}\boldsymbol{\sigma}\times\boldsymbol{e}$, where E is the proton energy, $V_c(r)$ is the Coulomb potential due to an uniformly charged sphere of radius R and $x = (r - r_0A^{\frac{1}{3}})/a$, $x' = (r - r_0'A^{\frac{1}{3}})/a'$.

V (MeV)	W (MeV)	V_s (MeV)	C	C'	r_0 (fm)	r_0' (fm)	a (fm)	a' (fm)	R_c (fm)
−59	−9	−6.6	0.2	0.05	1.20	1.428	0.65	0.704	1.20

The experimental data have been fitted with (3) using as free parameters β and a normalization constant. The curves in Fig. 1 are the results of the calculation. The K_0^2 standard deviation has been evaluated from the standard deviation characterizing the

(9) J. J. GRIFFIN: *Phys. Rev.*, **127**, 1248 (1962).

approximate β-value obtained by minimizing

$$\chi^2 = \sum_i \left(\frac{W(\theta_i) - W_{\exp}(\theta_i)}{\varepsilon(\theta_i)}\right)^2 \tag{4}$$

neglecting second-order terms in β.

This procedure allows to estimate the K_0^2 standard deviation, due to the experimental errors on the values $W_{\exp}(\theta_i)$, within 10%. The systematic errors that derive from the use of (3) can be evaluated by means of the exact expression (1). This procedure suggests that the K_0^2 obtained with formula (3) have to be lowered by about two units.

In Fig. 3 the values of K_0^2 are reported with those obtained from the reaction ^{206}Pb(α, f) (10). The good agreement between the various data shows that, to a good

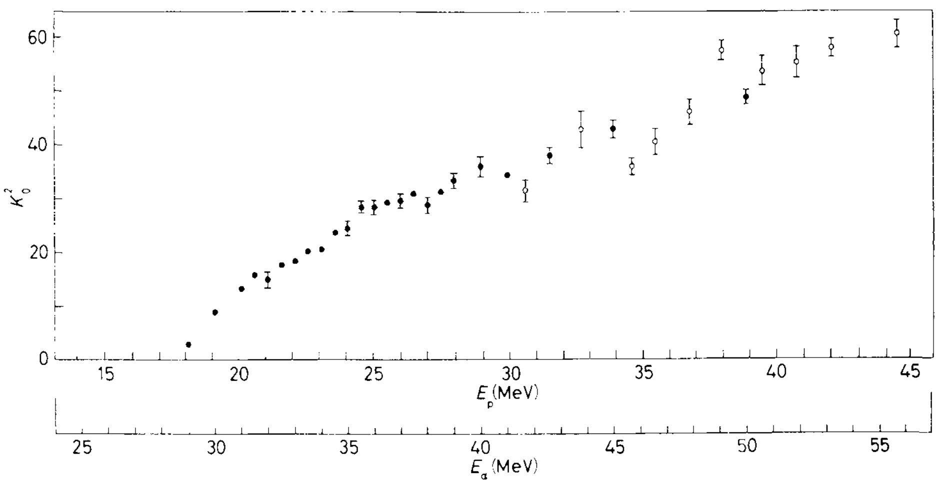

Fig. 3. – Energy dependence of the anisotropy parameter K_0^2 for the fission nucleus ^{210}Po: ○ $\alpha + {}^{206}$Pb ● $p + {}^{209}$Bi.

approximation, the angular distributions of fission fragments, for the same compound nucleus and the same excitation energy, are characterized by the same K_0^2 independent of the entrance channel of the fission process. A more detailed analysis of these and other results is in progress.

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(10) L. G. Moretto, R. C. Gatti, S. G. Thompson, J. R. Huizenga and J. O. Rasmussen: *Phys. Rev.*, **178**, 1845 (1969).