Gauge Invariance and Regge-Pole Sum Rules for Pion Photoproduction (*).

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(ricevuto il 22 Marzo 1967)

It has been already remarked elsewhere (1) that in the case of a scattering amplitude involving a conserved vector current, superconvergence sum rules of the kind obtained by De Alfaro *et al.* (2) are a direct consequence of gauge invariance and of the assumption of unsubtracted dispersion relation for some of the invariant amplitudes.

It has also been shown (2) in the case of π-ρ scattering that the conjectured high-energy behavior can also be obtained by a Regge-pole model.

The purpose of this paper is to show that in the case of pion photoproduction, the sets of sum rules which can be obtained in these two ways are not equivalent and to discuss the Regge-pole sum rules.

It is well known (3) that for the photoproduction of pions with real photons the scattering amplitude can be written in general as $T=\sum_{i=1}^{8} N_i B_i(s,t,u)$ where the spin matrices N_i are defined as in (3). Gauge invariance then implies the following relations among the B's:

$$(1a)\qquad (s-u)B_2 = 2(t-m_\pi^2)B_3\,,$$

$$(1b)\qquad B_5+\tfrac{1}{4}(u-s)B_6+\tfrac{1}{2}(t-m_\pi^2)B_8=0\,.$$

(*) Work supported in part by the U.S. Atomic Energy Commission.

(1) D. Amati and R. Jengo: *Phys. Lett.*, **24**, B 108 (1967).

(2) The method for obtaining fixed-t sum rules using unsubtracted dispersion relations or Regge asymptotic behavior is due to V. De Alfaro, S. Fubini, G. Rossetti and G. Furlan: *Phys. Lett.*, **21**, 576 (1966); and L. D. Soloviev: Dubna preprint (1965). There are many other strong-interatction sum rules from asymptotic behavior which have been discussed previously. See M. L. Goldberger, H. Miyazawa and R. Oehme: *Phys. Rev.*, **99**, 986 (1955); G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu: *Phys. Rev.*, **103**, 1343 (1957); R. G. Sachs: *Phys. Rev.*, **126**, 2256 (1962); A. P. Balachandran, P. G. O. Freund and C. R. Schumacher: *Phys. Rev. Lett.*, **12**, 209 (1964); A. P. Bakachandran: *Journ. Math. Phys.*, **5**, 614 (1964); *Phys. Rev.*, **134**, B 197 (1964); A. P. Balachandran and F. N. von Hippel: *Ann. of Phys.*, **30**, 446 (1964); A. P. Balachandran: *Ann. of Phys.*, **30**, 476 (1964); *Phys. Rev.*, **137**, B 177 (1965); *Nuovo Cimento*, **42**, 804 (1966); A. P. Balachandran and J. J. Loeffel: to be published.

(3) J. S. Ball: *Phys. Rev.*, **124**, 2014 (1961). The notation of this paper will be used throughout.

If one now assumes that B_3, B_5 and B_8 satisfy unsubtracted dispersion relations, then clearly B_2 and B_6 must be superconvergent. In terms of the CGLN [4] invariant amplitudes the resulting sum rules can be written as

$$\int_{M^2}^{\infty} \mathrm{Im}\, A_2^{(-)}(s,t)\,\mathrm{d}s = 0\,, \tag{2a}$$

$$\int_{M^2}^{\infty} \mathrm{Im}\, A_4^{(-)}(s,t)\,\mathrm{d}s = 0\,, \tag{2b}$$

once their crossing properties have been taken into account.

Had one started with the meson off the mass shell, one would have obtained on the r.h.s. of eq. (1*a*) an equal-time commutator, which can be evaluated assuming an algebra of currents. Going back on the mass shell one would again obtain the sum rules (2*a*) and (2*b*). This is indeed the way they were first derived by MUKUNDA and RADHA [5].

On the other hand one can look at the asymptotic behaviour of the A's as given by a Regge-pole model. This can be readily done by means of the partial-wave analysis in the crossed channel which is also given in Ball's paper [6].

The result is that at high energies $A_i(s,t)\sim s^{\alpha_i(t)-1}$, where $\alpha_i(t)$ is the leading trajectory. Now for the amplitudes A_1, A_2 and A_4 this trajectory is that of a vector meson for which it is generally accepted that $0\leqslant\alpha(0)\leqslant 1$. Consequently the Regge-pole model does not suggest superconvergence for either A_2 or A_4 in contradiction with the sum rules (2*a*) and (2*b*).

However we observe that for A_3 and for the linear combination A_1+tA_2 the leading trajectory is coupled respectively to a triplet $\mathcal{N}\overline{\mathcal{N}}$ state with parity $(-1)^{J+1}$ or to the singlet of the $\mathcal{N}\overline{\mathcal{N}}$ system. This is possible only for the pion trajectory for which $\alpha(t)<0$ for $t\leqslant 0$. Hence Regge-pole dominance gives the following superconvergence sum rules:

$$\int_{M^2}^{\infty} \mathrm{Im}\, A_3^{(0)}(s,t)\,\mathrm{d}s = 0\,, \tag{3a}$$

$$\int_{M^2}^{\infty} \mathrm{Im}\, A_3^{(+)}(s,t)\,\mathrm{d}s = 0\,, \tag{3b}$$

$$\int_{M^2}^{\infty} \mathrm{Im}\left(A_1^{(-)}(s,t) + tA_2^{(-)}(s,t)\right)\mathrm{d}s = 0\,, \tag{3c}$$

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(4) G. CHEW, M. GOLDBERGER, F. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1345 (1957).

(5) N. MUKANDA and T. K. RADHA: *Nuovo Cimento*, **44**, 723 (1966); see also S. GASIOROWICZ: *Phys. Rev.*, **146**, 1067 (1966).

(6) A detailed analysis is given by G. KRAMER and P. STICHEL: *Zeits. f. Phys.*, **178**, 519 (1964).

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where we have written only the isospin parts which give nontrivial sum rules. As usual we can try to saturate these sum rules by means of the lowest-lying states, *i.e.* the nucleon and the (3, 3) isobar.

In this drastic approximation eq. (3*a*) has a contribution only from the nucleon intermediate state, for which

$$\operatorname{Im} A^{(0)}_{3\mathcal{N}}(s, t) = gF^s_2 \pi\, \delta(s - M^2)\,,$$

where g is the $\pi\mathcal{N}\mathcal{N}$ coupling constant and F^s_2 is the static isoscalar magnetic form factor of the nucleon. Hence eq. (3*a*) yields the condition

$$gF^s_2 = 0$$

in good agreement with the small experimental value of F^s_2.

In the case of eq. (3*b*) the nucleon and the (3, 3) resonance give the following contributions to $\operatorname{Im} A^{(+)}_3$:

$$\operatorname{Im} A^{(+)}_{3\mathcal{N}}(s, t) = \frac{1}{2}(\mu'_p - \mu_n)\frac{e}{2M} g\pi\, \delta(s - M^2)\,, \tag{4}$$

$$\begin{aligned}\operatorname{Im} A^{(+)}_{3(3,3)}(s, t) = \frac{e\lambda_1}{3m_\pi}\sqrt{\frac{2}{3}}\pi\delta(s - M^{*2})\Bigg\{&\frac{3C_1}{2(M^{*2} - M^2)}\left(1 - \frac{C_2}{C_1}\frac{M^* + M}{m_\pi}\right)(m^2_\pi - t) + \\ &+ C_2(M^* + M)\frac{3(M^{*2} - M^2 + m^2_\pi) + (M^* + M)^2}{4M^{*2}m_\pi} - C_1\left(3 - \frac{M^* + M}{2M^*}\right)\Bigg\}\,,\end{aligned} \tag{5}$$

where M^* is the isobar mass and λ_1, C_1 and C_2 are the coupling constants for $\pi\mathcal{N}\mathcal{N}^*$ and $\gamma\mathcal{N}\mathcal{N}^*$ respectively, as defined by GOURDIN and SALIN [7]. Since in the sum rule the coefficients of the different powers of t must be separately equated to zero, we obtain by substituting (4) and (5) into eq. (3*b*)

$$1 - \frac{C_2}{C_1}\frac{M^* + M}{m_\pi} = 0\,, \tag{6}$$

$$\begin{aligned}\frac{1}{2}(\mu'_p - \mu_n) = -\frac{2M}{g}\frac{\lambda_1}{3m_\pi}\sqrt{\frac{2}{3}}\Bigg\{&C_1\left(\frac{M^* + M}{2M^*} - 3\right) + \\ &+ C_2(M^* + M)\frac{3(M^{*2} - M^2 + m^2_\pi) + (M^* + M)^2}{4M^{*2}m_\pi}\Bigg\}\,.\end{aligned} \tag{7}$$

Equation (6) is equivalent to the condition

$$C_4 \equiv C_1 \frac{m^2_\pi}{M^{*2} - M^2}\left(1 - \frac{C_2}{C_1}\frac{M^* + M}{m_\pi}\right) = 0\,,$$

(7) M. GOURDIN and PH. SALIN: *Nuovo Cimento*, **27**, 193, 609 (1963).

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which is very well satisfied experimentally as discussed by GOURDIN and SALIN (7).

On the other hand from eq. (7) we obtain (8)

$$\frac{\mu'_p - \mu_n}{2} = 2.5 ,$$

which is not too far from the experimental value of 1.85.

Consider now the sum rule (3c). We want to show that it is impossible to saturate it in a conventional way with a finite number of s-channel resonances. Indeed it is possible to show (9) that for the invariant amplitudes A_1, A_3 and A_4 the nucleon-pole contribution is given only by the magnetic part of the $\gamma\mathcal{N}\mathcal{N}$ coupling, which is gauge invariant by itself.

However, in order to get a definite contribution to the amplitude A_2, which is given by the coupling to the photon of the orbital electric current $P_\mu \bar{u}u$ one is forced to consider also the contributions of the crossed nucleon graph and the pion exchange. This is due to the well-known fact that only the sum of the three Born diagrams is gauge invariant.

Explicitly one obtains

$$(8) \qquad \begin{cases} \operatorname{Im} A_1^{(-)\text{Born}}(s,t) = -\sqrt{2}g\dfrac{e}{2}\pi\delta(s-M^2)\,, \\[2ex] \operatorname{Im} A_2^{(-)\text{Born}}(s,t) = \dfrac{e\sqrt{2}}{t-m_\pi^2}\, g\pi\delta(s-M^2)\,. \end{cases}$$

On the other hand any s-channel resonance can be coupled to the $\gamma\mathcal{N}$ system only through the magnetic interaction. The contribution to all the invariant amplitudes is therefore well defined and the general form of $\operatorname{Im} A_i$ will be

$$(9) \qquad \operatorname{Im} A_i(s,t) = P_i(t)\pi\delta(s-\bar{M}^2)\,,$$

where $\bar{M}$ is the resonance mass and $P_i(t)$ is a polynomial in t, the order of which depends on the spin of the resonance.

Now our statement follows at once observing that the Born term (8) is equivalent to a polynomial of infinite order in t and cannot be compensated by a finite number of terms like (9).

However, we think that this is a shortcoming of the approximation used rather than a failure of the sum rule itself. From a physical point of view this can be understood if we remember that our sum rule was obtained by « Reggeizing » the

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(8) In obtaining this number we have corrected the values of C_1 and C_3 given in ref. (7) by a factor $\sqrt{2/3}$ as discussed by R. H. DALITZ and D. SUTHERLAND: *Phys. Rev.*, **146**, 1182 (1966). Since we are interested in the product λC this is the only correction relevant to our case.

(9) See *e.g.* H. ROLLNIK: *Proceeding of 1965 Easter School*, CERN 35-24.

exchange of the pion, while in our approximation gauge invariance implies that there is a contribution to A_2 coming from the exchange of an elementary pion.

After this work was completed the authors have been informed that L. K. PANDE at Trieste and M. B. HALPERN at Princeton have discussed analogous sum rules.

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It is a pleasure to thank the Theoretical High-Energy Group of Syracuse University for very interesting and helpful discussions, and Dr. A. P. BALACHANDRAN for reading the manuscript.

Photoproduction of ρ and f Mesons at Energies up to 5.8 GeV.

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(*Nuovo Cimento*, **48** A, 262 (1967))

In the φ and φ_H distribution of Fig. 5 the labelling of the horizontal scale should *not* be 0, 90, 180 degrees, but 0, 180, 360 degrees.

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