Particle Mixing and the Breaking of Charge Symmetry in the Λ-$\mathcal{N}$ Interaction (*).

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Summary. — Electromagnetic mixing of isotopic-spin eigenstates leads to important charge-symmetry-breaking (CSB) elements in the Λ-$\mathcal{N}$ interaction. The most important mixing is that for Σ^0-Λ, which allows coupling of the (dominant) $T=1$ component of physical π^0 and ρ^0 to the physical Λ. π^0-η and ρ^0-ω-φ mixing give additional contributions. A single-meson-exchange CSB potential model is constructed from these mixings, with the help of the octet model of SU_3 to relate meson-baryon couplings. The difference between Λ bindings in the mirror hypernuclei $^4He_\Lambda$ and $^4H_\Lambda$ may be explained with this model, in which vector-meson exchange is more important than pseudoscalar-meson exchange. Predictions of the model for Λ-$\mathcal{N}$ scattering and for binding energies of possible excited states of the four-body hypernuclei are given.

1. – Introduction.

Those charge-symmetry-breaking (CSB) contributions to the Λ-$\mathcal{N}$ interaction which arise from electromagnetic mixing of isotopic-spin eigenstates are discussed in this paper. It appears that single-vector-meson exchange between an $\mathcal{N}$ and a physical Λ may lead to the largest of these CSB effects and thereby provide the most important contribution to the difference ΔB_Λ between the binding energies of the Λ in the mirror hypernuclei $^4He_\Lambda$ and $^4H_\Lambda$. Σ^0-Λ and π^0-η mixing and that part of ΔB_Λ due to exchange of physical π^0 between $\mathcal{N}$ and physical Λ have already been calculated by DALITZ and VON HIPPEL (1).

The observed difference between the binding energies of the four-body

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(1) R. H. DALITZ and F. VON HIPPEL: *Phys. Lett.*, **10**, 153 (1964).

hypernuclei [2,3]

$$(1) \qquad \Delta B_\Lambda = B_\Lambda(^4\mathrm{He}_\Lambda) - B_\Lambda(^4\mathrm{H}_\Lambda) = (0.30 \pm 0.14)\ \mathrm{MeV}$$

indicates the presence of a component of the strong Λ-$\mathcal{N}$ interaction which is not charge-symmetric. In fact, if this interaction were charge-symmetric, Coulomb effects would lead to a negative ΔB_Λ. Consideration of the difference between the r.m.s. radii of the nucleon distributions in the core nuclei due to Coulomb repulsion in ^{3}He and consideration of the additional Coulomb energy associated with the compression of the nucleon core in $^4\mathrm{He}_\Lambda$ [4,5] lead to the estimate

$$(2) \qquad (\Delta B_\Lambda)_{\text{Coulomb}} \approx -0.45\ \mathrm{MeV}\ .$$

With (2), the CSB component of the Λ-$\mathcal{N}$ interaction would be required to account for a ΔB_Λ about 0.45 MeV larger than that in (1).

The estimate (2) was made with the help of the «naive model» of DALITZ and THACKER [6], with which they reproduced the measured r.m.s. radii of the charge and magnetic moment distributions in ^{3}He and ^{3}H remarkably well. We took the value given in ref. [6] for the radius of the central region in these nuclei within which the wave functions of all nucleons are assumed to be the same. We then obtained the following values for the r.m.s. radii of the various nucleon distributions in the three-body nuclei: $a_p(^3\mathrm{He}) = 1.66$ fermi, $a_n(^3\mathrm{He}) = 1.49$ fermi; $a_p(^3\mathrm{H}) = 1.46$ fermi, $a_n(^3\mathrm{H}) = 1.59$ fermi. The r.m.s. radius of ^{3}He was then taken to be $R(^3\mathrm{He}) = [2a_p^2/3 + a_n^2/3]^{\frac{1}{2}}$; for $R(^3\mathrm{H})$, p and n are interchanged in the preceding formula. The results are $R(^3\mathrm{He}) = 1.61$ fermi and $R(^3\mathrm{H}) = 1.55$ fermi; the difference between these radii leads to most of (2). For core radii appropriate to the four-body hypernulcei (including core compression) and Λ-$\mathcal{N}$ interaction of range $(2M_\pi)^{-1}$ without a hard core, [4] we find $\mathrm{d}B_\Lambda/\mathrm{d}R \approx -2.5\ B_\Lambda$/fermi for Λ-nucleus potentials of Gaussian form [7].

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[2] M. RAYMUND: *Nuovo Cimento*, **32**, 555 (1964).

[3] A recent analysis of binding-energy data for the four-body hypernuclei by the Brussels-Dublin-London Collaboration led to $\Delta B_\Lambda = (0.16 \pm 0.17)$ MeV [D. H. DAVIS: private communication (1965)]. Use of this value instead of (1) would reduce by about 0.1 MeV the value of ΔB_Λ to be explained by the present model.

[4] R. H. DALITZ and B. W. DOWNS: *Phys. Rev.*, **111**, 967 (1958).

[5] R. C. HERNDON, Y. C. TANG and E. W. SCHMID: *Phys. Rev.*, **137**, B 294 (1965).

[6] R. H. DALITZ and T. W. THACKER: *Phys. Rev. Lett.*, **15**, 204 (1965).

[7] This value of $\mathrm{d}B_\Lambda/\mathrm{d}R$ was obtained by differentiation of the interpolation formula of J. M. BLATT and J. D. JACKSON: *Phys. Rev.*, **76**, 18 (1949) relating the well depth parameter to the binding energy for a Gaussian potential; for an exponential potential, the result is essentially the same. Our result is about twice as large as the value used in ref. [1].

We estimate that, in the absence of CSB effects, $B_\Lambda(^4\mathrm{H}_\Lambda)$ would be about 2.4 MeV [8]. With this we have $\mathrm{d}B_\Lambda/\mathrm{d}R \approx -6$ MeV/fermi; and, with the radius difference above, $(\Delta B_\Lambda)_{\text{Coulomb}} \approx -0.35$ MeV. Finally, the nucleon core is compressed by the bound Λ [4,5] and this leads to an increase in the Coulomb energy of $^4\mathrm{He}_\Lambda$ over that in $^3\mathrm{He}$. For a core compression of 14 percent obtained in ref. [5] with hard-core potentials, this effect produces an additional $(\Delta B_\Lambda)_{\text{Coulomb}} \approx -0.10$ MeV. These Coulomb contributions, taken together give (2) [9].

Our discussion of the origin of ΔB_Λ will be limited to those contributions which arise from isotopic-spin mixing for Σ^0 and Λ and for the neutral pseudoscalar and vector mesons. Some of these (see first paragraph) and some other less important contributions have been discussed by DALITZ and VON HIPPEL [1]. The largest of the latter comes from baryon mass differences in intermediate states of the two-pion-exchange Λ-$\mathcal{N}$ interaction and explains about 0.05 MeV of ΔB_Λ, leaving about 0.7 MeV of the central value discussed above to be explained [3].

2. – Particle mixing.

Nonzero off-diagonal matrix elements of the electromagnetic mass-splitting operator imply that some neutral mass eigenstates (physical states, designated by a superscript $\sim$) are mixtures of isotopic-spin eigenstates (designated by the usual particle symbols alone) [1]. That is [10],

$$\tilde{\Lambda} = \Lambda - c\Sigma^0 . \tag{3a}$$

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[8] With (1) and (2), our estimate of the central value of ΔB_Λ to be explained by CSB effects is ≈ 0.75 MeV. In the absence of CSB effects, one-half of this would be added to the measured $B_\Lambda(^4\mathrm{H}_\Lambda)$ and subtracted from the measured $B_\Lambda(^4\mathrm{He}_\Lambda)$. These additions are considerably larger than the errors on the measured binding energies (≈ 0.1 MeV). This should be taken into account when the four-body hypernuclei are included in analyses of the binding energy data of the s-shell hypernuclei; CSB effects cancel out in the three- and five-body hypernuclei.

[9] Coulomb effects were not considered in the binding-energy analyses in ref. [5]. On account of Coulomb repulsion, the nucleon core of $^4\mathrm{He}_\Lambda$ is slightly less compressible than that of $^4\mathrm{H}_\Lambda$. This reduces the Coulomb energy contribution to $(\Delta B_\Lambda)_{\text{Coulomb}}$ but increases the contribution due to the difference between (compressed) core radii. These effects nearly compensate one another; and a direct calculation [similar to those reported in ref. [4]] including both effects also led to (2).

[10] We omit over-all normalization factors in (3a), (4a) and (5a); the differences between these and unity are much less than other uncertainties in the problem.

The octet model of SU_3 leads to [1,11]

$$c \approx 0.01_2 \,. \tag{3b}$$

Similarly [1,11,12],

$$\tilde{\pi}^0 = \pi^0 + c'\eta \qquad \tilde{\eta} = \eta - c'\pi^0 \tag{4a}$$

with

$$c' \approx 0.01_1 \,. \tag{4b}$$

Isotopic-spin mixing among the vector mesons is complicated by φ-ω mixing:

$$\begin{cases} \tilde{\varrho}^0 = \varrho^0 - c''\omega + d''\varphi \\ \tilde{\omega} = a''\omega - b''\varphi + e''\varrho^0 \\ \tilde{\varphi} \;= a''\varphi + b''\omega + h''\varrho^0 \,, \end{cases} \tag{5a}$$

in which ϱ^0 and φ are pure members of a unitary octet and ω is a pure unitary singlet. The large φ-ω mixing in (5a) is believed to be a consequence of medium-strong interactions [13]; and the mixing of ρ^0 with φ and ω is presumably electromagnetic mixing. The off-diagonal elements of the electromagnetic mass-splitting operator can be expected to be of order e^2 times those of the medium-strong mass-splitting operator [13]. We therefore take the coefficients

$$a'' = 0.77 \qquad b'' = 0.64 \tag{5b}$$

to be the same as those obtained by diagonalizing the $(\text{mass})^2$ matrix in the absence of ρ^0-mixing [14]. The electromagnetic mixings are then calculated as

(11) C. H. CHAN and A. Q. SARKER: *Nuovo Cimento*, **36**, 1402 (1965).

(12) We ignore the complication of X^0-η mixing recently discussed by R. H. DALITZ and D. G. SUTHERLAND: *Nuovo Cimento*, **37**, 1777 (1965). The way in which this kind of mixing enters the present problem is indicated below in our discussion of vector meson mixing. The effect of the unitary singlet X^0 in CSB will be considerably less than that of the unitary singlet ω because the $X^0 - \pi^0$ mass difference is much larger than the $\omega - \rho^0$ mass difference.

(13) L. E. PICASSO, L. A. RADICATI, D. P. ZANELLO and J. J. SAKURAI: *Nuovo Cimento*, **37**, 187 (1965).

(14) R. H. DALITZ: *Proceedings of the Sienna International Conference on Elementary Particles* (Bologna, 1963), p. 171; J. J. SAKURAI: *Phys. Rev.*, **132**, 434 (1963).

perturbations, the coefficients being [15]

$$
(5c)\quad
\begin{cases}
c'' \approx \dfrac{a''^2\langle\varrho^0|\delta M^2|\omega\rangle - a''b''\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\omega}-M^2_{\tilde\rho^0}} + \dfrac{b''^2\langle\varrho^0|\delta M^2|\omega\rangle + a''b''\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\varphi}-M^2_{\tilde\rho^0}} \\[2ex]
d'' \approx \dfrac{a''b''\langle\varrho^0|\delta M^2|\omega\rangle - b''^2\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\omega}-M^2_{\tilde\rho^0}} - \dfrac{a''b''\langle\varrho^0|\delta M^2|\omega\rangle + a''^2\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\varphi}-M^2_{\tilde\rho^0}} \\[2ex]
e'' \approx \dfrac{a''\langle\varrho^0|\delta M^2|\omega\rangle - b''\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\omega}-M^2_{\tilde\rho^0}} \\[2ex]
h'' \approx \dfrac{b''\langle\varrho^0|\delta M^2|\omega\rangle + a''\langle\varrho^0|\delta M^2|\varphi\rangle}{M^2_{\tilde\varphi}-M^2_{\tilde\rho^0}} \,.
\end{cases}
$$

The estimate $\langle\varrho^0|\delta M^2|\omega\rangle \approx 0.3_6\cdot 10^4\,(\text{MeV})^2$ follows from the analysis of a baryon mass formula in ref. [13]. The matrix element

$$\langle\varrho^0|\delta M^2|\varphi\rangle = [M^2_{\rho^0} - M^2_{\rho^+} + M^2_{K^{*+}} - M^2_{K^{*0}}]/\sqrt{3}$$

can be obtained from SU_3 in the same way as that for π^0-η mixing [1,11,16]. In the absence of an established mass difference between the K* mesons, we take this difference to be zero; then the observed masses of the ρ mesons [17] lead to $\langle\varrho^0|\delta M^2|\varphi\rangle = (-0.3 \pm 0.7)\cdot 10^4\,(\text{MeV})^2$. Since this can have either sign, and since the estimate of $\langle\varrho^0|\delta M^2|\omega\rangle$ in ref. [13] is based upon the assumption that the dominant electromagnetic mixing is that for ρ^0-ω, we take $\langle\varrho^0|\delta M^2|\varphi\rangle = 0$ for the calculations of this paper. Then (5c) become

$$(5d)\qquad c''\approx 0.05 \qquad d''\approx 0.04 \qquad e''\approx 0.06 \qquad h''\approx 0.00_5\,.$$

The large values of c'', d'' and e'' in (5d) are due to the smallness of $(M^2_{\tilde\omega} - M^2_{\tilde\rho^0})$. If $\langle\varrho^0|\delta M^2|\varphi\rangle$ has the central value deduced above from SU_3 instead of zero as we have assumed, these coefficients would be almost twice as large; in this case h'' would be much smaller than its value in (5d).

([15]) In first-order perturbation theory, the denominators in (5c) would involve the masses after medium-strong mixing but before electromagnetic mixing, rather than physical masses. We estimate that the difference between these is about 5 percent for the $\tilde\omega - \tilde\rho^0$ denominator and about $\frac{1}{2}$ percent for $\tilde\varphi - \tilde\rho^0$; these differences are negligible for the numerical estimates given in (5d).

([16]) S. Okubo and B. Sakita: *Phys. Rev. Lett.*, **11**, 50 (1963).

([17]) M. Roos: *Nucl. Phys.*, **52**, 1 (1964); *Phys. Lett.*, **8**, 1 (1964).

3. – CSB potentials.

The potentials corresponding to exchange of single $\tilde{\pi}^0$ [1] and single $\tilde{\eta}$ between $\tilde{\Lambda}$ and $\mathcal{N}$ have the CSB components

$$V_{\tilde{\Lambda}\mathcal{N}\tilde{\pi}^0}(\boldsymbol{x}) = \tau_3(\mathcal{N})[-2cg_{\Lambda\Sigma\pi} + c'g_{\Lambda\Lambda\eta} + c^2c'g_{\Sigma\Sigma\eta}]g_{\text{pp}\pi^0}v_{\text{ps},\tilde{\pi}^0}(\boldsymbol{x})\,, \tag{6}$$

$$V_{\tilde{\Lambda}\mathcal{N}\tilde{\eta}}(\boldsymbol{x}) = \tau_3(\mathcal{N})[-2cc'^2g_{\Lambda\Sigma\pi} - c'g_{\Lambda\Lambda\eta} - c^2c'g_{\Sigma\Sigma\eta}]g_{\text{pp}\pi^0}v_{\text{ps},\tilde{\eta}}(\boldsymbol{x})\,. \tag{7}$$

The nucleon isospin operator $\tau_3(\mathcal{N})$ gives $+1$ for $\mathcal{N} = \text{p}$ and -1 for $\mathcal{N} = \text{n}$. The g's are pseudoscalar coupling constants appropriate to charge-independent interactions. The function $v_{\text{ps},\mu}(\boldsymbol{x})$ is the potential corresponding to exchange of a single neutral pseudoscalar meson of mass μ, divided by the product of pseudoscalar coupling constants.

The CSB components of the potentials corresponding to exchange of single $\tilde{\rho}^0$, $\tilde{\omega}$ and $\tilde{\varphi}$ between $\tilde{\Lambda}$ and $\mathcal{N}$ are

$$\begin{aligned} V_{\tilde{\Lambda}\mathcal{N}\tilde{\rho}^0}(\boldsymbol{x}) = \tau_3(\mathcal{N})\{&[-2cg_{\Lambda\Sigma\rho^0} - c''g_{\Lambda\Lambda\omega} - c^2c''g_{\Sigma\Sigma\omega} + d''g_{\Lambda\Lambda\varphi} + c^2d''g_{\Sigma\Sigma\varphi}]\cdot \\ &\cdot[g_{\text{pp}\rho^0}v_{vv,\tilde{\rho}^0}(\boldsymbol{x}) + f_{\text{pp}\rho^0}v_{vt,\tilde{\rho}^0}(\boldsymbol{x})] + \\ &+[-2cf_{\Lambda\Sigma\rho^0} - c''f_{\Lambda\Lambda\omega} - c^2c''f_{\Sigma\Sigma\omega} + d''f_{\Lambda\Lambda\varphi} + c^2d''f_{\Sigma\Sigma\varphi}]\cdot \\ &\cdot[f_{\text{pp}\rho^0}v_{tt,\tilde{\rho}^0}(\boldsymbol{x}) + g_{\text{pp}\rho^0}v_{tv,\tilde{\rho}^0}(\boldsymbol{x})]\} \end{aligned} \tag{8}$$

$$\begin{aligned} V_{\tilde{\Lambda}\mathcal{N}\tilde{\omega}}(\boldsymbol{x}) = \tau_3(\mathcal{N})\{&[-2ce''^2g_{\Lambda\Sigma\rho^0} + a''e''g_{\Lambda\Lambda\omega} + \\ &+ a''c^2e''g_{\Sigma\Sigma\omega} - b''e''g_{\Lambda\Lambda\varphi} - b''c^2e''g_{\Sigma\Sigma\varphi}][g_{\text{pp}\rho^0}v_{vv,\tilde{\omega}}(\boldsymbol{x}) + f_{\text{pp}\rho^0}v_{vt,\tilde{\omega}}(\boldsymbol{x})] + \\ &+[-2ce''^2f_{\Lambda\Sigma\rho^0} + a''e''f_{\Lambda\Lambda\omega} + a''c^2e''f_{\Sigma\Sigma\omega} - b''e''f_{\Lambda\Lambda\varphi} - b''c^2e''f_{\Sigma\Sigma\varphi}]\cdot \\ &\cdot[f_{\text{pp}\rho^0}v_{tt,\tilde{\omega}}(\boldsymbol{x}) + g_{\text{pp}\rho^0}v_{tv,\tilde{\omega}}(\boldsymbol{x})]\} \end{aligned} \tag{9}$$

$$\begin{aligned} V_{\tilde{\Lambda}\mathcal{N}\tilde{\varphi}}(\boldsymbol{x}) = \tau_3(\mathcal{N})\{&[-2ch''^2g_{\Lambda\Sigma\rho^0} + b''h''g_{\Lambda\Lambda\omega} + b''c^2h''g_{\Sigma\Sigma\omega} + \\ &+ a''h''g_{\Lambda\Lambda\varphi} + a''c^2h''g_{\Sigma\Sigma\varphi}][g_{\text{pp}\rho^0}v_{vv,\tilde{\varphi}}(\boldsymbol{x}) + f_{\text{pp}\rho^0}v_{vt,\tilde{\varphi}}(\boldsymbol{x})] + \\ &+[-2ch''^2f_{\Lambda\Sigma\rho^0} + b''h''f_{\Lambda\Lambda\omega} + b''c^2h''f_{\Sigma\Sigma\omega} + a''h''f_{\Lambda\Lambda\varphi} + a''c^2h''f_{\Sigma\Sigma\varphi}]\cdot \\ &\cdot[f_{\text{pp}\rho^0}v_{tt,\tilde{\varphi}}(\boldsymbol{x}) + g_{\text{pp}\rho^0}v_{tv,\tilde{\varphi}}(\boldsymbol{x})]\}\,. \end{aligned} \tag{10}$$

The g's and f's are vector and tensor coupling constants, respectively, for charge-independent interactions defined, for example, by the interaction Lagrangian given in ref. [18]. The functions $v(\boldsymbol{x})$ are neutral-vector-meson-exchange potentials

[18] R. A. Bryan and B. L. Scott: *Phys. Rev.*, **135**, B 434 (1964).

divided by the coupling-constant products; the subscripts vv, tt, vt and tv indicate vector-vector, tensor-tensor, vector-tensor and tensor-vector interactions, the first subscript referring to the $\tilde{\Lambda}$ vertex.

Of the meson-baryon couplings relevant to (6)–(10), only $g_{\text{NN}\pi^0}$ is well known, and reasonable estimates exist for some of the vector-meson couplings ([19]); for the rest, we rely on the octet model of SU_3. In the couplings involving members of meson octets, there appear F-D mixing parameters α_i ([22]) ($i = ps$, v or t, for pseudoscalar, vector or tensor): $\alpha = 0$ means pure F-type coupling; $\alpha = 1$, pure D-type coupling. The couplings of the octet mesons are then

$$g_{\Sigma\Lambda\pi} = -g_{\Lambda\Lambda\eta} = g_{\Sigma\Sigma\eta} = 2\alpha_{ps} g_{\text{NN}\pi^0}/\sqrt{3}\,; \tag{11a}$$

$$g_{\Lambda\Sigma\rho} = -g_{\Lambda\Lambda\varphi} = g_{\Sigma\Sigma\varphi} = 2\alpha_v g_{\text{NN}\rho^0}/\sqrt{3} = 0\,, \tag{11b}$$

in which we have assumed pure F-type vector coupling for the vector mesons ([23]);

$$f_{\Lambda\Sigma\rho} = -f_{\Lambda\Lambda\varphi} = f_{\Sigma\Sigma\varphi} = 2\alpha_t f_{\text{NN}\rho^0}/\sqrt{3}\,. \tag{11c}$$

The pseudoscalar mixing parameter is believed to lie in the range $(\frac{1}{2} \div 1)$ ([24-26]); we take

$$\alpha_{ps} = \tfrac{3}{4}\,. \tag{12a}$$

Studies of electromagnetic form factors of nucleons indicate that the tensor coupling of isoscalar vector mesons to nucleons is small or zero ([27]). According to SU_3, $f_{\mathcal{N}\mathcal{N}\varphi} = (3 - 4\alpha_t) f_{\text{NN}\rho^0}/\sqrt{3}$; we therefore take

$$\alpha_t = \tfrac{3}{4}\,. \tag{12b}$$

([19]) See, for example, ref. ([18,20,21]) and other references cited there.

([20]) R. F. DASHEN and D. H. SHARP: *Phys. Rev.*, **133**, B 1585 (1964).

([21]) S. COLEMAN and H. J. SCHNITZER: *Phys. Rev.*, **134**, B 863 (1964).

([22]) M. GELL-MANN: *Phys. Rev.*, **125**, 1067 (1962).

([23]) See, for example, M. GELL-MANN: *The Eightfold Way: A Theory of Strong Interaction Symmetry*, California Institute of Technology Laboratory Report CTSL-20 (1961), reproduced in M. GELL-MANN and Y. NE'EMAN: *The Eightfold Way* (New York, 1964), p. 11.

([24]) A. W. MARTIN and K. C. WALI: *Phys. Rev.*, **130**, 2455 (1963).

([25]) J. J. DE SWART and C. K. IDDINGS: *Phys. Rev.*, **130**, 319 (1963).

([26]) B. W. DOWNS and R. J. N. PHILLIPS: *Nuovo Cimento*, **36**, 120 (1965). With a one-boson-exchange model for hypernuclear forces, the zero-energy singlet scattering length for Λ-Λ was less negative than that for Λ-$\mathcal{N}$ (as required by results of phenomenological analyses) for $\alpha_{ps} \leqslant \frac{1}{2}$; $\alpha_{ps} \geqslant \frac{3}{4}$ is indicated in ref. ([24,25]).

([27]) See, for example, the remarks and references cited in ref. ([18]).

The remaining couplings in (6)–(10) involve the unitarity singlet ω, which is assumed to couple the same to all baryons. For this isoscalar vector meson, we therefore take $f_{\Lambda\Lambda\omega} = f_{\Sigma\Sigma\omega} = f_{\mathcal{N}\mathcal{N}\omega} = 0$ [27].

On account of the magnitudes of the mixing coefficients c, c', c'', d'', e'' and h'', the important terms in (6)–(10) are those which contain only one of these as a factor. With the couplings given in the preceding paragraph, the leading terms in the CSB potentials are then

$$V_{\widetilde{\Lambda}\mathcal{N}\widetilde{\pi}^0}(\boldsymbol{x}) = -\tau_3(\mathcal{N})\,[\sqrt{3}(2c+c')/2]\,g^2_{\mathrm{pp}\pi^0}\,v_{\mathrm{ps},\widetilde{\pi}^0}(\boldsymbol{x}) \tag{13}$$

$$V_{\widetilde{\Lambda}\mathcal{N}\widetilde{\eta}}(\boldsymbol{x}) = -\tau_3(\mathcal{N})\,[-\sqrt{3}c'/2]\,g^2_{\mathrm{pp}\pi^0}\,v_{\mathrm{ps},\widetilde{\eta}}(\boldsymbol{x}) \tag{14}$$

$$V_{\widetilde{\Lambda}\mathcal{N}\widetilde{\rho}^0}(\boldsymbol{x}) = -\tau_3(\mathcal{N})\,\{[c''g_{\Lambda\Lambda\omega}]\,[g_{\mathrm{pp}\rho^0}v_{vv,\widetilde{\rho}^0}(\boldsymbol{x}) + f_{\mathrm{pp}\rho^0}v_{vt,\widetilde{\rho}^0}(\boldsymbol{x})] + [\sqrt{3}\,(2c+d'')f_{\mathrm{pp}\rho^0}/2][f_{\mathrm{pp}\rho^0}v_{tt,\widetilde{\rho}^0}(\boldsymbol{x}) + g_{\mathrm{pp}\rho^0}v_{tv,\widetilde{\rho}^0}(\boldsymbol{x})]\} \tag{15}$$

$$V_{\widetilde{\Lambda}\mathcal{N}\widetilde{\omega}}(\boldsymbol{x}) = -\tau_3(\mathcal{N})\,\{[-a''e''g_{\Lambda\Lambda\omega}]\,[g_{\mathrm{pp}\rho^0}v_{vv,\widetilde{\omega}}(\boldsymbol{x}) + f_{\mathrm{pp}\rho^0}v_{vt,\widetilde{\omega}}(\boldsymbol{x})] + [-\sqrt{3}b''e''f_{\mathrm{pp}\rho^0}/2]\,[f_{\mathrm{pp}\rho^0}v_{tt,\widetilde{\omega}}(\boldsymbol{x}) + g_{\mathrm{pp}\rho^0}v_{tv,\widetilde{\omega}}(\boldsymbol{x})]\} \tag{16}$$

$$V_{\widetilde{\Lambda}\mathcal{N}\widetilde{\varphi}}(\boldsymbol{x}) = -\tau_3(\mathcal{N})\,\{[-b''h''\,g_{\Lambda\Lambda\omega}]\,[g_{\mathrm{pp}\rho^0}v_{vv,\widetilde{\varphi}}(\boldsymbol{x}) + f_{\mathrm{pp}\rho^0}v_{vt,\widetilde{\varphi}}(\boldsymbol{x})] + [-\sqrt{3}a''h''f_{\mathrm{pp}\rho^0}/2]\,[f_{\mathrm{pp}\rho^0}v_{tt,\widetilde{\varphi}}(\boldsymbol{x}) + g_{\mathrm{pp}\rho^0}v_{tv,\widetilde{\varphi}}(\boldsymbol{x})]\}\,. \tag{17}$$

A classification of the terms in (13)–(17) can be made on the basis of the particle-mixing from which they arise. The terms in (13) and (15) containing the factor c depend upon Σ^0-Λ mixing and stand alone. There is considerable cancellation among the other terms, which arise from meson mixing [28]: the terms in (13) and (14) containing c', which come from π^0-η mixing, tend to cancel one another [29]; there are similar cancellations among those terms in (15)–(17) not containing c, which come from ρ^0-ω-φ mixing. On account of these cancellations, the dominant particle mixing for CSB in the Λ-$\mathcal{N}$ interaction is Σ^0-Λ.

For the coupling constants in (13)–(17) we take

$$g_{\mathrm{pp}\pi^0} = 3.8\,; \tag{18a}$$

$$g_{\mathrm{pp}\rho^0} = 1/\sqrt{2}\,, \qquad f_{\mathrm{pp}\rho^0} = \sqrt{2}\,; \tag{18b}$$

$$g_{\Lambda\Lambda\omega} = 6\,; \tag{18c}$$

or

$$g_{\Lambda\Lambda\omega} = -5\,. \tag{18d}$$

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[28] This kind of cancellation was noted in the $\mathcal{N}$-$\mathcal{N}$ problem by Y. NOGAMI and Y. P. VARSHNI: *Nuovo Cimento*, **25**, 218 (1962).

[29] Since exchange of $\widetilde{\eta}$ was not considered in ref. [1], this partial cancellation was not noted there.

The pion coupling (18a) is usual ($g^2_{\text{pp}\pi^0} = 14.4$). The ρ^0 couplings (18b) are conservative, being close to the smallest empirical estimates ([19]). The ω couplings (18c) and (18d) were obtained from the isoscalar coupling constant given in ref. ([18]): $g^2_{\mathcal{N}\mathcal{N}\tilde{\omega}} + 0.30 g^2_{\mathcal{N}\mathcal{N}\tilde{\varphi}} = 21.5$. Considering only the medium-strong ω-φ mixing with the coefficients (5b), we expressed this in terms of $g_{\mathcal{N}\mathcal{N}\omega}$ and $g_{\mathcal{N}\mathcal{N}\varphi}$; then we used SU_3 to get $g_{\mathcal{N}\mathcal{N}\varphi} = \sqrt{3} g_{\text{pp}\rho^0}$ for pure F-type coupling. Finally, use of the value of $g_{\text{pp}\rho^0}$ in (18b) led to $g_{\mathcal{N}\mathcal{N}\omega} \approx 6$ or ≈ -5; and the assumption that a unitary singlet couples the same to all baryons gives (18c) and (18d). The analysis of nucleon isoscalar form factors in ref. ([20]) indicates that the negative sign for $g_{\mathcal{N}\mathcal{N}\omega}$ may be the correct one.

The Λ-$\mathcal{N}$ interactions in the four-body hypernuclei are predominantly in relative S-states. We therefore consider only the central components of the functions $v(\boldsymbol{x})$ appearing in (13)–(17). The leading terms in a nonrelativistic approximation to these potential functions are ([30])

$$v_{\text{ps},\mu}(x) = \mu \left\{ \left[\frac{\mu^2}{12 M_\Lambda M_\mathcal{N}} \right] \boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N}) \right\} \frac{\exp[-\mu x]}{\mu x}\,, \tag{19}$$

$$v_{\text{vv},\mu}(x) = \mu \left\{ \left[1 + \frac{\mu^2 (M_\Lambda + M_\mathcal{N})^2}{8 M_\Lambda^2 M_\mathcal{N}^2} \right] + \left[\frac{\mu^2}{6 M_\Lambda M_\mathcal{N}} \right] \boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N}) \right\} \frac{\exp[-\mu x]}{\mu x}\,, \tag{20a}$$

$$v_{\text{tt},\mu}(x) = \mu \left\{ \left[\frac{\mu^2}{4 M_\Lambda M_\mathcal{N}} \right] + \left[\frac{2}{3} + \frac{\mu^2 (M_\Lambda^2 + M_\mathcal{N}^2)}{12 M_\Lambda^2 M_\mathcal{N}^2} \right] \boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N}) \right\} \frac{\exp[-\mu x]}{\mu x}\,, \tag{20b}$$

$$v_{\text{vt},\mu}(x) = \mu \left\{ \left[\frac{\mu}{2 M_\mathcal{N}} + \frac{\mu^3 (M_\Lambda + M_\mathcal{N})}{16 M_\Lambda^2 M_\mathcal{N}^2} \right] + \left[\frac{\mu}{3 M_\Lambda} + \frac{\mu^3}{24 M_\Lambda M_\mathcal{N}^2} \right] \boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N}) \right\} \frac{\exp[-\mu x]}{\mu x}\,, \tag{20c}$$

$$v_{\text{tv},\mu}(x) = \mu \left\{ \left[\frac{\mu}{2 M_\Lambda} + \frac{\mu^3 (M_\Lambda + M_\mathcal{N})}{16 M_\Lambda^2 M_\mathcal{N}^2} \right] + \left[\frac{\mu}{3 M_\mathcal{N}} + \frac{\mu^3}{24 M_\Lambda^2 M_\mathcal{N}} \right] \boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N}) \right\} \frac{\exp[-\mu x]}{\mu x}\,. \tag{20d}$$

4. – ΔB_Λ and other results.

With (19) and (20), the CSB potentials (13)–(17) are of the form

$$-\tau_3(\mathcal{N}) [A + B\boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N})] \exp[-\mu x]/(\mu x).$$

([30]) In the vector-meson-exchange potential functions, small terms containing $(\mu/M)^4$ or the $\Lambda - \mathcal{N}$ mass difference as a factor have been neglected; compare, for example, eq. (20a) with eq. (7) of ref. ([26]).

The net contribution of such a potential to $B_\Lambda(^4\mathrm{He}_\Lambda)$ is

$$(A + 3B)\langle\psi|\exp[-\mu x]/(\mu x)|\psi\rangle$$

and the negative of this to $B_\Lambda(^4\mathrm{H}_\Lambda)$. Therefore the contribution to ΔB_Λ of the CSB potential for an exchanged meson of mass μ is

$$(\Delta B_\Lambda)_\mu = (2A_\mu + 6B_\mu)\langle\psi|\exp[-\mu x]/(\mu x)|\psi\rangle \; . \tag{21}$$

The square of the wave function ψ in (21) is taken to be

$$\psi^2 = N^2\big(\exp[-\beta^2 R^2]\varphi^2_\Lambda(r) f^2(x)\big) \; , \tag{22}$$

normalized to unity, in which $\boldsymbol{x} = \boldsymbol{r} - \boldsymbol{R}$. The factor $\exp[-\beta^2 R^2]$ is the nucleon distribution function for the three-nucleon cores; we take $\beta^2 = 0.75\ (\text{fermi})^{-1}$, which corresponds to a (compressed) core radius $\langle R^2\rangle^{\frac{1}{2}} = 1.41$ fermi ([31]). $\varphi_\Lambda(r)$ is the wave function of the Λ with respect to the center of mass of the nucleon core; for this we take $\varphi_\Lambda(r) = \exp[-ar^2] + y\exp[-br^2]$ with $a = 0.277\ (\text{fermi})^{-2}$, $b = 0.045\ (\text{fermi})^{-2}$ and $y = 0.366$, as given in ref ([1]). Two forms were taken for the Λ-$\mathcal{N}$ correlation function:

$$f_1(x) = 1 \tag{23a}$$

in which correlations are ignored; and

$$f_2(x) = \begin{cases} 0 & x \leqslant c \\ 1 - \exp[-\gamma(x - c)] & x > c \; , \end{cases} \tag{23b}$$

with core radius $c = 0.4$ fermi and $\gamma = 5.45\ (\text{fermi})^{-1}$ ([32]).

The results of the calculations are given in Table I for the particle-mixing coeffcients (3*b*), (4*b*), (5*b*) and (5*d*) and for the coupling constants (18) ([33]). The parenthetical entries under $(\Delta B_\Lambda)_\mu$ are the values which come from the terms in (13) and (15) which are proportional to c and arise from Σ^0-Λ mixing,

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([31]) This implies a core compression of about 11 percent, somewhat less than that obtained in ref. ([5]); see Sect. 1.

([32]) This Λ-$\mathcal{N}$ correlation function was used in a variation calculation of Λ binding in nuclear matter, and $\gamma = 5.45\ (\text{fermi})^{-1}$ is the optimum value of the variation parameter: B. W. DOWNS and M. E. GRYPEOS: *Λ Binding in Nuclear Matter* (*Nuovo Cimento* to be published).

([33]) The expectation values in the third column of Table I were evaluated in connection with ref. ([34]).

([34]) B. W. DOWNS and R. J. N. PHILLIPS: *Nuovo Cimento*, **41**, A 374 (1966).

as discussed below eq. (17). The values of ΔB_Λ are then

$$(24a)\qquad \Delta B_\Lambda = \begin{cases} 0.87\ (0.67)\ \text{MeV for } f_1 \\ 0.39\ (0.22)\ \text{MeV for } f_2 \end{cases}$$

with the $\Lambda\Lambda\omega$ coupling (18c), and

$$(24b)\qquad \Delta B_\Lambda = \begin{cases} 0.47\ (0.67)\ \text{MeV for } f_1 \\ 0.14\ (0.22)\ \text{MeV for } f_2 \end{cases}$$

with (18d); the parentheses have the same meaning as in Table I. The values of $(\Delta B_\Lambda)_\mu$ for vector-meson exchanges depend critically upon the large $\Lambda\Lambda\omega$ coupling; but the total vector-meson contribution to ΔB_Λ is considerably less sensitive to this coupling because of the systematic cancellations discussed after eq. (17).

TABLE I. – *Contributions to* ΔB_Λ *for* $\Lambda\Lambda\omega$ *couplings* (18c) *and* (18d).

μ	$\langle\psi\|\exp[-\mu x]/(\mu x)\|\psi\rangle$		$(\Delta B_\Lambda)_\mu$ (MeV)			
			Eq. (18c)		Eq. (18d)	
	f_1	f_2	f_1	f_2	f_1	f_2
$\tilde{\pi}^0$	$2.10\cdot10^{-1}$	$1.68\cdot10^{-1}$	0.11 (0.07)	0.09 (0.06)	0.11 (0.07)	0.09 (0.06)
$\tilde{\eta}$	$7.70\cdot10^{-3}$	$3.11\cdot10^{-3}$	−0.08	−0.03	−0.08	−0.03
$\tilde{\rho}^0$	$3.16\cdot10^{-3}$	$8.53\cdot10^{-4}$	5.55 (0.60)	1.50 (0.16)	−1.69 (0.60)	−0.46 (0.16)
$\tilde{\omega}$	$2.84\cdot10^{-3}$	$7.22\cdot10^{-4}$	−4.45	−1.13	2.03	0.52
$\tilde{\varphi}$	$1.33\cdot10^{-3}$	$2.06\cdot10^{-4}$	−0.26	−0.04	0.10	0.02

CSB effects in Λ-$\mathcal{N}$ scattering have been discussed in ref. [34] in terms of phenomenological potentials; we calculated the effect of our CSB potentials in the same way here [35]. The charge-symmetric (CS) potentials were taken to be those of ref [5], which have a hard-core radius 0.4 fermi, and singlet and triplet scattering lengths −2.89 fermi and −0.70 fermi, respectively. To these were added the CSB potentials described here. The zero-energy scattering

[35] In ref. [34], phenomenological CSB potentials required to explain all of $(\Delta B_\Lambda)_{CSB} \approx 0.75$ MeV were used in scattering calculations; here we used only one-meson-exchange CSB potentials which are supposed to explain most, but not all, of $(\Delta B_\Lambda)_{CSB}$. In all calculations of this paper, the nucleon mass was taken to be the average of neutron and proton masses so that one-meson-exchange CSB effects would not be diluted by other CSB effects.

lengths a and effective ranges r_0 of the total interactions are given in Table II, the CSB potentials being designed by $\Lambda\Lambda\omega$ coupling (18c) or (18d). Results

TABLE II. – *Zero-energy scattering parameters.*

Potential		Singlet		Triplet	
		a (fermi)	r_0 (fermi)	a (fermi)	r_0 (fermi)
CS		−2.89	1.95	−0.70	3.72
CS + (18c)	Λ-p	−2.66	1.91	−0.76	3.80
	Λ-n	−3.15	1.97	−0.65	3.86
CS + (18d)	Λ-p	−2.60	1.92	−0.71	3.86
	Λ-n	−3.22	1.97	−0.69	3.58

are given for both Λ-p and Λ-n; the former can be measured directly, and a method for measuring the latter in final-state interactions was given in ref. ([34]). The results in Table II have the qualitative features of case (a2) in ref. ([34]), which assumes a CSB potential of the form $-\tau_3(\mathcal{N})B\boldsymbol{\sigma}(\Lambda)\cdot\boldsymbol{\sigma}(\mathcal{N})\exp[-\mu x]/(\mu x)$.

The contribution of a CSB potential to Λ binding in possible $J=1$ excited states of the four-body hypernuclei is

$$[\delta B_\Lambda^*({}^4\mathrm{He}_\Lambda^*)]_\mu = (A_\mu - B_\mu)\langle\psi'|\exp[-\mu x]/(\mu x)|\psi'\rangle,$$

and the negative of this for ${}^4\mathrm{H}_\Lambda^*$. Although ψ' is not the same as the ψ in (22) (because of differences in Λ binding and compressed nucleon-core radii), values obtained with (22) should give a good estimate. For $g_{\Lambda\Lambda\omega}=6$, δB_Λ^* is negligible; for $g_{\Lambda\Lambda\omega}=-5$, $\delta B_\Lambda^*({}^4H_\Lambda^*)$ is about 0.1 MeV for the correlation function f_1 and about half this for f_2. These results probably do not indicate an observable effect unless $(B_\Lambda^*)_{\mathrm{CS}}$ is close enough to zero that CSB would make ${}^4\mathrm{H}_\Lambda^*$ bound and ${}^4\mathrm{He}_\Lambda^*$ unbound; and this does not appear to be the case ([5]).

5. – Remarks.

For $g_{\Lambda\Lambda\omega}=6$, our results (24$a$) bracket the value $\Delta B_\Lambda \approx 0.7$ MeV to be explained by single-meson exchanges, as discussed in Sect. **1** ([3]); for $g_{\Lambda\Lambda\omega}=$ $=-5$, (24b) falls somewhat short of reproducing the required ΔB_Λ. Values intermediate between those obtained with f_1 and f_2 could be obtained (a) with a core radius smaller than 0.4 fermi or (b) with a soft core such as that corresponding to (charge-symmetric) vector-meson exchange [36]. As pointed out following eqs. (18), the ppρ^0 couplings (18b) are conservative; for example, values $\sqrt{2}$ times as large as these can also be reconciled with experimental data [27]. Use of ppρ^0 couplings $\sqrt{2}$ times those in (18b) lead to values of ΔB_Λ approximately twice as large as those in (24) in each case (including those in parentheses), providing greater opportunities for explaining the empirical ΔB_Λ.

Increasing the ppρ^0 couplings (18b) by a factor of $\sqrt{2}$ also has the effects of increasing the shifts in scattering lengths from the CS values given in Table II by about (50÷100) percent and of nearly doubling the largest values of $\delta B^*_\Lambda(^4\mathrm{H}^*_\Lambda)$ described below Table II.

It is clear from Table I and eq. (24) that CSB can be dominated by vector-meson-exchanges and, in particular, by the ρ^0-exchange contribution arising from Σ^0-Λ mixing. The CSB from pseudoscalar-meson exchanges is less important. The $\tilde{\pi}^0$-$\tilde{\eta}$ cancellation discussed following eq. (17) reduces the pseudoscalar contribution considerably: for equal pseudoscalar coupling constants, $\tilde{\eta}$-exchange is about $2\frac{1}{2}$ times as effective as $\tilde{\rho}^0$-exchange with f_1 and about 25% more effective with f_2, as was noted in ref. [26].

Really large effects in Λ-p scattering were obtained in ref. [34] with a spin-independent phenomenological CSB potential. Such a potential could be approximated by the present model only if the vector couplings of ρ^0 to Λ, Σ^0 and $\mathcal{N}$ were very much larger than the tensor couplings (see eq. (20a)), instead of $g_{\Lambda\Sigma\rho^0} = 0$ and $g_{\mathrm{pp}\rho^0} = \frac{1}{2} f_{\mathrm{pp}\rho^0}$ as we have assumed.

Single-meson-exchange contributions to CSB in the Σ^0-$\mathcal{N}$ interaction are similar to those in the Λ-$\mathcal{N}$ interaction. The Σ^0-$\mathcal{N}$ CSB potentials can be obtained from (6)–(10) by interchanging Λ and Σ^0 and by replacing the mixing coefficient c by $-c$. On account of the rapid conversion of Σ^0 to Λ, however, CSB for Σ^0-$\mathcal{N}$ is much less accessible than for Λ-$\mathcal{N}$.

CSB in the Λ-$\mathcal{N}$ interaction is dominated by Σ^0-Λ mixing, which does not play a role in the $\mathcal{N}$-$\mathcal{N}$ interaction. The contribution of single-meson exchanges to $\mathcal{N}$-$\mathcal{N}$ CSB is therefore considerably smaller than that to Λ-$\mathcal{N}$ CSB [36].

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[36] B. W. DOWNS and Y. NOGAMI: *Meson Mixing and the Charge Asymmetry of the $\mathcal{N}$-$\mathcal{N}$ Interaction* (to be published).

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RIASSUNTO (*)

La mescolanza elettromagnetica di autostati dello spin isotopico porta ad importanti elementi che rompono la simmetria della carica (CSB) nell'interazione Λ-$\mathcal{N}$. La mescolanza più importante è quella per Σ^0-Λ, che consente l'accoppiamento della componente (dominante) $T = 1$ dei π^0 e ρ^0 fisici al Λ fisico. Le mescolanze π^0-η e ρ^0–ω–φ danno ulteriori contributi. Da queste mescolanze si costruisce un modello di potenziale CSB di scambio di un solo mesone, con l'aiuto del modello di ottetto della SU_3 per collegare gli accoppiamenti mesone-barione. La differenza fra i legami del Λ negli ipernuclei speculari $^4He_\Lambda$ e $^4H_\Lambda$ può essere spiegata con questo modello, in cui lo scambio di mesoni vettoriali predomina sullo scambio di mesoni pseudoscalari. Si espongono le predizioni del modello per lo scattering Λ-$\mathcal{N}$ e per le energie di legame dei possibili stati eccitati degli ipernuclei a quattro corpi.

(*) *Traduzione a cura della Redazione.*