Schrödinger's Equation as a Consequence of Zitterbewegung.

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Summary. – The Schrödinger equation is derived assuming that any particle is subject to a flickering motion (zitterbewegung) independent of external forces. The motion should be as that of a rocket emitting random bursts of jets with isotropic distribution and also subject to the external, deterministic forces.

In the past there have been two attempts to obtain the Schrödinger equation by a classical stochastic process [1].

The first attempt is due to NELSON [2] who assumed a Brownian stochastic process for any particle. However, this process has a white spectral density and, therefore, it cannot be frictionless because of the fluctuation-dissipation theorem and, mainly, because any charged oscillator (like an electron moving around a nucleus) with charge e, mass m, proper frequency ω_0, baricentric velocity $\boldsymbol{v}$ is subject to a friction force F_f given by the Einstein-Hopf formula [3]

$$\boldsymbol{F}_f = -\tfrac{4}{3}\pi^2 \boldsymbol{v} e^2 (mc^2)^{-1} [G(\omega_0) - \tfrac{1}{3}\omega_0 (\mathrm{d}G/\mathrm{d}\omega)_{\omega_0}] . \tag{1}$$

With $G(\omega_0) = \text{constant}$, eq. (1) gives a braking force. Consequently, the velocity of a charged oscillator should decrease exponentially with time and we could not have a motion by inertia. In order to overcome this drawback NELSON had to assume an equation of motion different from Newton's and implying forward and backward derivatives. This is not classical physics and Nelson's attempt has only to be considered as a mathematical connection between Schrödinger equation and other *ad hoc* postulated equations [3].

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(1) I disregard all the attempts in which an imaginary time or other imaginary quantities are introduced at the beginning.
(2) E. NELSON: *Phys. Rev.*, **150**, 1079 (1966).
(3) See C. CAVALLERI: *Phys. Rev. D*, **23**, 363 (1981), where also a criticism of stochastic mechanics and in particular of the paper of ref. (2) is given. An extension of eq. (1) to a free charge has been obtained by A. RUEDA: *Phys. Rev. A*, **23**, 2020 (1981).

The second attempt was fraught with a great hope because starting from a fundamental theory called stochastic electrodynamics [4] (SED). Its spectral density is proportional to ω^3 (where ω is the angular frequency) and is the only one to be relativistically invariant and therefore allowing a motion by inertia. This can also be verified by eq. (1) because, for $G(\omega) \propto \omega^3$ it is $\boldsymbol{F}_i = 0$ (note that any other spectrum gives $\boldsymbol{F}_i \neq 0$). SED gives results in agreement with quantum mechanics (QM) and even with quantum electrodynamics (QED) for all the linear systems [5]. However, it fails in the case of nonlinear systems [6].

The stochastic process proposed in this paper is given by the zitterbewegung (as foreseen in Dirac's theory and also in Pauli's) together with the transverse energy [3,7] of QED. A macroscopic model of it can be obtained by a rocket having a main jet perpendicular to the velocity $\boldsymbol{v}_s$ so that, by this jet only, it would perform a circular orbit. But the rocket has also many other nozzles with almost isotropic directions emitting random bursts which produce a diffusive motion for the centre of the orbit which is no longer a perfect circle. The absolute value $|\boldsymbol{v}_s|$ of the velocity is taken to be constant (and equal to the light speed for the electron [8-10]) and the diffusive motion of the centre of the perturbed orbit is equivalent to the transverse energy of QED. The random bursts also produce a fluctuation of the axis of the roughly circular orbit and this completes the description of an electron endowed with spin. If we represent the instantaneous spin value by a vector of constant magnitude (both speed and average radius of the spin circular orbit are constant) and randomly fluctuating direction and we keep the first tip of this vector fixed, the second tip (arrow) of the spin vector is uniformly distributed on a sphere. If there is an external magnetic field $\boldsymbol{B}$, an up- or down-spin corresponds to a distribution of the arrow on a semi-sphere whose symmetry axis is parallel or antiparallel to $\boldsymbol{B}$, respectively. This is just what required for violating the Bell inequalities and explaining the Einstein-Podolski-Rosen paradox [11].

The random electromagnetic fields of SED are considered as real (and coinciding with the zero-point field of QED taken in realistic sense), but their effect should only

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(4) For a review of the results of SED see M. SURDIN: *Ann. Inst. Henri Poincaré*, **15**, 203 (1971) See also T. H. BOYER: *Phys. Rev. D*, **11**, 790, 809 (1975).

(5) For the most recent achievements see T. H. BOYER: *Phys. Rev.*, **18**, 1238 (1978); P. CLAVERIE and S. DINER: *Int. J. Quantum Chem.*, **2**, Suppl. 1, 41 (1977).

(6) P. CLAVERIE and S. DINER: *Int. J. Quantum Chem.*, **12**, Suppl. 1, 41 (1977); P. CLAVERIE: *Proceedings of « Einstein Centennial Symposium on Fundamental Physics »*, edited by S. M. MOORE, J. G. RODRIGUEZ-VARGAS, A. RUEDA and G. VIOLINI (Bogotà, 1979); in *Dynamical Systems and Microphysics*, edited by A. BLAQUIERE, F. FER and A. MARZOLLO, CISM Courses and Lectures No. 261 (Springer-Verlag, New York, N. Y., 1980), p. 111; P. CLAVERIE, L. PESQUERA and F. SOTO: *Phys. Lett. A*, **80**, 113 (1980); T. W. MARSHALL and P. CLAVERIE: *J. Math. Phys. (N. Y.)*, **21**, 1819 (1980).

(7) D. HESTENES: *J. Math. Phys. (N. Y.)*, **16**, 568 (1975); *Am. J. Phys.*, **47**, 399 (1979), and references therein. The works of Hestenes are very important for showing in the pure environment of QM that the ground-state kinetic energy is completely determined by the electron spin even in the Schrödinger equation. The latter represents en electron in an eigenstate of spin which, in our model, corresponds to an isotropic distribution of spin axes (unpolarized spin).

(8) G. CAVALLERI and G. SPINELLI: *Nuovo Cimento B*, **39**, 93 (1977); G. CAVALLERI: *Nuovo Cimento B*, **55**, 392 (1980).

(9) A. RUEDA and G. CAVALLERI: *Nuovo Cimento C*, **6**, 239 (1983).

(10) A. O. BARUT and A. J. BRACKEN: *Phys. Rev. D*, **23**, 2454 (1981); A. O. BARUT and N. ZANGHI: *Phys. Rev. Lett.*, **52**, 2009 (1984). A motion with the velocity of light has also been introduced by N. CUFARO PETRONI and J. P. VIGIER: *Int. J. Theor. Phys.*, **18**, 807 (1979) although not for spin. These authors consider a lattice in the Minkowski space and then assume that any particle undergoes a random walk at the speed of light between the points of the lattice. This procedure also implies jumps towards negative times for antiparticles or, conversely, particle-antiparticle transitions. Moreover, they do not consider the presence of external forces whose actions on such particles would be difficult to be treated.

(11) I. PITOWSKY: *Phys. Rev. Lett.*, **48**, 1299 (1982); *Phys. Rev. D*, **27**, 2316 (1983).

give the radiative corrections which are neglected in this paper. I also neglect the radiation damping term whose dissipated power should balance, on the average, that absorbed from the random fields of SED. In other words the flickering motion due to the self-reaction is here considered as much more intense and as a constant, therefore independent of the external forces, also.

The assumed zitterbewegung (or spin motion) has already given good results for both a gravitational theory ([8]) and the origin of the cosmic rays ([9]) and can be thought of as given by a self-reaction although, at present, it is not explainable in classical terms. Yet any difficulty is displaced from the atomic to the elementary-particle level. This is satisfactory because atomic physics is governed by the well-known electromagnetic interaction, while the structure of the elementary particles or their internal motion like that of spin are still unknown. In any case, once assumed the spin motion all the procedure leading to the Schrödinger equation is purely classical and allows us to conceive atoms intuitively by a realistic picture which is forbidden by the orthodox interpretation of QM.

In this paper an unpolarized spin is considered, *i.e.*, as said, with the arrows of the spin axis isotropically distributed ([12]). Moreover, in the present model, the electron has a self-reaction so as to behave like a rocket with jets and has no friction. There is, therefore, a velocity potential φ for the velocity $\langle \boldsymbol{v} \rangle$, of the centre of the spin orbit, *i.e.* for the electron mean velocity $\langle \boldsymbol{v} \rangle$, where the average is taken over a time interval long compared to the spin revolution period T and short compared to an atomic revolution period $T_a \simeq \alpha^{-2} T \simeq 1.88 \cdot 10^4 \, \mathrm{T}$. We can, therefore, write

$$\langle \boldsymbol{v} \rangle = \nabla \varphi \,. \tag{2}$$

The problem must be treated statistically by the probability density $\varrho(r, t)$ obtained, as usual, by an ensemble average. Also all the properties of a fluid can be derived by the kinetic theory with the use of ϱ even if a single particle only is considered, provided the interactions with the stochastic envirommente are taken into account. In a fluid the interactions are due to the other molecules, while in our case of a single electron they are due to the fields and to the effects of the random jets emitted by our rocket-like electron. What is important is to obtain the average energy E per unit mass, including the internal energy U. Indeed, as for a generic fluid with density ϱ, it has been proved ([13]) that the equations of motion can be derived by demanding that the action

$$\mathscr{A} = \int \mathrm{d}t \, \mathrm{d}^3 r \, L = \int \mathrm{d}t \, \mathrm{d}^3 r (\partial_t \varphi + E) \varrho \,, \tag{3}$$

be stationary.

In the following we look for E. To this aim we formalize the zitterbewegung assumption by requiring that the rocketlike electron performs a stochastic motion with mean free path λ and a speed distribution $f(v)$ independent of external forces and, therefore, constant in time. (The simplest example, in accordance with Dirac zitterbewegung ([10]), is that, in the absence of external forces, the electron performs a circular motion of diameter λ at the speed of light c.)

([12]) For a polarized spin what is actually measured in a stochastic model is the mean square root of the component $\langle L_x^2 \rangle^{\frac{1}{2}}$ of the spin angular momentum $\langle L^2 \rangle^{\frac{1}{2}}$ along the $\boldsymbol{B}$ direction. Statistically $\langle L_x^2 \rangle = \langle L_y^2 \rangle = \langle L_z^2 \rangle = \langle L^2 \rangle / 3$ and that is why the absolute value of the spin is $\sqrt{3}$ times the observable component, thus explaining a QM result.

([13]) R. L. SELIGER and G. B. WHITHAN: *Proc. R. Soc. London, Ser. A*, **305**, 1 (1968); E. A. SPIEGEL: *Physica (Utrecht) D*, **1**, 236 (1980).

If there is a concentration gradient $\nabla\varrho$ a diffusion flow is present. In the case of plane symmetry with $\varrho(\boldsymbol{r}, t)$ on a plane α and $\varrho(\boldsymbol{r} - \boldsymbol{\lambda}, t - \lambda/v)$ on another plane parallel to α and a mean free path λ apart from α, the net current density flow $\boldsymbol{J}$ perpendicular to α is given by the mean value $\boldsymbol{J}_0$ (due to drift or inertia) plus the difference between the diffusion flows between the two planes and due to the different ϱ

$$J = J_0 + [\varrho(\boldsymbol{r} - \boldsymbol{\lambda}, t - \lambda/v) - \varrho(r, t)]v\,, \tag{4}$$

the absolute value v of the velocity being assumed constant. A first-order expansion gives

$$\varrho(\boldsymbol{r} - \boldsymbol{\lambda}, t - \lambda/v) \simeq \varrho(r, t) - \boldsymbol{\lambda}\cdot\nabla\varrho - (\lambda/v)\,\partial_t\varrho\,. \tag{5}$$

The contribution of the last term in the very short time λ/v is negligible and (5) in (4) gives

$$J = J_0 - v\boldsymbol{\lambda}\cdot\nabla\varrho\,. \tag{6}$$

In the general case, instead of considering the net effect due to the two opposite flow densities coming from the two parallel planes, we take a sphere of radius λ and we consider the contributions due to the free flights coming from the sphere surface

$$\boldsymbol{J}_1(r) = \boldsymbol{J}_0 + \int_0^{2\pi}\frac{\mathrm{d}\psi}{2}\int_0^{\pi}\frac{\mathrm{d}\theta}{2}\sin\theta\,\varrho(\boldsymbol{r} - \boldsymbol{\lambda}, t - \lambda/v)\,\boldsymbol{v}\,. \tag{7}$$

Still by (5), where the last term is neglected and assuming the direction of $\nabla\varrho$ as the symmetry axis from which the nutation angles θ of $\boldsymbol{v} = v\,(\boldsymbol{i}\sin\theta\cos\psi + \boldsymbol{j}\sin\theta\sin\psi + \boldsymbol{k}\cos\theta)$ are measured, we get, since $\nabla\varrho$ is parallel to $\boldsymbol{v}$,

$$\boldsymbol{J}_1(\boldsymbol{r}) = \boldsymbol{J}_0 - \int_0^{\pi}\mathrm{d}\theta\,\tfrac{1}{2}\sin\theta\,v\lambda\cos^2\theta\,\nabla\varrho = \boldsymbol{J}_0 - \tfrac{1}{3}\lambda v\nabla\varrho\,. \tag{8}$$

As in the kinetic theory of gases we denote by D^* the coefficient of $\nabla\varrho$:

$$D^* = \tfrac{1}{3}\lambda v\,. \tag{9}$$

In a Brownian motion $D^* = D$, where $D = \langle\Delta y^2\rangle/2\,\Delta t$ is the transversal-diffusion coefficient, Δt being a convenient time interval ([14]). In our non-Markovian stochastic process with infinite memory ($\boldsymbol{J}_0$ in eq. (8) depends on all the preceding history) $\langle\Delta y^2\rangle \propto$ $\propto (\Delta t)^2$ and the diffusion coefficient would depend on Δt. We call D^* inertial spreading coefficient which turns out to be constant because $v = \text{const}$. With a velocity distribution function depending on external forces D^* would be no longer a constant. Consequently, the assumption of the zitterbewegung plays a fondamental role.

Let us now consider a second equal volume $\mathrm{d}V_2$ adjacent to that considered $\mathrm{d}V_1$ above and along the $\nabla\varrho$ direction. In a Brownian motion the $\boldsymbol{J}$ relevant to this second elementary volume would differ from the value (8) by infinitesimal terms. On the con-

([14]) M. Lax: *Rev. Mod. Phys.*, **38**, 359, 541 (1966).

trary, in our motion with inertia the diffusion effect in the first volume is remembered in the second and we have

$$\boldsymbol{J}_2 = \boldsymbol{J}_0' - D^*\nabla\varrho = (\boldsymbol{J}_0 - D^*\nabla\varrho) - D^*\nabla\varrho = \boldsymbol{J}_0 - 2D^*\nabla\varrho\,. \tag{10}$$

In other words, the pure effect of the diffusion due to the concentration gradient is the same as for $\mathrm{d}V_1$, but the average value $\boldsymbol{J}_0$ in its surroundings is different from that $\boldsymbol{J}_0$ of $\mathrm{d}V_1$. The reason is the absence of friction which implies a motion by inertia with a consequent memory in a cell $\mathrm{d}V_2$ of what occurred in the nearby cell $\mathrm{d}V_1$.

Consequently, if we consider an elementary volume including the two considered volumes we have an average diffusion inside it. Let $\langle\boldsymbol{v}\rangle$ be the centre-of-mass velocity of the probability relevant to this elementary volume. We recall that $\langle\boldsymbol{v}\rangle$ is obtained by an ensemble average and is *not* the actual velocity $\boldsymbol{v}$ of the particle. If $\langle\boldsymbol{v}_1\rangle = \boldsymbol{J}_0 - D^*\nabla\varrho$ is the ensemble average velocity of the first volume $\mathrm{d}V_1$ and $\langle\boldsymbol{v}_2\rangle = \boldsymbol{J}_0 - 2D^*\nabla\varrho$ the ensamble average velocity of $\mathrm{d}V_2$ and, if $\mathrm{d}V_1 = \mathrm{d}V_2$, it is

$$\varrho\langle\boldsymbol{v}\rangle = \tfrac{1}{2}\varrho(\langle\boldsymbol{v}_1\rangle + \langle\boldsymbol{v}_2\rangle) = \boldsymbol{J}_0 - 1.5D^*\nabla\varrho\,. \tag{11}$$

The diffusion velocity relevant to this centre-of-mass system is, therefore, $0.5D^*\nabla\varrho$ for the first volume $\mathrm{d}V_1$ and $-0.5D^*\nabla\varrho$ for the second volume $\mathrm{d}V_2$. The constant $0.5D^*$ depends on our choice of a single mean free path for the lengths of both $\mathrm{d}V_1$ and $\mathrm{d}V_2$. This should be proved by solving the relevant Boltzmann equation with an expansion of $\varrho(\boldsymbol{r}, \boldsymbol{v})$ in Legendre polynomials truncated after three terms, at least. The usual P_1 approximation (two terms) confirms the choice of a single λ for the diffusion current given by (8). In any case, we can write for the ensemble relative velocity $\langle\boldsymbol{v}_{r1}\rangle = \langle\boldsymbol{v}_1\rangle - \langle\boldsymbol{v}\rangle$ of the centre of mass of $\mathrm{d}V_1$ with respect to the centre-of-mass velocity $\langle\boldsymbol{v}\rangle$ of $\mathrm{d}V_1 + \mathrm{d}V_2$

$$\langle\boldsymbol{v}_{r1}\rangle = C\nabla\varrho/\varrho\,, \tag{12}$$

where C is an unknown constant (like λ and D^*). The same expression with reversed sign holds for $\langle\boldsymbol{v}_{r2}\rangle = \langle\boldsymbol{v}_2\rangle - \langle\boldsymbol{v}\rangle$.

Consequently, the kinetic energy of the volume $\mathrm{d}V = \mathrm{d}V_1 + \mathrm{d}V_2$, which is equal to the translational kinetic energy (as if all the masses have the centre-of-mass velocity) plus the kinetic energy relative to the centre of mass (König's theorem), can be written as

$$\frac{m}{2}\,\mathrm{d}^3r\,\langle\boldsymbol{v}\rangle^2 + \frac{m}{4}\,\varrho\,\mathrm{d}r_1\left(\frac{C}{\varrho}\nabla\varrho\right)^2 + \frac{m}{4}\,\varrho\,\mathrm{d}^3r_2\left(-\frac{C}{\varrho}\nabla\varrho\right)^2, \tag{13}$$

where only averaged quantities are considered because they are the only ones which are observed, as in the motion of a generic fluid for which eq. (3) applies.

Again note that the second and third terms of eq. (13) would disappear in a Brownian motion since the local average velocity $\langle\boldsymbol{v}\rangle$ already includes the diffusion velocity which differs by a negligible amount for two adjacent elementary volumes whose centres are separated by some mean free paths λ.

If the external, macroscopic force $\boldsymbol{F}(r)$ per unit mass is conservative and can, therefore, be written as $m\varrho\,\mathrm{d}^3r\boldsymbol{F} = -m\varrho\,\mathrm{d}^3r\nabla V$ the total energy of the considered volume is given by

$$m\varrho\,\mathrm{d}^3r\left[\frac{1}{2}\langle v^2\rangle + \frac{1}{2}\left(\frac{C}{\varrho}\nabla\varrho\right)^2 + V\right] = m\varrho\,\mathrm{d}^3rE\,, \tag{14}$$

which is equal to that of a fluid with internal energy per unit mass $U = 2^{-1}(C\nabla\varrho/\varrho)^2$ given by the kinetic energy relative to the centre of mass.

Having obtained the total, average energy E per unit mass, we vary (3) with the E given by (14).

If we use (2) and set the functional derivative of $\mathscr{A}$ with respect to φ equal to zero, *i.e.* $\partial L/\partial\varphi - \partial_\alpha[\partial L/\partial(\partial_\alpha\varphi)] = 0$ with $\partial_\alpha = \partial/\partial x_\alpha$ and $\alpha = 0, 1, 2, 3$, which in our case reduces to $\partial_t[\partial L/\partial(\partial_t\varphi)] + \partial_s[\partial L/\partial(\partial_s\varphi)] = 0$ with $s = 1, 2, 3$ we obtain

$$\partial_t\varrho + \nabla\cdot(\varrho\nabla\varphi) = 0\,. \tag{15}$$

This is the continuity equation expressing the conservation of the particles number and which could, therefore, be obtained also directly.

Setting the functional derivative of $\mathscr{A}$ with respect to ϱ equal to zero gives

$$\partial_t\varphi + \frac{1}{2}(\nabla\varphi)^2 + V + C^2\left[\frac{1}{2}\left(\frac{\nabla\varrho}{\varrho}\right)^2 - \frac{\nabla^2\varrho}{\varrho}\right] = 0\,. \tag{16}$$

By the transformation $\psi = \varrho^{\frac{1}{2}}\exp[i\varphi/2C]$ it is easy to show that eqs. (15) and (16) are equivalent to the complex equation

$$-C^2\nabla^2\psi + \psi V/2 = iC\partial_t\psi\,, \tag{17}$$

which is equal to the Schrödinger equation provided $C = \hbar/2m$. The value of the constant has to be assumed, as SCHRÖDINGER did, to get agreement with experiments.

The central point of this derivation is the diffusion current with respect to the local centre of mass. This has never been understood by the authors who tried to derive the Schrödinger equation by a stochastic model. Indeed all speak of a diffusion current $-D\nabla\varrho$ with respect to the *laboratory*, which is meaningless in our motion by inertia, where a velocity with respect to the laboratory depends on all the preceding history of the particle. Moreover, once introduced the mean local velocity there is no longer a place for another local velocity with respect to the laboratory. Finally, it is the diffusion around the local centre of mass with a velocity proportional to what is known as diffusion velocity which justifies, via König's theorem, the local statistical independence of $\langle\boldsymbol{v}\rangle$ with respect to $\pm C\nabla\varrho$. This was assumed without reason by SANTOS ([15]) who already wrote the variational principle (3). Moreover, SANTOS considered a Brownian stochastic process which cannot avoid friction.

In principle, although no author used it, the diffusion around the local centre of mass could be used in stochastic electrodynamics (SED) whose process allows a motion by inertia. However, the inertial spreading coefficient given by (9) and therefore the C appearing in (12)-(14) are constant in SED only for linear forces and that is why SED gives good results for linear problems but fails for nonlinear ones. To show this, let us consider a local accelerated observer S fixed to the local centre of mass C_M of an elementary probability cloud $\varrho\,\delta V$ and apply the virial theorem to the particles of the ensemble around C_M and inside δV. In C_M there is perfect balance between $m\boldsymbol{a}$ and the external

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([15]) E. SANTOS: *Nuovo Cimento B*, **59**, 65 (1969). Also P. D. RASKIN: *Found. Phys.*, **8**, 31 (1978) had similar ideas. He, too, uses a Brownian motion and, in spite of the presence of friction, he considers a conserved internal kinetic energy which he subtracts from (instead of adding to) the other energies. Moreover, he uses different variations andh is resultant eq. (24*a*) misses the term $(C\,\Delta\varrho/\varrho)^2$, so that his eqs. (24*a*) and (24*b*) are *not* equivalent to the Schrödinger equation.

force $\boldsymbol{F}(C_M)$. In a point $\delta\boldsymbol{r}$ apart from C_M the force is $\boldsymbol{F} = \boldsymbol{F}(C_M) + \delta\boldsymbol{r}\cdot\nabla\boldsymbol{F}$. What is effective in S is $\boldsymbol{F} - m\boldsymbol{a} = \delta\boldsymbol{r}\cdot\nabla\boldsymbol{F}$, which for a linear $\boldsymbol{F} = -A\boldsymbol{r}$ is still linear, so that its local energy is $\langle A\,\delta r^2/2\rangle$ to be equated to the local kinetic energy which, for fixed $\langle\delta r^2\rangle$, turns out to be independent of the distance from the attractive centre. For nonlinear systems $K = \langle\delta r^2(\partial F/\partial r)/2\rangle$, where $\partial F/\partial r$ depends on $\boldsymbol{r}$. For a Coulomb force $\partial F/\partial r \propto r^{-3}$ and K would strongly depend on r, hence on t. That is why the results of SED without zitterbewegung are different from those of QM for nonlinear systems.

The assumption of the zitterbewegung as a stochastic motion without friction and with a time-independent velocity distribution has led to the Schrödinger equation (17). However, for getting full agreement with nonrelativistic quantum mechanics we must add the theory of measurement. To this regard, it is not usually emphasized that there are two kinds of observations, one perturbing and the other nonperturbing the system under examination. For instance the observations of spontaneous decays (either of excited atoms or of radioactive nuclei) are nonperturbing. Indeed in this case the observer simply counts the emitted particles whose emission rate is independent of the distance between the source and the detectors. On the contrary in the stimulated decay the observer sends some particles into the system and we do not have in nature something less perturbing than a photon or an electron to observe another electron. The above point of view, typical of SED and in general of all classical physics, coincides with the probabilistic interpretation ([16]) of QM which avoides the paradoxes like Schrödinger's cat. In this case it is not the observation which makes the state superposition of alive and dead cats precipitate on a given pure state. We simply have a probability of having the cat either alive or dead even before the measurement. The latter simply ascertains what there is already.

The other requirements to get agreement with QM are automatically satisfied, since we have constructed ψ so that $\psi\psi^* = \varrho$ and ϱ is our starting quantity. Being ϱ a probability density it is obviously normalized, *i.e.* $\int \mathrm{d}^3r\,\psi\psi^* = 1$. Since ψ is not observable, the other postulates of von Neumann are unnecessary.

([16]) R. G. NEWTON: *Am. J. Phys.*, **42**, 1029 (1980).