Multiple-Scattering Corrections to Pomeranchukon Exchange.

S. FRAUTSCHI (*) and B. MARGOLIS (**)
CERN - Geneva

(ricevuto il 10 Luglio 1968)

When the Pomeranchukon trajectory was introduced to fit the forward peaks of high-energy elastic scattering, it was initially assigned a slope $\alpha'_p(t) \approx 1\,(\text{GeV})^{-2}$ at small momentum transfers t. Comparison of subsequent pp, $\bar{\text{p}}$p, πp and Kp experiments with single-Pomeranchukon exchange models, however, indicated that although the *other* well-established trajectories seem to have a slope of this order, the Pomeranchukon trajectory fit best when assigned a small or even zero slope α' [1].

In the present paper we return to this question of the Pomeranchukon slope, taking account of multiple-Pomeranchukon exchange corrections (*i.e.* multiple scattering), and incorporating the current theoretical idea that the leading trajectories may be approximately linear over the whole observed range of t. Remarkably, the inclusion of multiple scattering resurrects the possibility that the Pomeranchukon has a « normal » slope ($\alpha'_p \approx 1\,(\text{GeV})^{-2}$). The model with this slope is consistent with the gross features of pp, $\bar{\text{p}}$p, πp and Kp elastic-scattering data at present energies and makes the following distinctive predictions concerning future higher-energy experiments:

i) $d\sigma/dt$ will continue falling as energy increases at all but the smallest values of t, rather than reaching a plateau as it would if $\alpha'_p = 0$.

ii) The total cross-section will eventually rise. We estimate $\sigma^{\text{tot}}_{\text{pp}}$ will rise from 40 mb at 30 GeV, to ≈ 50 mb at infinite energy.

iii) The ratio $\text{Re}\,A(0)/\text{Im}\,A(0)$ for the forward nonflip amplitude, which is generally negative at present energies, will cross over and become positive at higher energies.

Before proceeding further, we should acknowledge that multiple-scattering treatment of the elastic peak is an old idea. For example, AMATI *et al.* [2], ANSELM and

(*) Alfred P. Sloan Foundation Fellow. Permanent address: Physics Department, California Institute of Technology, Pasadena, Cal.

(**) National Research Council of Canada Senior Research Fellow. On leave from Institute of Theoretical Physics, McGill University, Montreal.

(1) See, for example, G. F. CHEW: *Comm. Nucl. Part. Phys.*, **1**, 121 (1967).

(2) D. AMATI, M. CINI and A. STANGHELLINI: *Nuovo Cimento*, **30**, 193 (1963); A. A. ANSELM and I. T. DYATLOV: *Phys. Lett.*, **24**, B 479 (1967). In particular, ANSELM and DYATLOV have stressed that $\alpha'_p \approx 1\,(\text{GeV}/c)^{-2}$ fits the pp data.

DYATLOV [2], and CONTOGOURIS [3] have treated multiple exchange of the moving Pomeranchukon trajectory, while a number of authors [4-6] have discussed multiple scattering for a fixed spin exchange (*i.e.* $\alpha_p(t) \equiv 1$). In all these cases multiple scattering takes over at large momentum transfers and causes $d\sigma/dt$ to fall off less rapidly there, in agreement with the general features noted in nuclear physics by GLAUBER [7]. The new technical feature of our model is that it is the first calculation based on a Reggeized single exchange which makes full use of the Glauber formalism for generating multiple-exchange corrections [8].

The assumptions made in our model are as follows:

i) We consider only the helicity nonflip amplitude $A(s, t)$. This seems reasonable because nonflip scattering is the dominant elastic process at small t, and we shall find that even large-t processes are dominated by successive exchanges which individually involve $|t| \leqslant 0.5\ (\text{GeV})^2$.

ii) We assume a straight-line trajectory,

$$\alpha_p(t) = 1 + t\alpha' . \tag{1}$$

The coupling is assumed such that the single Pomeranchukon exchange contribution to A is

$$A_{\text{pole}}(s, t) = c\left(\frac{s}{s_0}\exp[-i\pi/2]\right)^{\alpha_p(t)} . \tag{2}$$

Concerning the omission of the signature zeros at $\alpha_p = -1, -3, \ldots$ in eq. (2), we note that these could only affect momentum transfers of $|t| \geqslant 2\ (\text{GeV})^2$ in a single exchange, which (as noted above) are not important for multiple scattering.

iii) We represent A by Glauber's formula [7]

$$A(s, t) = ik\sqrt{s}\int \frac{d^2b}{2\pi}\left(1 - \exp[2i\delta(b)]\right)\exp[i\boldsymbol{b}\cdot\boldsymbol{q}] , \tag{3}$$

where b is the impact parameter, δ the corresponding phase shift, $\boldsymbol{q}$ the three-momentum transfer ($t = -\boldsymbol{q}^2$), and k the centre-of-mass momentum. This formula is valid at high energies and small angles.

---

(3) A. P. CONTOGOURIS: *Phys. Lett.*, **23**, 698 (1966).

(4) L. VAN HOVE: *Nuovo Cimento*, **28**, 798 (1963); W. COTTINGHAM and R. F. PEIERLS: *Phys. Rev.*, **137**, B 147 (1965); I. V. ANDREEV and I. M. DREMIN: *The large-angle elastic scattering at high energies*, Lebedev Institute preprint (1968).

(5) T. T. CHOU and C. N. YANG: *Phys. Rev. Lett.*, **20**, 1213 (1968); L. DURAND III and R. LIPES: *Phys. Rev. Lett.*, **20**, 637 (1968).

(6) C. B. CHIU and J. FINKELSTEIN: *A hybrid model for elastic scattering*, CERN preprint, TH. 892 (1968).

(7) R. J. GLAUBER: *Lectures in Theoretical Physics*, ed. W. E. BRITTEN *et al.*, vol. **1** (New York, 1959), p. 315; *High-Energy Physics and Nuclear Structure* (Amsterdam, 1967), p. 311.

(8) AMATI *et al.* used unitarity, the Regge input being identified with the inelastic contributions to the unitarity relation (our formalism does not involve this particular identification). CONTOGOURIS used Glauber's eikonal approximation, but in a somewhat different way than we do. The first suggestion that Regge terms be identified with the Born approximation to Glauber's multiple-scattering theory was made by R. C. ARNOLD: *Phys. Rev.*, **140**, B 1022 (1965); **153**, 1523 (1967). The model of Chiu and Finkelstein uses this identification, but with $\alpha_p(t)$ fixed at 1.

iv) We identify single Pomeranchukon exchange with the « Born approximation » to Glauber's formula:

$$A_{\text{pole}}(s, -\boldsymbol{q}^2) = ik\sqrt{s}\int \frac{\mathrm{d}^2 b}{2\pi}\left(-2i\delta(b)\right)\exp\left[i\boldsymbol{b}\cdot\boldsymbol{q}\right]. \tag{4}$$

What is involved in this, our most novel assumption, is the usual identification of the phase in Glauber's formula with single scattering, and the assumption that single scattering is given by A_{pole}, or more generally by the sum over all Regge poles. It follows that multiple scattering is represented by Regge cuts, and in this connection we note that Mandelstam's famous diagram (9) for the first cut (Fig. 1) does have precisely the form of a double scattering, with particle A for example separating into components a and b which successively scatter off components of particle B.

Given these assumptions, the consequences of the model are easy to compute. Recognizing that eq. (4) has the form of a Fourier transform, one inverts the transform and finds

$$2i\delta(b) = \int \frac{\mathrm{d}^2 q}{2\pi}\left[\frac{A_{\text{pole}}(s, -\boldsymbol{q}^2)}{-ik\sqrt{s}}\right]\exp\left[-i\boldsymbol{b}\cdot\boldsymbol{q}\right], \tag{5}$$

which, for A_{pole} as given in eq. (2), is

$$2i\delta(b) = -\frac{\xi}{\mu}\exp\left[-b^2/4\alpha'\mu\right] \tag{6}$$

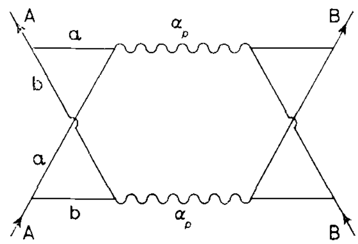

Fig. 1. – Diagram which, according to MANDELSTAM (9), gives a Regge cut. Straight lines represent particles, wavy lines represent the Pomeranchukon trajectory.

with $\xi = -c\sqrt{s}/2k\alpha' s_0$ and $\mu = \ln(s/s_0) - i\pi/2$. The strength parameter ξ is positive (from eq. (2) and the optical theorem, one sees that c is negative) and constant ($\sqrt{s}/k \to$ const at high energies). Inserting (6) into (3), expanding $e^{2i\delta}$ in a power series, and performing the resulting simple integrations over Gaussians, we find

$$A = 2ik\sqrt{s}\,\alpha'\,\xi\sum_{n=1}^{\infty}\frac{1}{n\,n!}(-\xi/\mu)^{n-1}\exp\left[\frac{t\alpha'\mu}{n}\right]. \tag{7}$$

Equation (7) expresses the amplitude in terms of three parameters (s_0, α', and the strength parameter ξ). We have evaluated the sum by computer, keeping s_0 at the conventional value 1 (GeV)^2, and varying ξ and α' to fit $\operatorname{Im} A(t=0) = (k\sqrt{s}/4\pi)\sigma_{\text{pp}}^{\text{tot}}$ and the general trend of the t-dependence of $\mathrm{d}\sigma_{\text{pp}}/\mathrm{d}t$. The best fit is obtained with $\xi = 7$ and $\alpha' = 0.82\,(\text{GeV})^{-2}$; it is compared with pp data (10-14) in Fig. 2a). Only data at

(9) S. MANDELSTAM: *Nuovo Cimento*, **30**, 1148 (1963).

(10) A. N. DIDDENS, E. LILLETHUN, G. MANNING, A. E. TAYLOR, T. G. WALKER and A. M. WETHERELL: *Phys. Rev. Lett.*, **9**, 111 (1962).

(11) D. HARTING, P. BLACKALL, B. ELSNER, A. C. HELMHOLZ, W. C. MIDDELKOOP, B. POWELL, B. ZACHAROV, P. ZANELLA, P. DALPIAZ, M. N. FOCACCI, S. FOCARDI, G. GIACOMELLI, L. MONARI, J. A. BEANEY, R. A. DONALD, P. MASON, L. W. JONES and D. O. CALDWELL: *Nuovo Cimento*, **38**, 60 (1965).

(12) G. COCCONI, V. T. COCCONI, A. D. KRISCH, J. OREAR, R. RUBINSTEIN, D. B. SCARL, B. T. ULRICH, W. F. BAKER, E. W. JENKINS and A. L. READ: *Phys. Rev.*, **138**, B 165 (1965).

(13) K. J. FOLEY, R. S. GILMORE, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, E. H. WILLEN, R. YAMADA and L. C. L. YUAN: *Phys. Rev. Lett.*, **15**, 45 (1965).

(14) J. V. ALLABY, A. N. DIDDENS, A. KLOVNING, E. LILLETHUN, E. J. SACHARIDIS, K. SCHLUPMANN and A. M. WETHERELL: *Phys. Lett.*, **27**, B 49 (1968).

$\theta_{c.m.} \leqslant 45°$ have been included in Fig. 2*a*) since the Glauber formalism does not apply to large angles. In general, the best fit presented in Fig. 2*a*) has about the right exponential fall-off at large t, « shrinks » suitably as s increases, and has a « break » correctly placed near $t = -1.2\,(\text{GeV})^2$. Of course secondary trajectories should also be added in the usual way; they are needed to reduce the bulge near $t = -1.2\ \text{GeV}/c^2$, and to make $\text{Re}\,A(0)/\text{Im}\,A(0)$ and the energy dependence of σ^{tot} come out right as discussed below. In the present paper, however, we limit ourselves to the simple Pomeranchukon model, which does represent the gross features of the data in terms of only three parameters, including *normal* α' and s_0.

Since the results of numerical calculations are by themselves rather untransparent, it may prove helpful to discuss briefly some analytical features of the model. The n-th term in eq. (7), which is identified physically with n-th-order scattering (n Pomeranchukon exchanges) since it arises from $(2i\delta)^n$, can be written

$$(8)\qquad A_n = \frac{\text{const}}{\mu^{n-1}}\left(\frac{s}{s_0}\exp\left[-i\pi/2\right]\right)^{1+t\alpha'/n}.$$

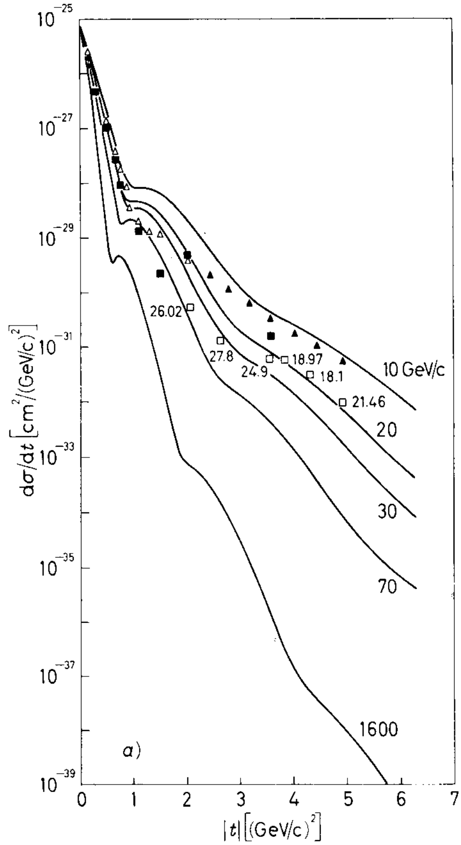

Fig. 2*a*. – Proton-proton scattering, calculated using formula (8). Experimental data from refs. [10-14]: ▲ 12.1 GeV/c, △ 12.4 GeV/c, ■ 18.4 GeV/c, $\alpha' = 0.82\,(\text{GeV/c})^{-2}$, $\xi = 7$, $s_0 = 1\,(\text{GeV})^2$.

For $n \geqslant 2$ this expression represents a « Regge cut » wich extends up to $\alpha_{\max}(t) = 1 + t\alpha'/n$ and has a discontinuity of the form $(\alpha - \alpha_{\max})^{n-2}$; we can see this by writing the contour integral around the cut as

$$(9)\qquad \int\limits_{-\infty}^{\alpha_{\max}} d\alpha(\alpha - \alpha_{\max})^{n-2}\left(\frac{s}{s_0}\exp\left[-i\pi/2\right]\right)^{\alpha} =$$

$$= \int\limits_{-\infty}^{\alpha_{\max}} d\alpha(\alpha - \alpha_{\max})^{n-2}\exp\left[\alpha\mu\right] = \frac{(n-2)!}{\mu^{n-1}}\left(\frac{s}{s_0}\exp\left[-i\frac{\pi}{2}\right]\right)^{\alpha_{\max}}.$$

Thus the mechanism by which multiple scattering takes over at large t or large s is the one expected [9,15] in Regge theory, that cuts extending up to $\alpha_{\max} = 1 + t\alpha'/n$ progressively dominate the pole at $\alpha = 1 + t\alpha'$. While it has been possible to ignore

[15] D. AMATI, S. FUBINI and A. STANGHELLINI: *Nuovo Cimento*, **26**, 896 (1962).

cut terms at small t as a first approximation (though even at $t=0$, they give corrections of order $\frac{1}{3}$), this approximation becomes completely untenable at large t for trajectories, such as the linear one we have assumed, which fall well below the branch points of the first few cuts. Numerically, we see from eq. (7) that for typical values such as $\ln(s/s_0)=4$ ($E_{\text{lab}}\approx 30$ GeV), $\xi=7$, and $\alpha'=0.82\ (\text{GeV}/c)^{-2}$, the transition from single to double exchange dominance occurs at $t\approx -0.6\ \text{GeV}/c^2$, from double to triple at $t\approx -1.8\ \text{GeV}/c^2$, and so forth. This is the basis for our statement that each individual scattering involves a relatively small momentum transfer (roughly $\sqrt{-t}\approx\frac{1}{2}$ GeV/c for typical cases).

At large t, the dominant exchange has an n of order

$$\bar{n}\approx\sqrt{\frac{-t\alpha'\ln(s/s_0)}{\ln|\bar{n}\mu/\xi|}}\,. \tag{10}$$

When $\bar{n}$ is large, one can approximate the sum over n in (7) by an integral over n. The integrand is strongly peaked at $n=\bar{n}$, and the dominant features of the integral can be estimated from the behaviour at this point ([16,17]). We find to a rough approximation that the magnitude of A is given by

$$|A|\propto\frac{s}{s_0}\exp\left[-2\sqrt{t\alpha'\ln\frac{s}{s_0}\ln|\xi/\bar{n}\mu|}\right]. \tag{11}$$

Thus the multiple-scattering corrections convert the Gaussian momentum transfer dependence $A_{\text{pole}}\propto\exp[-a(\sqrt{-t})^2]$, characteristic of small t, into the exponential dependence $A\propto\exp[-b\sqrt{-t}]$ characteristic of large-t data ([2,16]). Or, to put it another way, they convert an input which falls off faster than the Cerulus-Martin bound ([18]) into an output lying just within the bound. Concerning the s-dependence, by rewriting eq. (11) in the form

$$|A|\propto\left(\frac{s}{s_0}\right)^{1-2[t\alpha'\ln^{-1}(s/s_0)\ln|\xi/\bar{n}\mu|]^{\frac{1}{2}}}, \tag{12}$$

we see that at large t one can speak of an «effective α»

$$\alpha_{\text{eff}}=1-2[t\alpha'\ln^{-1}(s/s_0)\ln|\xi/\bar{n}\mu|]^{\frac{1}{2}}, \tag{13}$$

which, for example, reaches $\alpha_{\text{eff}}\approx -1.6$ at $t=-8\ (\text{GeV})^2$ for 30 GeV pp scattering with the parameters cited above.

Although our model is qualitatively successful in representing the pp data, it does not follow that α'_p is necessarily large, since good fits exist in which α'_p small or zero and secondary trajectories are responsible for the observed shrinking ([19]). However, models

---

([16]) G. COCCONI: *Interpretation of the transverse-momentum distribution of particles in high-energy hadron collisions*, CERN Internal Report NP 68-17 (1968) (to be published in *Nuovo Cimento*).

([17]) S. FRAUTSCHI, O. KOFOED-HANSEN and B. MARGOLIS: to be published.

([18]) F. CERULUS and A. MARTIN: *Phys. Lett.*, **8**, 80 (1964); T. KINOSHITA: *Phys. Rev. Lett.*, **12**, 256 (1964); A. MARTIN: *Nuovo Cimento*, **37**, 671 (1965).

([19]) W. RARITA, R. J. RIDDELL, C. B. CHIU and R. J. N. PHILLIPS: *Phys. Rev.*, **165**, 1615 (1968).

with $\alpha'_p \approx 1\ (\mathrm{GeV}/c)^{-2}$ and $\alpha'_p = 0$ do differ distinctively in the energy range available to the next generation of accelerators and colliding beams.

i) In models with a fixed spin-one exchange, an asymptotic plateau is eventually reached above which the fixed spin exchange dominates and $d\sigma/dt$ is essentially energy-independent. CHIU and FINKELSTEIN [6], and (with different assumptions) ABARBANEL, DRELL and GILMAN [20], have estimated the rate at which this condition is approached, and find that asymptotics should be reached at laboratory energies of the order of $(100 \div 200)$ GeV. If $\alpha'_p \approx 1\ (\mathrm{GeV}/c)^{-2}$ on the other hand, $d\sigma/dt$ keeps falling, as indicated by the predictions in Fig. 2a), for 70 GeV (Serpukhov energy) and 1600 GeV (CERN colliding-beam equivalent laboratory energy).

ii) For $\alpha' = 0$, σ^{tot} approaches a constant asymptotically, whereas in our model it can be deduced by combining the optical theorem $\sigma^{\mathrm{tot}} = (4\pi/k\sqrt{s})\,\mathrm{Im}\,A(0)$ with eq. (7):

$$\sigma^{\mathrm{tot}} = 8\pi\alpha'\,\xi \sum_{n=1}^{\infty} \frac{1}{n\,n!}\,\mathrm{Re}\,(-\,\xi/\mu)^{n-1}\,. \tag{14}$$

To get an idea of what happens, consider the first two terms:

$$\sigma^{\mathrm{tot}} = \mathrm{const}\left[1 - \frac{\xi \ln(s/s_0)}{4[\ln^2(s/s_0) + \pi^2/4]} + O(\xi^2)\right]. \tag{15}$$

We see that σ^{tot} *rises* logarithmically as s increases. At present energies this rise is masked by secondary trajectories, but since their contribution decreases like $s^{-\frac{1}{2}}$ the logarithmic rise should eventually take over. The physical mechanism operating here is well known [21]: the second-order term contributes a Glauber shadow, and in the Regge-cut theory the shadowing decreases like $\ln s$. The amount σ^{tot} rises will depend on the secondary trajectories, which we have omitted, but to obtain a first approximation we insert the parameters for our best pp fit into eq. (14) and obtain a rise from $\sigma^{\mathrm{tot}}_{\mathrm{pp}} = 40$ mb at 30 GeV to ≈ 55 mb in the asymptotic limit. A crude estimate of the probable effect of secondary trajectories reduces this asymptotic value to ≈ 50 mb.

iii) The higher-order terms in eq. (7) also contribute a real part to $A(0)$ as a result of the phase variation introduced by nonzero α'. For example, through second order one has

$$\mathrm{Re}\,A(0) = \frac{\pi k\sqrt{s}\,\alpha'\,\xi^2}{4\ln^2(s/s_0) + \pi^2} + O(\xi^2)\,. \tag{16}$$

This real part is falling (relative to $\mathrm{Im}\,A(0)$) like $\ln^2(s/s_0)$ and has $\mathrm{Re}\,A(0)/\mathrm{Im}\,A(0)$ positive [6] and of order 0.1 at 30 GeV. In order to fit the present data, which show $\mathrm{Re}\,A(0)/\mathrm{Im}\,A(0)$ negative, one must again call upon secondary trajectories. These secondary contributions, however, fall off like $s^{-\frac{1}{2}}$, so at higher energies $\mathrm{Re}\,A(0)/\mathrm{Im}\,A(0)$ is expected to change sign.

---

[20] H. ABARBANEL, S. DRELL and F. GILMAN: *Phys. Rev. Lett.*, **20**, 280 (1968).
[21] B. M. UDGAONKAR and M. GELL-MANN: *Phys. Rev. Lett.*, **8**, 346 (1962).

We have also worked out the consequences of our model for $\bar{\text{p}}\text{p}$ ([13]) and πp ([22,23]) scattering (Fig. 2*b*)). Here α' should be kept the same. We have again used the same s_0 for simplicity, while varying ξ to fit the magnitude of σ^{tot} in each case (for $\bar{\text{p}}\text{p}$ this represents a change in the model, the large $\xi_{\bar{\text{p}}\text{p}}$ now referring to the combined coupling of all trajectories—taken to have common s_0 and α'—at the energy considered in Fig. 2*b*)). The model produces the correlation between large σ^{tot} and narrow peak width expected on elementary grounds ($\theta \sim \hbar/pR \sim \hbar/p\sqrt{\sigma/\pi}$). The mechanism is that larger σ^{tot} implies greater coupling strength ξ, and this increases the double-scattering correction, which is acting to reduce the peak width. More precisely, if we write $A = a(1 - b|t| + O(t^2))$ at small t, the first two terms in eq. (7) give

$$b = \mu\alpha'[1 + \xi/8\mu + O(\xi^2)]\,, \tag{17}$$

which increases with ξ. In a similar way ([6,24]), when secondary trajectories are introduced which make $\sigma^{\text{tot}}_{\text{pp}}$ differ from $\sigma^{\text{tot}}_{\bar{\text{p}}\text{p}}$, etc., Regge-cut corrections can give the « cross-over phenomenon » between the $d\sigma/dt$ of pp and $\bar{\text{p}}\text{p}$, $\pi^+\text{p}$ and $\pi^-\text{p}$, and K^+p and K^-p without hypothesizing zeros in the couplings of the secondary trajectories. Secondary trajectories *are* needed to fit the observed decrease of σ^{tot}, the negative sign of $\text{Re}\,A(0)/\text{Im}\,A(0)$, and the striking πp dip at $t \simeq -3\,\text{GeV/c}^2$ which has been seen up through $p_{\text{lab}} = 8\,\text{GeV/c}$ at least ([23]). In connection with this dip, we recall that in addition to breaks or dips associated with transitions from n-exchange dominance to $(n+1)$-exchange dominance ([5-7]), other dips associated with definite integer or half-integer α are possible ([25,26]). The first kind persists at high energies (though shifting position slowly if α_p varies with t) and is not necessarily associated with any particular value of α; the second kind can shine forth only at relatively low t (or u) where single exchange is important, and will disappear as the energy rises (disappearing rapidly if another pole lies higher, slowly if only cuts lie higher). It remains to be seen if the $t = -3\,(\text{GeV})^2$ dip can be explained in this way ([26]).

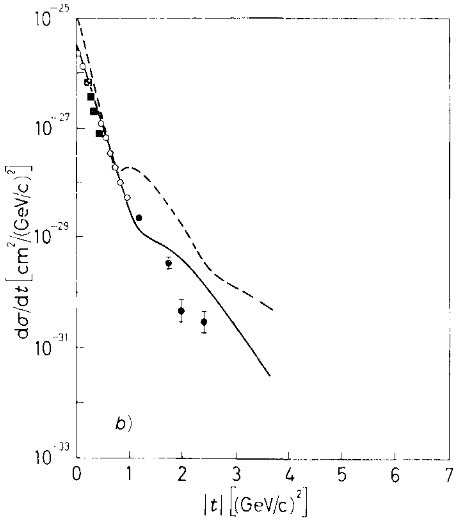

Fig. 2*b*. – πp and $\bar{\text{p}}\text{p}$ elastic scattering, calculated using formula (8). Experimental data from ref. ([13,22,23]); – – – $\bar{\text{p}}\text{p}$, $\xi = 10$, $p_{\text{lab}} = 16\,\text{GeV/c}$, $\alpha' = 0.82\,(\text{GeV/c})^{-2}$, $s_0 = 1\,(\text{GeV})^2$; —— $\pi^+\text{p}$, $\xi = 4$, $p_{\text{lab}} = 12.5\,\text{GeV/c}$; ○ $\pi^+\text{p}$, 18 GeV/c; ● $\pi^+\text{p}$, 12 GeV/c; ■ $\bar{\text{p}}\text{p}$, 15.91 GeV/c.

We conclude with several comments and points for further study.

([22]) K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell and L. C. L. Yuan: *Phys. Rev. Lett.*, **11**, 425 (1963).

([23]) J. Orear, R. Rubinstein, D. B. Scarl, D. B. White, A. D. Krisch, W. R. Frisken, A. L. Read and H. Ruderman: *Phys. Rev.*, **152**, 1162 (1966).

([24]) V. Barger and L. Durand III: *Phys. Rev. Lett.*, **19**, 1295 (1967).

([25]) G. Höhler, J. Baacke, H. Schlaile and P. Sonderegger: *Phys. Lett.*, **20**, 79 (1966); F. Arbab and C. Chiu: *Phys. Rev.*, **147**, 1045 (1966); S. Frautschi: *Phys. Rev. Lett.*, **17**, 722 (1966).

([26]) V. Barger and R. J. N. Phillips: *Phys. Rev. Lett.*, **20**, 564 (1968).

i) In the Glauber tratment of elastic scattering, taking $A(t)$ pure imaginary produces deep dips at the transitions from single-scattering to double-scattering dominance, etc., due to interference between the two terms. The dips may be filled in by adding a phase (*i.e.* by giving A a real part which does not vanish at the same points as $\operatorname{Im} A$), or by adding helicity-flip amplitudes. In our model the signature factor automatically introduces a t-dependent phase, and this is the mechanism which converts the dips into mere breaks. But when double scattering takes over at sufficiently small t, as happens at very high energy or when ξ is especially large, the phase has less chance to develop between $t = 0$ and t_{break}, and the dip is less completely filled in. Thus dips can be seen in the 1600 GeV pp curve (Fig. 2*a*)), and in the $\bar{\text{p}}$p curve (Fig. 2*b*)).

ii) Historically, the original assignment of a large slope to α_p failed when it was discovered that although the pp peak « shrank », the πp peak was essentially energy-independent and the $\bar{\text{p}}$p peak actually expanded. Secondary trajectories could account for these differences, but only if α_p' was small [19]. In our multiple-scattering mode, the shrinking is numerically about the same at small t as in a single-pole model with the same (large) α_p', so we must explain why we believe the secondary trajectories will have a greater capacity to counteract the shrinking this time. One reason is that the simple inverse connection between σ^{tot} and peak width, mentioned before eq. (17), together with the appreciable rise of σ^{tot} from 10 to 30 GeV in our model, would already lead one to expect shrinking. When the secondary trajectories are added to make a flat $\sigma^{\text{tot}}_{\text{pp}}$, and falling $\sigma^{\text{tot}}_{\bar{\text{p}}\text{p}}$ and $\sigma^{\text{tot}}_{\pi\text{p}}$, this physical source of shrinking will be removed, or even reversed to expansion in the case of $\bar{\text{p}}$p and πp, and crude estimates indicate that the physically observed behaviour will not be so difficult to obtain. Another way to put it is that the secondary trajectories must be more strongly coupled in our model since they must make up the difference between falling $\bar{\text{p}}$p and πp total cross-sections and *rising* multiple Pomeranchukon-exchange contributions.

iii) Empirically some reactions, such as $\pi\mathcal{N}$ charge exchange, can be described fairly well over the range $0 \leqslant |t| \leqslant 1\ (\text{GeV})^2$ with a single-pole exchange. In these cases multiple exchange again produces cuts extending to the right of the poles by the usual amount, so we have to explain why multiple exchange is more important for the elastic peak than for these cases. Fortunately several groups [27-29] have studied the example of the ρ-exchange contribution to the helicity-flip $\pi^-\text{p} \to \pi^0\text{n}$ amplitude, and all agree that the double-scattering correction associated with (ρ+Pomeranchukon) exchange is indeed quite small near $t = 0$. The reason [28,29] is the change of sign in the ρ coupling at $t = -0.6\ (\text{GeV})^2$, which produces cancellations in the double-scattering correction. Chiu and Finkelstein [28] have found a similar situation in backward $\pi^+\text{p} \to \text{p}\pi^+$ due to the zero in nucleon-trajectory exchange at $u = -0.2\ (\text{GeV})^2$. Thus it appears that the relative success of simple pole models at small momentum transfer in certain cases can be understood.

iv) Recently there has been considerable discussion about conspiracies at $t = 0$, and about factorization of Regge-pole couplings at $t = 0$, at the « cross-over point », at points where $\alpha = 0$, etc. In general, Regge cuts do not factor and do not have a definite conspiring or evasive behaviour (*i.e.* a definite Toller quantum number M). Thus inclusion of Regge cuts reopens all these subjects; for example absorption models

---

[27] P. G. O. Freund and P. J. O'Donovan: *Phys. Rev. Lett.*, **20**, 1329 (1968).
[28] C. Chiu and J. Finkelstein: private communication.
[29] G. Cohen-Tannoudji: private communication on the work of the Saclay group.

of np charge-exchange which include second-order scattering easily turn the forward dip associated with evasive π exchange into a narrow forward peak ([30]). The interesting question then arises: do *all* Regge poles have the lowest possible Toller quantum numbers $M=0$ (bosons) and $M=\frac{1}{2}$ (fermions), with the higher M contributions needed to fit experiment coming exclusively from cuts, or does M merely take on low values for the *top few* trajectories, as is the case with various other quantum numbers. To study this phenomenologically one can assume definite quantum numbers for the poles, calculate the multiple-scattering corrections (which will automatically include some conspiring terms corresponding to $M \geqslant 1$), and see if the amount of conspiracy thus introduced is sufficient to explain the data consistently.

v) If $\alpha'_p \approx 0.8\,(\text{GeV})^{-2}$, one expects a 2^+ particle with mass ≈ 1100 MeV. There are several possible candidates, such as the $f^0(1250\,\text{MeV})$ and the recently discovered 1085 MeV resonance ([31]), whose possible connection to the Pomeranchukon has been discussed by JOHNSON ([32]). Note that although the usual analysis ([33]) gives strong reasons for associating the f^0 with P' rather than the Pomeranchukon, the whole question of the P' trajectory is reopened when cuts are present.

* * *

We have profited by discussions with Drs. L. BERTOCCHI, C. B. CHIU, G. COCCONI, A. N. DIDDENS and J. FINKELSTEIN. We are grateful for computational assistance given by Mr. K. S. KÖLBIG of the CERN Applied Mathematics Group. Finally, we wish to thank Profs. L. VAN HOVE and J. PRENTKI for the hospitality extended to us in the Theoretical Study Division at CERN.

([30]) E. M. HENLEY and I. J. MUZINICH: *Phys. Rev.*, **136**, B 1783 (1964); G. A. RINGLAND and R. J. N. PHILLIPS: *Phys. Lett.*, **12**, 62 (1964).

([31]) C. WHITEHEAD, J. G. MCEWAN, R. J. OTT, D. K. AITKEN, G. BENNET and R. E. JENNINGS: *Nuovo Cimento*, **53** A, 817 (1968).

([32]) R. C. JOHNSON: *Nucl. Phys.*, **5**, B 673 (1968).

([33]) B. R. DESAI and P. G. O. FREUND: *Phys. Rev. Lett.*, **16**, 622(1966).