IL NUOVO CIMENTO VOL. LVI A, N. 4 21 Agosto 1698

Connection between Phenomenological Fits for High-Energy Data (*).

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(ricevuto il 26 Giugno 1968)

Arguments have been given by KRISCH (1) in favour of a phenomenological formula fitting high-energy elastic data over the whole angular interval. This formula is a superposition of three Gaussians in $\beta k_\perp$ (k, θ c.m. variables, β = velocity of the c.m. protons). The various slopes in this formula are connected to the sizes of inner and inner domains of interaction. Whereas the gross features in the behaviour of high-energy data are well reproduced (we will ignore the fine-structure effects recently found in p-p (2)), this formula has some shortcomings: *A*) on the phenomenological side, the data for the largest value of $k_\perp$ show a tendency to deviate from Krisch's formula exhibiting a less pronounced fall-off; *B*) from the theoretical point of view it is not clear why there should be only three domains of interaction whereas one would rather expect an infinity of them as we get closer to the center of the target proton or, equivalently, as the energy of the incoming proton increases; *C*) the more serious theoretical drawback is that Krisch's formula violates the general bound of Cerulus and Martin (3) valid under very weak requirements of analyticity and boundedness.

This last complication was absent in the phenomenological formula previously proposed by OREAR (4).

In this paper we will show that if we assume that the three-Gaussian formula of ref. (1) is replaced by an infinite series of Gaussians (in which case we assume an infinity of domains of interactions thus accounting for point *B*) above), such that the first few terms reproduce Krisch's formula, automatically this accounts also for points *A*) and *C*).

This leads us to a four-parameter integral formula that we propose as a phenom-

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(*) Supported in part by USAF-EOAR Grant 68-0015.

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(1) See A. D. KRISCH: *Phys. Rev. Lett.*, **19**, 1149 (1967), where references to previous papers can be found. In this paper the data analysed refer to $d\sigma^\dagger/dt$ for p-p elastic scattering, $d\sigma^\dagger/dt$ being the p-p differential cross-section from which the effects due to particle identity have been removed. In the present paper we shall refer to $d\sigma^\dagger/dt$ although the same arguments could be repeated for the usual $d\sigma/dt$.

(2) See for instance ALLABY *et al.*: Cern Report 68-7 and the references quoted there.

(3) F. CERULUS and A. MARTIN: *Phys. Lett.*, **8**, 80 (1964).

(4) J. OREAR: *Phys. Rev. Lett.*, **12**, 112 (1964); *Phys. Lett.*, **13**, 190 (1964).

enological tool to fit high-energy data (fine-structure effects (2) excluded). The dominant contribution to this formula as $k_\perp \to \infty$ exhibits Orear's behaviour (4) thus restoring nonviolation of the Martin-Cerulus bound (3). Amazingly enough, when the actual numbers are used, this asymptotic behaviour (eq. (10)) has a slope very close to Orear's one.

According to what said above we assume

$$X = \frac{\mathrm{d}\sigma^\dagger}{\mathrm{d}t}\bigg/\left(\frac{\mathrm{d}\sigma^\dagger}{\mathrm{d}t}\right)_{t=0} = C\sum_{n=0}^{\infty}\frac{a^{-n}}{(cn+1)^\nu}\exp\left[-\frac{bx^2}{cn+1}\right], \qquad c>0,\ a>1,\ x\equiv\beta k_\perp, \tag{1}$$

where C is a normalization coefficient

$$C^{-1} = \sum_{n=0}^{\infty}\frac{a^{-n}}{(cn+1)^\nu} \tag{2}$$

and a, b, c, ν are four real parameters to be adjusted (b being essentially the slope of the diffraction peak).

In a forthcoming paper (5) we shall discuss how to sum and how to evaluate the asymptotic behaviour as $x\to\infty$ of the general series in (1). Here we limit ourselves to notice that choosing (6)

$$\nu = \tfrac{7}{2}, \qquad a = 5, \qquad c = 2, \qquad b = 10\ (\mathrm{GeV}/c)^{-2}, \tag{3}$$

we get an excellent fit over the whole angular interval to all high-energy data as seen in Fig. 1 (where the experimental points are as in ref. (1)).

With the above value $\nu = \frac{7}{2}$ we get

$$X = -\frac{C}{b^3}\frac{\mathrm{d}^3}{\mathrm{d}(x^2)^3}\sum_{n=0}^{\infty}\frac{a^{-n}}{\sqrt{cn+1}}\cdot\exp\left[-\frac{bx^2}{cn+1}\right]. \tag{4}$$

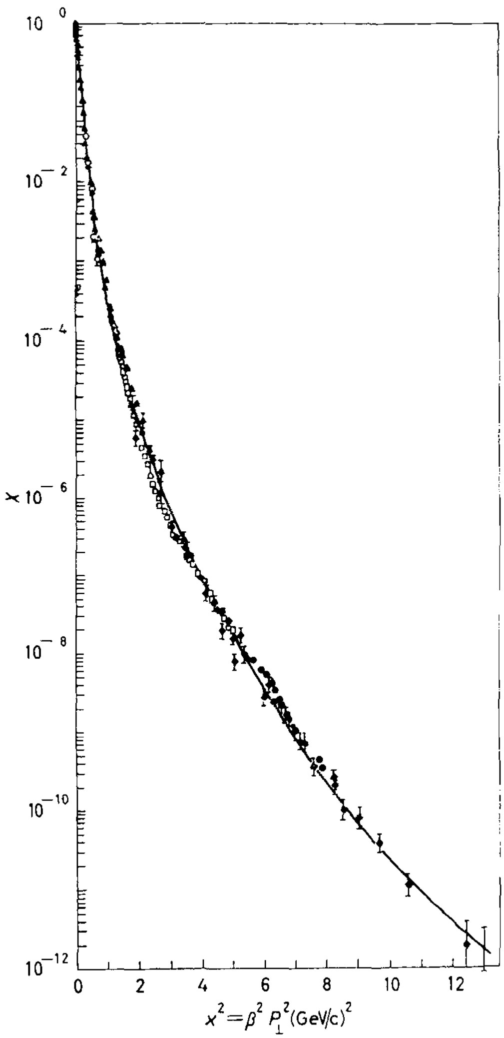

Fig. 1. – Plot of $X = (\mathrm{d}\sigma^\dagger/\mathrm{d}t)/(\mathrm{d}\sigma^\dagger/\mathrm{d}t)_{t=0}$ as given by eq. (4) *vs.* x^2 for p-p elastic data taken as in ref. (1).

(5) H. FLEMING, A. GIOVANNINI and E. PREDAZZI: to be published.

(6) It should be stressed that the values given in eq. (3) for the parameters are not obtained from a best fit of the experimental points but from a simple trial procedure, and by asking that the first few terms reproduce Krisch's formula.

Using now

$$\exp\left[-\frac{x^2}{4\alpha}\right] = 2\sqrt{\frac{\alpha}{\pi}}\int_0^\infty dy \cos xy \exp[-\alpha y^2]\,, \tag{5}$$

the series in eq. (4) can be summed and we obtain

$$X = -\frac{C}{b^3\sqrt{\pi b}}\frac{d^3}{d(x^2)^3}\int_0^\infty dy \cos xy \left[\exp[y^2/4b] - \frac{1}{a}\exp\left[-\frac{y^2(c-1)}{4b}\right]\right]^{-1}. \tag{6}$$

Upon derivation with respect to x^2, from eq. (6) we get a simple three-parameter formula which could be used for numerical fits to high-energy data due to the very rapid convergence of its integrand.

Owing to the parity of the integrand in eq. (6) we can also evaluate X by using the Cauchy theorem which allows us to obtain an Orear-like series for X. The poles of the integrand are located at the intercepts of the parametric curves

$$\begin{cases} -u^2 + v^2 = \dfrac{4b}{c}\ln a\,, \\[2ex] uv = \dfrac{4b}{c}m\pi\,, \end{cases} \qquad y = u + iv,\; m = 0, \pm 1, \pm 2, \ldots, \tag{7}$$

and we then get

$$X = -\frac{C}{b^3}a^{1/c}\frac{d^3}{d(x^2)^3}\left\{\sqrt{\frac{\pi}{c\ln a}}\exp\left[-2\sqrt{\frac{b\ln a}{c}}\,x\right] + \sqrt{\frac{\pi}{b}}\sum_{m=1}^\infty \frac{\exp[-F_+(m)x]}{[\ln^2 a + 4m^2\pi^2]^{\frac{1}{2}}}\cdot\right.$$
$$\left.\cdot\left[F_+(m)\cos\left(F_-(m)x - 2m\frac{\pi}{c}\right) - F_-(m)\sin\left(F_-(m)x - 2m\frac{\pi}{c}\right)\right]\right\}, \tag{8}$$

where

$$F_\pm(m) = \left\{\frac{2b}{c}\left[\sqrt{\ln^2 a + 4m^2\pi^2} \pm \ln a\right]\right\}^{\frac{1}{2}}. \tag{9}$$

It is clear from eqs. (8), (9) that the form (8) is an ideal tool to evaluate X as $x \to \infty$ since the series converges very rapidly with increasing m, contrary to what we had in eq. (1). We then get (with $c = 2$)

$$X \underset{x\to\infty}{\simeq} C\frac{\sqrt{\pi b a}}{8b^3}\exp\left[-x\sqrt{2b\ln a}\right]\left\{\frac{2b\ln a}{x^3} + 3\frac{\sqrt{2b\ln a}}{x^4} + \frac{3}{x^5}\right\}. \tag{10}$$

With the values (eq. (3)) $b = 10$, $a = 5$, from eq. (10) we find that the leading term as $x \to \infty$ is dominated by $\exp[-\beta k_\perp/0.173]$ to be compared with $\exp[-k_\perp/0.158]$ in Orear's fit. Therefore the two agree within 10%. Moreover at $x^2 = 12$ the value obtained from the only term in eq. (10) practically coincides with the one obtained summing up the first eight terms in (1), whereas at $x^2 = 4$ the value obtained from (10) is off by 20%, and is therefore already a good approximation.

Summarizing the results (valid in the particular case $\nu = \frac{7}{2}$ in eq. (1)), eqs. (1), (6) and (8) are different expressions for the same quantity. Equation (1) is a Krisch-like form and few terms provide a good approximation for small x thus establishing a connection with diffraction, whereas eq. (8) is particularly useful for large x showing that the Cerulus-Martin [3] bound is not violated and therefore restoring nonviolation of anayticity and boundedness. Also, eq. (1) shows that the contributions coming from inner and inner domains of interactions are *essential* to avoid violation of the Cerulus-Martin bound. At the same time the agreement with experiments is excellent (Fig. 1) and moreover the procedure may shed some light on the long debated question of whether or not small and large angles are to be attributed to inherently different mechanisms or (as is implicit in the present approach) they can be reconciled with one another [7].

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One of us (H.F.) acknowledges a fellowship from the Brazilian Conselho Nacional de Pesquisas.

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[7] After this work was written, we received a paper by C. COCCONI (CERN, NP Int. Report 68-17) in which, starting from very different motivations, similar results are obtained.