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On the Effect of n-p Tensor Forces in $^3H_\Lambda$ - II.

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Summary. — A study is made of the effect of the tensor part of the neutron-proton interaction in hypertriton. It is found that the corrections obtained for the volume integral of the Λ-nucleon interaction result in appreciable variation of the parameters of the Λ-nucleon potential as derived from studies of light hypernuclei.

1. – Introduction.

Among the known hypernuclei the lightest one $^3H_\Lambda$ has a particular position, since the low number of particles allows detailed variational analyses with two-body central interparticle forces (1,2) or even with hard-core (3) or three-body forces (4). A small noncharge-symmetric term in the Λ-$\mathcal{N}$ interaction has no practical effect in $^3H_\Lambda$. Such a term seems to be required by some experimental data (5); it is also predictable theoretically (6) with a spin-dependence of opposite sign for a proton and a neutron. Since in hypertriton the n-p pair has spin 1, the first perturbative correction to the energy due to this term is zero; even a more accurate variational calculation shows that it is not important (7).

(1) R. H. Dalitz and B. W. Downs: *Phys. Rev.*, **110**, 958 (1958).
(2) R. H. Dalitz and B. W. Downs: *Phys. Rev.*, **114**, 593 (1959).
(3) B. W. Downs, D. Smith and T. Truong: *Phys. Rev.*, **129**, 2730 (1963).
(4) J. D. Chalk III and B. W. Downs: *Phys. Rev.*, **132**, 2727 (1963).
(5) M. Raymund: *Nuovo Cimento*, **32**, 555 (1964).
(6) R. H. Dalitz and B. W. Downs: *Phys. Rev.*, **111**, 967 (1958).
(7) S. Rosati: unpublished.

In a study of $^3H_\Lambda$ it may be of interest to take into account also the tensorial part of the neutron-proton interaction. A previous work ([8]) has investigated the role of such a force, but any definite conclusion was excluded due to the particularly simple trial wave functions used. A re-examination of the problem is made with more flexible trial functions containing a larger number of variational parameters.

2. – n-p potential and wave functions.

We follow the formulation of the problem as given in I and we limit ourselves to pointing out the essential differences with respect to I.

We assume the following neutron-proton potential:

$$V_{n,p}(\boldsymbol{r}) = -(W_1 + W_2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_p) \frac{\exp[-kr]}{r} - W_3 \frac{\exp[-\eta r]}{r} S_{n,p}(\boldsymbol{r}) , \tag{1}$$

where $\boldsymbol{\sigma}$ is the vector Pauli matrix and

$$S_{n,p}(\boldsymbol{r}) = \frac{3}{r^2} (\boldsymbol{\sigma}_n \cdot \boldsymbol{r})(\boldsymbol{\sigma}_p \cdot \boldsymbol{r}) - \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_p .$$

FEHSBACH and SCHWINGER ([9]) have shown that with this n-p potential it is possible to fit the properties of the deuteron and the low-energy n-p scattering data. Here we consider six sets of parameters (see Table I) selected from ref. ([9]); five sets are the ones just considered in I (*); the new one, the sixth, was shown by PEASE and FESHBACH ([10]) to also yield a correct value for the binding energy of the triton.

In the main, our calculations have been performed with the potential of type *b*) of Table I which practically coincides in our case with the more general Hall and Powell's one ([11]). The other five cases are considered to see how the results may depend on the actual choice.

The trial wave function of the hypertriton is chosen to have the form

$$\psi_{^3H_\Lambda} \propto f(r, s, t)\varphi_{S_2} + x g(r, s, t)\varphi_{D_1} , \tag{2}$$

([8]) B. BARSELLA and S. ROSATI: *Nuovo Cimento*, **20**, 914 (1961) (this work will be indicated hereafter by I).

([9]) H. FESHBACH and J. SCHWINGER: *Phys. Rev.*, **84**, 194 (1951).

(*) The values given for W_3 in Table I of I must be multiplied by a factor 3.

([10]) R. L. PEASE and H. FESHBACH: *Phys. Rev.*, **88**, 945 (1952).

([11]) H. H. HALL and J. L. POWELL: *Phys. Rev.*, **90**, 912 (1953).

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TABLE I. – *Neutron-proton potential parameters.*

Neutron-proton potential parameters	W_1+W_2 (MeV·fermi)	W_3 (MeV·fermi)	k (fermi^{-1})	η (fermi^{-1})	$\lvert x \rvert$
a)	34.46	62.97	0.7402	0.7236	0.205
b)	46.92	50.64	0.8425	0.6524	0.195
c)	49.82	50.76	0.9050	0.6524	0.198
d)	82.46	18.57	1.0341	0.3619	0.145
e)	75.11	27.96	1.0341	0.4708	0.167
f)	55.44	40.59	0.8471	0.5882	—

where φ_{S_2} and φ_{D_1} are the normalized spin functions for the S and the D waves, respectively (*); r is the neutron-proton distance, s [t] the Λ-proton [neutron] distance; the square of the coefficient x gives directly the percentage of the D wave. We have chosen the radial functions f and g as follows

$$f(r, s, t) \propto (\exp[-\alpha_1 s] + A \exp[-\alpha_2 s])(\exp[-\alpha_1 t] + A \exp[-\alpha_2 t])(\exp[-\beta_1 r] + B \exp[-\beta_2 r]) , \tag{3}$$

$$g(r, s, t) \propto r^2(\exp[-\gamma_1 s] + C \exp[-\gamma_2 s])(\exp[-\gamma_1 t] + C \exp[-\gamma_2 t])(\exp[-\delta_1 r] + D \exp[-\delta_2 r]) . \tag{4}$$

The function (2) with f as given by (3), but with $x=0$, coincides with the one used by DALITZ and DOWNS ([2]) which results in a fairly good description both of the asymptotic behaviour and the correlations between the three particles. For this reason we have chosen the D radial function g with a completely similar dependence on the interparticle distances.

As a result of the small value of the Λ binding energy the $^3H_\Lambda$ can be pictured as a Λ bound to a deuteron not very much distorted. Consequently if the functions (2), (3) and (4) are to accurately describe the hypertriton we must obtain a good description of the free deuteron with the wave function

$$\psi_d = \frac{1}{\sqrt{1+x^2}} \{C_1(\exp[-\beta_1 r] + B \exp[-\beta_2 r])\sigma_j + x C_2 (\exp[-\delta_1 r] + D \exp[-\delta_2 r]) S_{jk}\sigma_k\} \chi^0 , \tag{5}$$

where C_1 and C_2 are the normalization constants for the S and D radial functions, χ^0 is the singlet function for the n, p pair, and $S_{jk} = 3r_j r_k - \delta_{jk} r^2$.

(*) The complete form of these spin functions is given in Appendix A of I.

Using the function (5) we have performed variational calculations so as i) to check our treatment of the deuteron core of hypertriton and ii) to have a reasonable starting-point for the parameters related to the n-p side in the wave function for $^3H_\Lambda^*$. The results for the various n-p potentials are listed

TABLE II. – *Binding energy B_d^* and wave function parameters of the deuteron for the potentials of Table I.*

Neutron-proton potential	β_1 (fermi^{-1})	β_2 (fermi^{-1})	B	δ_1 (fermi^{-1})	δ_2 (fermi^{-1})	D	B_d^*	$\lvert x \rvert$
a)	0.313	0.800	3.425	4.937	1.603	0.087	−2.15	0.194
b)	0.305	0.814	3.633	4.667	1.523	0.086	−2.08	0.183
c)	0.306	0.833	3.793	4.775	1.553	0.087	−2.11	0.183
d)	0.348	1.064	2.893	3.745	1.213	0.076	−2.18	0.136
e)	0.328	0.973	3.293	4.158	1.343	0.078	−2.16	0.155
f)	0.328	0.862	3.008	4.390	1.426	0.081	−2.21	0.173

in Table II where B_d^* represents the estimate obtained for the deuteron binding energy. The values obtained for B_d^* are reasonably near to the experimental value the difference being between 0.02 and 0.15 MeV, according to the different potentials of Table I. We may argue that the r-side dependence in the hypertriton radial functions (3), (4) is not completely suitable; however, we can take into account the main effect of this fact by assuming B_d^*, in the hypertriton variational calculations, to be the true deuteron binding energy.

3. – Three-body variational calculations and results.

The calculations performed are straightforward generalizations of the corresponding ones in I, so we limit ourselves to pointing out a simplification not used in I. As the total kinetic energy commutes with the orbital angular momentum, its averaged value, calculated with the functions (2), does not contain any S-D mixed term; moreover the S radial function (3) and the D one (4) have a product form

$$\prod_{i=1}^{3} g_i(r_i) \qquad [(r_1, r_2, r_3) \equiv (r, s, t)].$$

It is therefore possible to use the result obtained in Appendix of ref. ([12]) with appreciable reduction in the numerical calculations.

([12]) J. W. MURPHY and S. ROSATI: *Nucl. Phys.*, to be published.

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TABLE III. — *Λ-nucleon interaction in* $^3H_\Lambda$. *The instrinsic range* 1.4843 fermi *corresponds to exchange of two pions; the value* 0.8411 fermi *corresponds to exchange of a* K-*meson.*

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Yukawa potential shape with intrinsic range 1.4843 fermi														
B_Λ MeV	α_1 fermi^{-1}	α_2 fermi^{-1}	A	β_1 fermi^{-1}	β_2 fermi^{-1}	B	γ_1 fermi^{-1}	γ_2 fermi^{-1}	C	δ_1 fermi^{-1}	δ_2 fermi^{-1}	D	$\|x\|$	$\overline{U}_2$ MeV·fermi^3
0	0.083	0.70	1.81	0.316	0.847	2.90	0.70	0.082	0.57	5.2	1.58	0.082	0.184	651
	0.047*	0.59*	2.13*	0.380*	1.13*	2.21*	—	—	—	—	—	—	0	621*
0.25	0.106	0.75	1.84	0.316	0.844	2.90	0.85	0.100	0.56	5.1	1.58	0.085	0.186	701
	0.111*	0.80*	1.67^*_5	0.380*	1.14*	2.14*	—	—	—	—	—	—	0	672*
1.00	0.157	0.87	1.81	0.325	0.845	2.77	0.67	0.137	0.51	5.2	1.62	0.087	0.188	772
	0.184*	0.98*	1.42*	0.393*	1.17*	1.94*	—	—	—	—	—	—	0	738*

Yukawa potential shape with intrinsic range 0.8411 fermi														
B_Λ MeV	α_1 fermi^{-1}	α_2 fermi^{-1}	A	β_1 fermi^{-1}	β_2 fermi^{-1}	B	γ_1 fermi^{-1}	γ_2 fermi^{-1}	C	δ_1 fermi^{-1}	δ_2 fermi^{-1}	D	$\|x\|$	$\overline{U}_2$ MeV·fermi^3
0	0.090	0.70	2.43	0.314	0.835	3.19	1.11	0.090	0.43	4.9	1.56	0.085	0.188	421
	0.093*	1.35^*_5	2.29*	0.382*	1.13*	2.22*	—	—	—	—	—	—	0	407*
0.25	0.116	1.19	2.46	0.322	0.840	3.10	0.99	0.112	0.47	4.9	1.56	0.087	0.189	440
	0.158*	1.38*	1.97*	0.382*	1.13^*_5	2.24*	—	—	—	—	—	—	0	422*
1.00	0.175	1.35	1.43	0.332	0.840	3.04	0.97	0.150	0.42	5.0	1.60	0.085	0.193	465
	0.239*	1.66*	1.74*	0.393*	1.15*	2.16*	—	—	—	—	—	—	0	444.5*

We come now to the problem of evaluating the minimum volume integral $\overline{U}$ of the Λ-$\mathcal{N}$ potential necessary to give the required binding energy. The minimum has to be found by varying all the variational parameters. For fixed values of all the other parameters, the optimum value of the mixing parameter x, as shown in I, can be readily evaluated: so we have a minimization problem in a twelve-dimensional space. In a first stage of the calculation, starting with one of the parameters we varied it by plus or minus a corresponding step. Having chosen for the parameter considered that value which gives the smallest volume integral, the same operation was repeated for the next parameter, and so on. In the neighbourhood of the minimum we changed the procedure. We changed one parameter at a time by multiples of a certain quantity so as to find its corresponding best variational value. When all the parameters had been considered the operation was again repeated with the varying quantity reduced by one-half the preceeding value, etc. This process was stopped when the fluctuations in the minimum were judged to be unimportant.

We have performed a first group of calculations on $^3H_\Lambda$ with the n-p potential of type *b*) and a Λ-$\mathcal{N}$ Yukawian potential shape. The results are collected in Table III, where we also report (marking with an asterisk) those obtained by DALITZ and DOWNS (2) using purely central forces. Three different values of the Λ binding energy are considered, namely $B_\Lambda = 0$, 0.25 and 1.00 MeV, so that it is quite possible to interpolate so as to obtain the results for any binding energy in the range considered (*).

TABLE IV. – *Results for the various* n-p *potentials of Table I and* Λ-$\mathcal{N}$ *Yukawa shape with intrinsic range* 1.4843 fermi.

Neutron-proton potential	*a*)	*b*)	*c*)	*d*)	*e*)	*f*)
$\overline{U}_2$ (MeV·fermi^3)	703	701	697	682	686	699

A second group of calculations were devoted to seeing how $\overline{U}_2$ changes when the different n-p potentials of Table I are used. We assumed $B_\Lambda = 0.25$ MeV and a Λ-$\mathcal{N}$ Yukawian potential corresponding to the exchange of two pions: the results obtained for the volume integral are listed in Table IV. Some of the preceding calculations were repeated using an exponential shape for the Λ-$\mathcal{N}$

(*) There is still a noticeable uncertainty in the Λ-d separation energy; the best present results give $B_\Lambda(^3H_\Lambda) = (0.21 \pm 0.2)$ MeV.

potential: the results are quite similar to those obtained with Yukawian shape, so they will not be discussed further.

4. – Conclusions.

For the parameters of the n-p potential (2) we have, in the main, concentrated on the choice *b*). This choice seems to be the preferable one, but it may be that, with suitable variations in the parameters, it is possible to obtain equivalently good results. In this respect it is interesting to observe that potentials like *a*), *c*) and *f*), having parameter values rather near to those of *b*), also give results, very near to the ones of potential *b*).

With respect to the case of central forces, the introduction of a tensorial term in the n-p potential gives some corrections to $\overline{U}_2$ which may be evaluated as 3 or 4% by inspection of Table III. These corrections are of some importance for the Λ-nucleon potential as actually derived from light hypernuclei data. Let us discuss briefly this point. We assume the value (*) $U_4 = 910$ MeV·fermi³ for the Λ-$\mathcal{N}$ volume integral in $^5He_\Lambda$ for the case of exchange of two pions and a Gaussian shape. From Table I of ref. (2), it is possible to evaluate how $\overline{U}_2$ changes assuming a Λ-$\mathcal{N}$ Gaussian potential shape instead of a Yukawian one. To fix the ideas, let us assume $B_\Lambda(^3H_\Lambda) = 0.25$ MeV. With the hypothesis of purely central forces we have (2) $\overline{U}_2 = 667$ MeV·fermi³ so that, using this result and the one for $\overline{U}_4$ we obtain

$$\overline{V}_a = 386.5 \text{ MeV}\cdot\text{fermi}^3\,, \qquad \overline{V}_p = 174.5 \text{ MeV}\cdot\text{fermi}^3\,,$$

where $\overline{V}_a$ and $\overline{V}_p$ indicate the integral over all the space of the Λ-$\mathcal{N}$ potential in the singlet and in the triplet spin states. When a tensor term is included in the n-p potential, we can give the estimate for a Gaussian shape: $\overline{U}_2 = 695$ MeV·fermi³, and consequently

$$\overline{V}_a = 407.5 \text{ MeV}\cdot\text{fermi}^3\,, \qquad \overline{V}_p = 167.5 \text{ MeV}\cdot\text{fermi}^3\,.$$

It is quite easy to repeat the evaluation for $\overline{V}_a$ and $\overline{V}_p$ for any desidered $B_\Lambda(^3H_\Lambda) \leqslant 1$ MeV, with conclusions very similar to the preceding ones.

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(*) See ref. (2); the errors quoted there for $\overline{U}_4$ are not essential for the present discussion.

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RIASSUNTO

Si studia l'effetto della parte tensoriale dell'interazione neutrone-protone nell'ipertritone. Le correzioni trovate per il volume integrale dell'interazione Λ-nucleone comportano variazioni apprezzabili nei parametri del potenziale Λ-nucleone derivato dallo studio degli ipernuclei leggeri.