IL NUOVO CIMENTO VOL. XXXIII, N. 2 16 Luglio 1964

Proton-Proton Elastic Scattering at High-Energies.

G. COCCONI
CERN - Geneva

(ricevuto il 28 Febbraio 1964)

Summary. — The behaviour of the high-energy cross-section for proton-proton elastic scattering as a function of proton energy and angle of emission is discussed. While small angle scattering is due to diffraction, large angle scattering seems the result of a statistical process. Several theoretical and experimental questions raised by these results are discussed in the concluding section.

1. – Introduction.

Recent measurements made in Brookhaven (1) at large angles and in the $(10 \div 30)$ GeV region, together with previous measurements performed at CERN (2) and in Brookhaven (3), give a rather complete experimental description of p-p elastic scattering up to the highest energies made available by the existing accelerators.

In Fig. 1 all the large-angle data and some of the small-angle ones are collected. The dashed curves are empirical and are supposed to describe the behaviour of the differential cross-section in the center of mass, $d\sigma/d\omega$, *vs.* the c.m. scattering angle, Θ, for fixed values of the incoming proton momentum in the laboratory system, p_0.

(1) G. COCCONI, V. T. COCCONI, A. D. KRISH, J. OREAR, R. RUBINSTEIN, B. D. SCARL, W. F. BAKER, E. W. JENKINS and A. L. READ: *Phys. Rev. Lett.*, **11**, 499 (1963); **12**, 132 (1964).

(2) A. N. DIDDENS, E. LILLETHUN, G. MANNING, A. E. TAYLOR, T. G. WALKER and A. M. WETHERELL: *Phys. Rev. Lett.*, **9**, 103 (1962).

(3) K. J. FOLEY, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, J. J. RUSSELL and L. C. L. YUAN: *Phys. Rev. Lett.*, **10**, 376, 543 (1963).

It is a remarkable fact that simple mathematical expressions can be found which, in certain intervals of Θ, describe rather satisfactorily the cross-sections as a function of some combinations of two variables that can be derived from

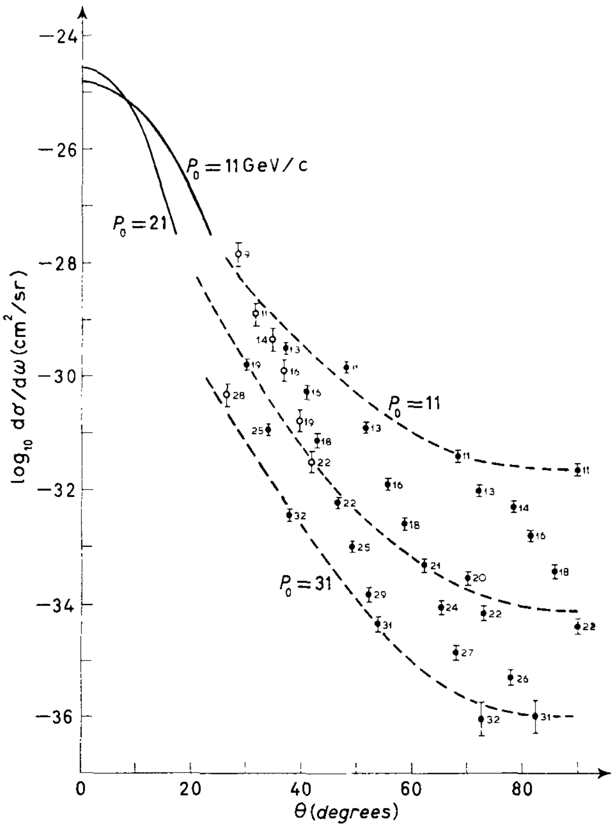

Fig. 1. – Differential cross-section of proton-proton elastic scattering *vs.* center-of-mass angle of emission. The numbers next to each point give the momentum of the protons in the laboratory, p_0, in GeV/c. The dashed lines illustrate the behaviour of the cross-sections as a function of Θ for some fixed values of p_0. ● Cornell-Brookhaven (1); ○ CERN (2); ——— Brookhaven (3).

Θ and p_0. Since a simple mathematical relation among physical quantities could be the consequence of a simple physical law, it is interesting to see what can be deduced in the present circumstances.

However, before proceeding further, few words must be said about the variables used. The unities of energy, momentum and cross-section being always GeV, GeV/c and cm² respectively, in the $(10 \div 30)$ GeV range of energy and in the case of p-p elastic scattering, the following expressions are valid within a few per cent:

3924

a) Total energy available in c.m. $w \approx 1.41\, p_0^{\frac{1}{2}}$ GeV.

b) c.m. momentum $p \approx 0.67\, p_0^{\frac{1}{2}}$ GeV/c.

c) (4-momentum transfer)^2, $-t = 2p^2(1-\cos\Theta) \approx 0.89\, p_0(1-\cos\Theta)(\text{GeV/c})^2$.

d) $-\dfrac{d\sigma}{dt} \approx \dfrac{7.0}{p_0}\dfrac{d\sigma}{d\omega}\ \text{cm}^2/(\text{GeV/c})^2$.

e) Optical theorem cross-section for $\sigma_{tot} = 40\cdot 10^{-27}\ \text{cm}^2$;

$$\frac{d\sigma}{d\omega}(\Theta = 0) = \left(\frac{K\sigma_{tot}}{4\pi}\right)^2 = 1.16\cdot 10^{-26}\, p_0\ \text{cm}^2/\text{sr} .$$

2. – Small-angle scattering.

The case of the small-angle scattering, typically $\Theta < 20°$, has already been discussed many times during the last few years. The final result can be summarized by the following expression:

$$\frac{d\sigma/d\omega}{(K\sigma_{tot}/4\pi)^2} = \exp[At] \qquad (-t < 1\ (\text{GeV/c})^2) . \tag{1}$$

All measured cross-sections are well fitted when $A \approx 9\ (\text{GeV/c})^{-2}$ [3]. The physical interpretation of this simple law is that it describes the diffraction scattering of the incoming proton by an absorbing sphere of radius $r = 2A^{\frac{1}{2}} \approx$ $\approx 1.2\cdot 10^{-13}$ cm. The cross-section of this sphere, $\pi r^2 = 45\cdot 10^{-27}\ \text{cm}^2$, agrees with the total p-p cross-section at these energies, $\sigma_{tot} \approx 40\cdot 10^{-27}\text{cm}^2$. It thus appears that diffraction scattering is the major phenomenon responsible for the dropping of the differential cross-section by a factor $\sim 10^{3.5}$ when Θ changes from zero to $\sim 20°$.

3. – Large-angle scattering.

What seems to be the main characteristic of large-angle scatterings is the flattening, for a fixed p_0, of $d\sigma/d\omega$ when $\Theta > 65°$ (see Fig 1). While a minimum at $\Theta = 90°$ is expected for the scattering of two identical particles, a « plateau » extending over about 40 % of the solid angle is peculiar.

Numerically, in the $\Theta > 65°$ region

$$\frac{d\sigma/d\omega(p_0 = 11\ \text{GeV/c})}{d\sigma/d\omega(p_0 = 31\ \text{GeV/c})} = \frac{d\sigma/d\omega(w = 4.75\ \text{GeV})}{d\sigma/d\omega(w = 7.75\ \text{GeV})} = 10^{4.4} , \tag{2}$$

a variation by a factor of 25 000 when the energy available changes by only 50 %.

3925

These two facts, quasi-isotropy over a large solid angle and strong energy-dependence of the cross-section, suggest a statistical interpretation. This means that proton scattering at large angles is interpreted as the result of the formation of a compound state which eventually decays with equal probability into a large number, N, of channels, all inelastic except one. In this case the probability of elastic scattering is $1/N$, and the differential elastic cross-section is given by the expression

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} \simeq \frac{\sigma_{\text{comp}}}{2\pi} \frac{1}{N} \tag{3}$$

where σ_{comp} is the total cross-section for the formation of the compound state (here assumed to be energy-independent).

Simple thermo-dynamical relations give then the « temperature » of the compound system. The total number of states is related to the enthropy of the system, $S(w)$, through the relation

$$N = \exp\,[S(w)]$$

and

$$\frac{\mathrm{d}S}{\mathrm{d}w} = \frac{1}{T}$$

correlates entropy, total energy and temperature. If the dependence of S on w is of the form

$$S = aw^n\,,$$

it follows from (3) that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} \propto \exp\,[-\,aw^n]$$

and that the temperature of the compound system is

$$T = \frac{1}{naw^{n-1}}\,. \tag{4}$$

The value of n characterizes the « gas » of the compound system. Two special cases are those with $n = \frac{1}{2}$ and $n = 1$. The first case corresponds to a Fermi gas, *i.e.*, to the case where in the constant volume compound system only one kind of particles is present, whose temperature increases when w increases; $n = 1$ corresponds to the case of a system in which, for w increasing, the number of possible kinds of particles increases so as to keep the energy per particle, and hence the temperature, constant.

3926

The measurements of $d\sigma/d\omega$ are not precise enough to determine the value of n. In fact, one can use either $n=1$ or $n=\frac{1}{2}$, or any other value around those, and fit reasonably well the cross-sections measured in the region $\Theta > 65°$.

However, independent of the value chosen for n, the temperature of the compound system as defined by relation (4) turns out to be the same, once the experimental ratio (2) of the cross-sections is introduced.

In fact, if

$$\frac{d\sigma/d\omega(w_1)}{d\sigma/d\omega(w_2)} = \exp[a(w_2^n - w_1^n)] = \exp[an\,\Delta w\,\langle w\rangle^{n-1}] = \exp[B]\,,$$

then

$$T = \frac{1}{na\,\langle w\rangle^{n-1}} = \frac{\Delta w}{B}$$

independent of n. With the experimental values (2), $\Delta w = 3$ GeV and $B = 4.4 \times 2.3 = 10.1$, one obtains

$$T = 3/10.1 = 0.30 \text{ GeV} = 2.15 m_\pi\,. \tag{5}$$

If one chooses $n=1$, the large-angle differential cross-sections satisfy the relation

$$\frac{d\sigma}{d\omega}(\Theta > 65°) = 10^{-24.60}\exp[-3.38w]\,. \tag{6}$$

In order to evaluate the actual number of states, N, a specific model must be introduced. Already more than one year ago FAST and HAGEDORN (4), prompted by a suggestion made by JONES (5), have tried to give a statistical explanation of large angle elastic scatterings and have computed $N(w)$ utilizing a statistical model developed by HAGEDORN. This model produces an essentially « constant temperature » because, in the compound system, beside the nucleons and the mesons, also the known excited states are counted separately. Accordingly, FAST and HAGEDORN find a total number of states roughly proportional to $\exp[aw]$, and precisely

$$N = \exp[3.30(w - 1.88)]\,. \tag{7}$$

It follows that

$$T = 1/3.3 = 0.30 \text{ GeV} = 2.18 m_\pi$$

a value close to that given in (5).

---

(4) G. FAST and R. HAGEDORN: *Nuovo Cimento*, **27**, 208 (1963); G. FAST, R. HAGEDORN and L. W. JONES: *Nuovo Cimento*, **27**, 856 (1963).

(5) L. W. JONES, K. W. LAI, M. L. PERL, S. TING, V. COOK, B. CORK and W. HOLLEY, *Proc. Geneva Conference* (1962), p. 591; L. W. JONES: *Phys. Lett.*, **8**, 287 (1964).

If (7) is now introduced in the expression (3) of the differential cross-section, together with one of the experimental values of the differential cross-section ($e.g.$, $d\sigma/d\omega(\Theta > 65°,\ w = 7.75\ \text{GeV}) \simeq 10^{-36.0}\ \text{cm}^2/\text{sr}$), one finds

$$\sigma_{\text{comp}} = 10^{-26.8}\ \text{cm}^2\,,$$

about 1/13 of the proton absorption cross-section; a reasonable number. So it appears that a concrete statistical model can well justify even quantitatively the large-angle elastic scattering of the protons.

4. – Intermediate angles.

Also for the angles ranging from 20° to 65°, simple mathematical expressions can be found fitting rather well the experimental points. However, the physical interpretation is not immediate and one is left with a choice among various possibilities.

a) A « universal curve » that fits very well all the experimental points with $\Theta > 20°$ is obtained by utilizing eq. (6) which has been found valid for large angles (*). In Fig. 2 the quantity

$$\frac{\log_{10}(d\sigma/d\omega) + 24.60}{w}$$

is plotted *vs.* Θ.

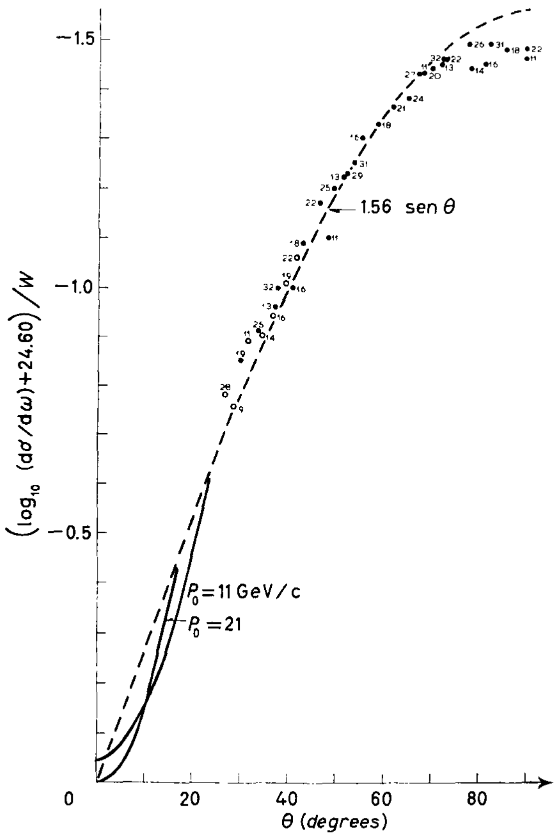

Fig. 2. – The data of Fig. 1 are here plotted *vs.* $(\log_{10}(d\sigma/d\omega)+24.60)/w$, where w is the total center-of-mass available energy, in GeV. The dashed line shows how the points at larger angles could be fitted by a sine function: ● Cornell-Brokhaven [1]; ○ CERN [2]; ——— Brookhaven.

(*) The « universal curve » proposed in reference [1], numerically does not differ substantially from that now proposed. However, the variables used here seem more appropriate for a general discussion.

3923

Practically all experimental points lie on a single curve which does not differ much from the sinusoidal that is plotted as a dashed line in the same figure. With a somewhat poorer fit all the experimental data with $\Theta > 20°$ then satisfy the relation

$$\frac{d\sigma}{d\omega} = 10^{-24.60} \exp[-3.59 w \sin\Theta] .$$

b) On the other hand, since (see Introduction) $w \simeq 2.10p$, the previous expression can also be written as follows:

$$\frac{d\sigma}{d\omega} = 10^{-24.60} \exp[7.55 p \sin\Theta] = 10^{-24.60} \exp\left[-\frac{p_\perp}{0.133}\right], \qquad (p_\perp \ll p) .$$

Actually, a slightly better fit with the transverse momentum variable is the following, proposed by Orear (6)

$$\frac{d\sigma}{d\omega} = 10^{-25.47} \exp\left[-\frac{p_\perp}{0.151}\right] .$$

It follows that the distribution of the transverse momenta of the scattered protons at a fixed proton momentum is of the form

$$(8) \qquad F(p_\perp)\, dp_\perp = \left(\frac{d\sigma}{d\omega}\frac{d\omega}{dp_\perp}\right) dp_\perp = \left(\frac{d\sigma}{d\omega}\frac{2\pi}{p}\operatorname{tg}\Theta\right) dp_\perp \propto p_\perp \exp\left[-\frac{p_\perp}{0.151}\right] dp_\perp .$$

This distribution is strangely similar to that found empirically for the momentum distribution of the inelastic secondaries produced in p-p interactions (7), namely,

$$(9) \qquad f(p_\perp)\, dp_\perp \propto p_\perp \exp[-p_\perp/0.17]\, dp_\perp .$$

c) However, these fits are not unique. For instance, a still rather good description of all the experimental points with $\Theta > 20°$ is given by the expression

$$\frac{d\sigma}{d\omega} = 10^{-14.30} \exp[-(10.40 + 7.36 \sin\Theta) w^{\frac{1}{2}}] ,$$

(6) J. Orear: *Phys. Rev. Lett.*, **12**, 112 (1964).
(7) G. Cocconi, L. J. Koester and D. H. Perkins: UCID-10022, p. 167 and Dec. 1961 Argonne Accelerator Users' Meeting.
(8) A. D. Krish: *Phys. Rev. Lett.*, **11**, 217 (1963), and private communication.
(9) R. Serber: *Phys. Rev. Lett.*, **10**, 358 (1963), and private communication.

i.e., an expression in which enters the square root of the c.m. energy. Other similar and *a priori* legitimate expressions can obviously be found.

d) KRISH (8) has pointed out that all experimental cross-sections, with Θ from 0° to 90°, can be described by the sum of three exponentials, each containing as a variable $\tau = p_\perp^2$. The three exponentials are interpreted by KRISH as the result of diffraction scattering of the protons by three regions of different dimensions.

e) Finally, SERBER (9) has shown that a purely absorbing Yukawa potential (essentially proportional to $1/r$) should give rise to a diffraction scattering that at angles $\Theta \geqslant 20°$ follows the law:

$$\frac{\mathrm{d}\sigma/\mathrm{d}t}{(\mathrm{d}\sigma/\mathrm{d}t)_{t=0}} \propto t^{-6}\,.$$

This law fits rather poorly the experimental points obtained at intermediate angles and even less well those at $\Theta > 65°$.

As a conclusion, it appears that an interpretation along simple lines of the behaviour of the intermediate angle cross-sections is not possible. Probably in this region the contributions of various phenomena, such as diffraction scattering, potential scattering and multiple, statistical scattering create a situation not easily resolvable.

We are of the opinion that the effect of the statistical scattering is felt even at angles much smaller than 65° and is responsible for the fact that an exponential law still fits well the experimental data.

In fact, it must be remembered that though the statistical model cannot predict any angular distribution, it is conceivable that, as in the case of secondary particle production (10), the conservation of angular momentum gives rise to a flat distribution at large angles accompanied by a preferential emission in the forward and backward directions, in qualitative agreement with what is observed.

5. – Conclusions.

Here we want to take seriously the hypothesis that elastic scattering at large angles is mostly produced not by a direct interaction, but by a series of statistically-incoherent multiple steps, *i.e.*, through the formation of a

(10) E. FERMI: *Phys. Rev.*, **81**, 683 (1951).

compound system. If we accept this interpretation we are confronted with the situation that the first time we look at a phenomenon where momentum transfers as large as $(3 \div 5)$ GeV/c are unmistakably involved, we do not find a direct process, a potential, a core, giving rise to them, but an indirect process.

As a matter of fact, the overall behaviour of nucleon-nucleon interaction at high energies suggests the absence of phenomena in which direct momentum transfers larger than 1 GeV/c or so are present. It is well known that at energies above 10^{11} eV the inelastic interaction of two nucleons often gives rise to secondaries that appear to diverge from two centers, the so-called fire balls, moving essentially in the direction of the two original nucleons ([11]). From these centers the secondaries emerge with momenta that correspond to a « temperature » of $\approx 2m_\pi$.

In reference ([12]), it was pointed out that these facts suggest the absence of truly high-energy components in nucleonic interactions. This point has been recently developed by WATAGHIN ([13]), FRAUTSCHI ([14]) and RAGHAVAN *et al.* ([15]), and WATAGHIN interprets it as an indication of structure in elementary particles. The elastic scattering results now specifically show that the large momentum transfers seem to occur through the indirect mechanism of multiple scatterings. Is the damping of large momentum transfers a general property of strong interactions?

Finally, it must be underlined that, independent of any interpretation, the similarity of the expressions (8) and (9) shows that, at least in first approximation, all the secondaries emerging from a p-p interaction, elastic protons as well as all kinds of particles produced inelastically, have in common the property that their transverse momentum distribution is of the form

$$f(p_\perp)\,\mathrm{d}p_\perp \propto p_\perp \exp\left[-p_\perp/p_{\perp 0}\right]\mathrm{d}p_\perp$$

with $< p_{\perp 0} \simeq 0.16$ GeV/c. A remarkable property which is hard to attribute to chance alone.

These speculations give actuality to several experimental questions.

a) The statistical interpretation of large-angle proton scattering should be confirmed by other independent evidence. ERICSON pointed out that, in

---

([11]) For an up-to-date discussion of the two-centre model see J. GIERULA: *Fortsch. der Phys.*, **11**, 109 (1963).

([12]) G. COCCONI: *Phys. Rev.*, **111**, 1699 (1958).

([13]) G. WATAGHIN: *Nuovo Cimento*, **25**, 1383 (1962); **30**, 483 (1963).

([14]) S. C. FRAUTSCHI: *Nuovo Cimento*, **28**, 409 (1963).

([15]) R. RAGHAVAN, T. N. RANGASWANNY and A. SUBRAMANIAN: *Nuovo Cimento*, **30**, 791 (1963).

analogy with the case of nuclear reactions produced via compound nucleus formation, if the plateau at $\Theta > 65°$ is correctly interpreted as the random decay of a compound system, accurate measurements of $d\sigma/d\omega$ in the $\Theta = 65° \div 90°$ region should show that the plateau really is not flat ([16]). Peaks and valleys should appear with separation of the order of few degrees, with peak to valley ratios as high as 10.

b) Can the conclusions based on evidence obtained from p-p interactions be considered valid for all strong interactions? We know already that the π-nucleon inelastic interactions have properties very similar to those of nucleon-nucleon collisions (limited transverse momenta of secondaries, average inelasticity). The same applied to high-energy K-meson interactions. As far as the elastic π-p process is concerned, the small-angle scattering is also essentially due to the diffraction of an absorbing sphere of about the same dimensions as that responsible for p-p scattering ([3]). No data, however, are available for intermediate and large angles, and obviously it will be very interesting to see what the situation is like there.

c) If it will turn out that even π-p scattering at large angles is mostly statistical, then it looks plausible that the small differences in behaviour observed between elastic p-p and elastic π-p scattering in the diffraction region are due to the existence, even in that region, of statistical factors and to the momentum dependence of the total cross-sections. In fact, the statistical factors tend to decrease the value of $d\sigma/d\omega$ as p_0 increases, and if the total cross-section is independent of p_0, as in the case of protons, the statistical factor will tend to give the impression that the diffraction pattern shrinks, *i.e.*, that the size of the interaction region giving rise to diffraction scattering increases as p_0 increases. If instead σ_{tot} decreases as p_0 increases, as is the case for π-p interactions, the effects of the statistical factor will be counterbalanced by that of the total cross-section and a more constant « size » will result. The same argument explains, at least qualitatively, the detailed behaviour of the elastic scattering observed for antiprotons and K-mesons.

d) If direct large momentum transfer processes do not occur in elastic scattering and in secondary production, is it meaningful to try other methods for probing more deeply into the nature of strong interactions? Are there phenomena in which large momentum transfers are produced directly? What is then necessary for experimenters to have, beams of higher intensity, because the cross-sections are small, or beams of higher energy, or both? The answer to these questions will probably influence the development of high-energy physics in the near future.

([16]) T. E. O. Ericson: CERN/TH 406 (Feb. 1964) and *Phys. Lett.* **4**, 258 (1963).

3932

* * *

The author feels indebted to many physicists of CERN for illuminating discussions. In particular to Professors L. VAN HOVE, T. E. O. ERICSON and V. F. WEISSKOPF.

RIASSUNTO

Si discute il comportamento osservato per lo scattering elastico protone-protone ad energie comprese fra 10 e 32 GeV. Lo scattering a piccoli angoli è dovuto principalmente a diffrazione; quello a grandi angoli sembra invece essere il prodotto di un processo statistico. Vengono quindi esaminati vari problemi teorici e sperimentali suggeriti dai risultati sopra descritti.