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π-α Scattering and the Pionic Form Factor (*).

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Summary. — Theoretical calculations are carried out on π^--α and π^+-α elastic scattering processes. Results are presented at a typical incident pion energy of 100 MeV. The results are sensitive to the pion's electromagnetic form factor since the form factor of the α-particle may be taken from electron scattering experiments. If the pion size is large the burden on the theoretical calculations is not great. However if the pion size is small the calculations must be refined. In any case the comparison of π^--α and π^+-α scattering is quite worth while and suggestions are made on how to carry it out.

One of the outstanding problems in modern high-energy physics is concerned with the determination of the structure of the pion. One possible description of the pion that is suitable for many purposes can be given in terms of its electromagnetic form factor. FRAZER [1] has indicated a possible method of determining the pion's form factor by scattering electrons from protons. However, the presence of the 3-3 resonance makes the results of this approach difficult to interpret CASSEL *et al.* [2] have used a more direct method in which

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(1) W. R. FRAZER: *Phys. Rev.*, **115**, 1763 (1959).

(2) D. CASSEL, M. BARTON, R. CRITTENDEN, V. FITCH and L. LEIPUNER: to be published. See also the Princeton University Ph. D. Thesis of D. CASSEL: *Form Factor of the π-Meson* (1954).

energetic pions are scattered against free electrons. The small value of the energy in the center-of-mass system militates against this method. Nevertheless an upper limit on the size of the pion has been obtained and is equal to 3 fermi. MAGLIĆ (3) has proposed the use of K-shell electrons of heavy elements to increase the c.m. energy in this method but the possible gain in using this technique appears to be small.

It is probable that all methods of finding a pionic size will be difficult to carry through. At least, this appears to be true for all methods with which we are familiar. Nevertheless we have devised a new and independent method that may have some merit and we have calculated the magnitude of the effects to be expected if the pion has a given r.m.s. size. With present pion beams it is likely that the initial experiments on the form factor of the pion of the type we propose would reveal only the pion's r.m.s. size but the method can be developed further to include the determination of the complete dynamic form factor of the pion. A pion «factory» making monoenergetic beams of 10^8 pion $cm^{-2}\,s^{-1}$ would definitely make our proposal workable. We suppose that the r.m.s. size of the positive and negative pions are the same, although in principle, there could be a difference.

Our idea is to scatter positive and negative pions elastically against the α-particle. From the difference in the measured values of the cross sections we hope to deduce the form factor of the pion. The explanation of the method is briefly as follows. Since the α-particle is an isoscalar body ($T=0$) the strong interaction scattering is expected to be the same for positive and negative pions. (We are presently ignoring any polarization effects in the pions due to the electromagnetic field of the α-particle because they are expected to be small. We are also neglecting possible small $T=1$ state contributions.) The electromagnetic interaction between the positive and negative pion and the α-particle is the same except for the important matter of sign. The addition and subtraction of the Coulomb amplitude to and from the strong interaction amplitude makes the two scattering cross-sections different from each other. In other words the interference between the Coulomb and strong interaction amplitudes accounts for an appreciable difference in the scattering cross-sections. In this method it is assumed of course that the electromagnetic form factor of the α-particle is known (4). A Gaussian form factor of r.m.s. size 1.65 fermi has been adopted for this purpose.

It is possible that the pion size is large enough so that in principle it could

(3) B. MAGLIĆ and G. COSTA: *Prospective pion-electron colliding beam experiments using K-shell of heavy atom as electron «Storage ring»*, to appear in the *Proceedings of the International Conference on High-Energy Physics at Dubna* (1964).

(4) R. HERMAN and R. HOFSTADTER: *High-Energy Electron Scattering Tables* (Stanford, 1960), p. 62.

be detected if the incident energy of the pion lies in the range (50 ÷ 200) MeV. In this range the phase shifts of the pion-nucleon scattering problem are known quite well, and only *s*- and *p*-phase shifts are necessary to describe the elastic scattering process. A typical experimental situation could involve a momentum transfer corresponding to $q^2 = 1.5$ fermi^{-2} at a pion scattering angle of 76° and an incident pion energy of 100 MeV. On the other hand it is well known that the Coulomb amplitude falls off very rapidly as the angle of scattering is increased. Thus it might be expected that at values of the momentum transfer (*e.g.* $q^2 = 1.5$ fermi^{-2}) where the effect of a pion size might be detectable the Coulomb process would already be so unimportant that its effect would be difficult to observe.

However, one may take advantage of the fact that at the larger angles required to reach a useful value of momentum transfer there is a minimum in the predicted strong interaction scattering of pions on He. The same type of minimum is actually found in the observed scattering of pions on Li and C ([5]) and is associated with a minimum in the pion-nucleon scattering behavior. It is possible therefore that near this minimum the Coulomb amplitude, though small, is *relatively* not so small when compared with the strong-interaction amplitude. If the small cross-sections near the minima can be measured, the difference in π^- and π^+ scattering will be sensitive to the Coulomb amplitude acting between the pions and the α-particle. Each Coulomb amplitude will be proportional to the product of the pion electromagnetic form factor and the electromagnetic form factor of the α-particle. Hence by knowing the α-particle form factor and measuring the π^+-α, π^--α scattering cross-sections we can find the pionic form factor.

We have made estimates of the effect to be expected by using an approximate multiple scattering theory of π-nucleus scattering previously discussed by one of the autors ([6]). Differences between free and bound π-nucleon two-body scattering amplitudes, off-the-energy-shell scattering, and nuclear correlations were neglected. The resulting single and double scattering terms were evaluated exactly, and a high-energy, small-angle approximation was used to estimate the third and higher order multiple scattering terms. The Coulomb effect is found to be rather large. Although we do not believe that the present calculations of the strong interaction scattering are reliable near the minimum they do indicate the order of magnitude of the effect to be expected from assumed pionic sizes of r.m.s. radius between 0 and 1.90 fermi. For example, it is possible that the present calculations may overestimate the depth of the minimum by a factor of two.

([5]) R. Williams, J. Rainwater and A. Pevsner: *Phys. Rev.*, **101**, 412 (1956); W. F. Baker, J. Rainwater and R. E. Williams: *Phys. Rev.*, **112**, 1763 (1958).

([6]) M. M. Sternheim: *Phys. Rev.*, **135**, B 912 (1964).

In the calculations we have approximated the effective r.m.s. radius of the α-particle by an appropriate amount, supposing that for the rough purposes of estimation the pion has a Gaussian density distribution. In this case the resultant r.m.s. radius may be obtained as the square root of the sum of squares of the pionic and α-particle radii. Only the Coulomb amplitude in the calculation contains the effect of a pionic size.

In Figs. 1 and 2 we give examples of the pertinent data for an incident

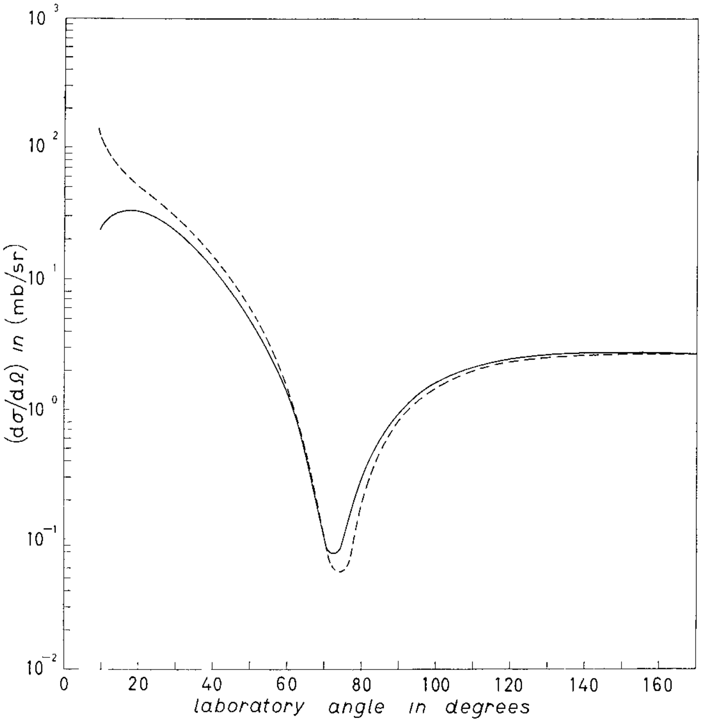

Fig. 1. – Elastic (π^--α) and (π^+-α) scattering cross-sections at 100 MeV incident pion energy. The pion is assumed to be a point in the calculations giving the above cross-sections: —— π^+; – – – π^-.

pion energy of 100 MeV. The size effect can be measured by examining the difference between π^- and π^+ scattering for zero pion size and the same quantity for various finite pion sizes. Near the minima the influence of a finite pion size of value 0.95 fermi is seen to be of the order of 16 % of the difference due to the point Coulomb effect for a typical cross-section of approximately 0.1 mb/sr. Near 66° the corresponding figures are about 14 % of the difference due to a point Coulomb effect and 4 mb/sr, respectively. Though such small differences would be difficult to detect, the experiment is perhaps not impossible at the present time. Of course, not only should one study the behavior of the π^--π^+

difference near the minimum but the variation of this difference over the whole angular interval deserves investigation in order to find the dynamic dependence

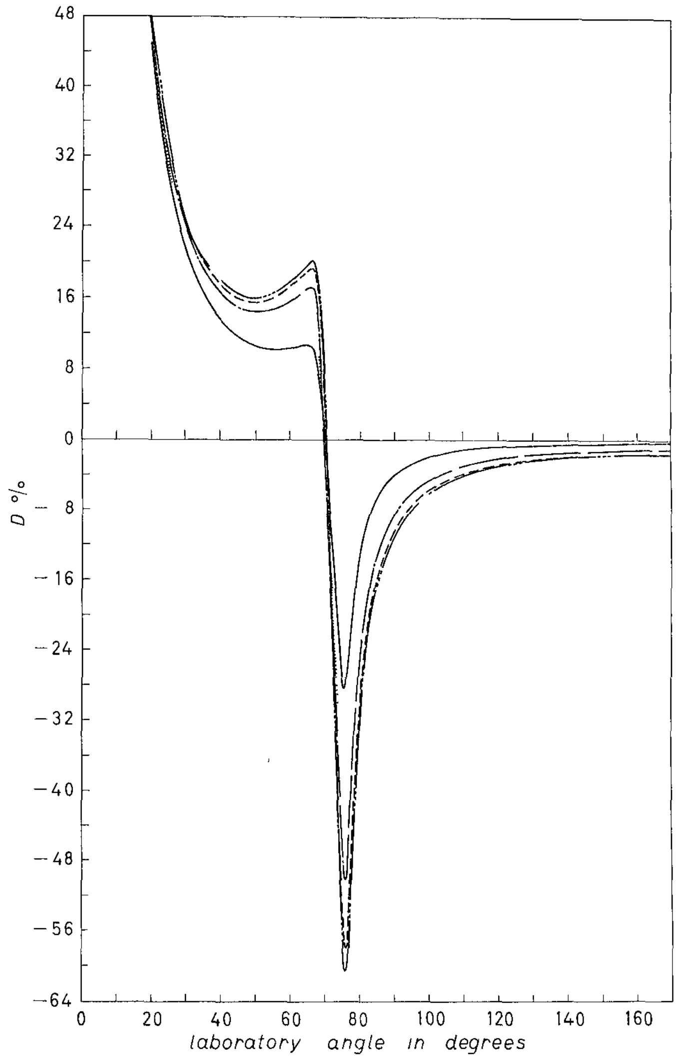

Fig. 2. – The percentage difference, $D = ((\pi^- - \pi^+)/\text{average}) \cdot 100$, is plotted at an incident pion energy of 100 MeV for various postulated sizes of the pion: —— 1.90 fermi; ····· 1.425 fermi; —·—·— 0.95 fermi; – – – 0.475 fermi; —··— 0.00 fermi.

of the pionic form factor. It is encouraging that experiments involving small differences can usually be done quite precisely. Moreover as the intensities of pionic beams increase this experiment will become easier. It is interesting to note that if the pion size should be very large, the difference between π^-

and π^+ scattering on helium would become small, *i.e.* the Coulomb effect would tend to vanish at all except the smallest angles. A large pion size would facilitate the experiments and not put a large burden on theory. On the other hand, a small pion size would require the development of a highly refined theory.

It may be expected that the theoretical calculations can be improved so that the sum of the cross-sections at the minimum may be known more reliably. In general, the sum of the experimentally measured π^- and π^+ cross-sections can serve as a valuable check on the calculations. Attempts are being made to obtain detailed optical model fits to existing α-nucleus scattering data with parameters differing only slightly from those calculated from π-nucleon phase shifts. Calculations of this kind will probably be needed to extract the pionic form factor from the experimental results.

It is possible to do this experiment at higher energies and smaller angles. However the present method of calculating pion-nucleus scattering has not yet been applied to this region.

In investigating the small influence of a pionic size it is possible that other effects could conceal the one we are looking for. For example, a contamination of an isotopic state, $T=1$, in the ground state $T=0$, could give rise to confusion in the difference-type experiment we are proposing. However, this effect is probably very small because of the high energy of any of the nearest excited states of He. It is also possible that the plane-wave approximation for the Coulomb scattering could be in error because of the simultaneous strong-interaction scattering. This effect has not yet been calculated but is likewise expected to be small.

We should like to point out that, apart from pion size considerations, $\pi^{\mp}$-α scattering is interesting in itself for the light it can throw on charge symmetry Coulomb effects. We feel that an experiment of this type would surely be rewarding. At this time an experiment might be carried out by using a long liquid helium target vessel and spark chambers as detectors. The angles could be observed in the spark chambers and the energy of the scattered pion could be determined by a range measurement or by a magnetic field, or perhaps both. A bubble chamber in also a possibility.

It may be possible to use the deuteron (*) as an equivalent isoscalar target. Here the looseness of the bond between the nucleons might make the approximations better and also the fact that fewer nucleons are present is certainly an advantage. However, we believe that the α-particle is more favorable because it is *a*) smaller in size and *b*) inelastic contributions to the scattering are relatively far away (>20 MeV) while in the deuteron they would be an order of magnitude closer.

(*) This possibility was suggested by Prof. L. I. SCHIFF.

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We wish to thank Prof. L. I. SCHIFF for his help in clarifying our ideas. We are also grateful to Prof. J. D. WALECKA and Dr. T. A. GRIFFY for their suggestions concerning possible effects competing with those of pionic size. We acknowledge the kind assistance of Mr. J. BREY in the computer progamming and the help of Mr. N. BETTINI and Mr. M. BAUM in preparing the figures.

RIASSUNTO (*)

Si fanno calcoli teorici sui processi di scattering elastico π^--α e π^+-α. Si presentano i risultati per una energia tipica del pione incidente di 100 MeV. I risultati sono influenzati dal fattore di forma elettromagnetico del pione, poichè il fattore di forma della particella α può essere dedotto dagli esperimenti di scattering dell'elettrone. Se la dimensione del pione è grande la complessità dei calcoli teorici non è grande. Invece se la dimensione del pione è piccola i calcoli devono essere raffinati. In ogni caso il confronto dello scattering π^--α e π^+-α è abbastanza interessante e si suggerisce come farlo.

(*) *Traduzione a cura della Redazione.*

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