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Quantum Physics of Single Systems.

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Summary. — By representing each occurrence of a closed quantum system by an operator which factors into a tensor product of a retarded and an advanced solution to the time-dependent Schrödinger equation, a local and objective description is obtained for each of the remote parts of an Einstein-Podolsky-Rosen–type situation.

Quantum theory and experiment agree in all the diverse situations for which the relevant physical interactions are known and the mathematical problems solvable. Certain difficulties still persist in the interpretation of single quantum systems (1). One of these is the global nature of quantum states, providing no local description for each of certain remote pairs of systems of the type suggested by EINSTEIN, PODOLSKY and ROSEN (2). Another one is that the applicability of quantum physics to all systems seems to require that there be either just one conscious observer in the Universe (3), or else a constant splitting of the Universe into many coexisting but noninteracting ones (4). Both these difficulties arise in the following thought experiment, of which this paper gives a local and objective interpretation, *i.e.* one in which all the effects of any external perturbation of an otherwise closed system propagate continuously in space-time, and in which human consciousness plays no greater role than it does in macroscopic physics.

(1) M. JAMMER: *The Philosophy of Quantum Mechanics* (New York, N. Y., 1974).
(2) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).
(3) E. P. WIGNER: *Amer. Journ. Phys.*, **31**, 6 (1963).
(4) H. EVERETT III: *Rev. Mod. Phys.*, **29**, 454 (1957).

A hydrogen atom with total-spin zero is ionized and its proton and electron are isolated in sealed boxes. This is done with negligible spin interactions, and the boxes are then taken far apart. Later on the electron box is opened by a quantum system F (*e.g.*, Wigner's friend) and the z-component of the electron spin is measured and recorded. Entirely within the relativistic present of this interaction between the electron and the system F, a person W (*e.g.* Wigner) opens the proton box, measures the x-component of proton spin and finds it, say, positive. Then W visits F, looks at the record of F's previous measurement of the z-component of the electron spin and finds also it, say, positive.

Standard quantum physics gives not only a consistent account of W's perceptions in this one situation, but also the relative frequency, $\frac{1}{4}$, for W's two observations to recur in a large number of similar situations. In this standard interpretation, W's perception of the result of the proton measurement changes the state of the total system from an eigenstate of total spin with eigenvalue 0 to a simultaneous eigenstate of the x-components of both proton and electron spin with eigenvalues plus and minus $\frac{1}{2}$, respectively. This interpretation provides no account, consistent with the assumption that quantum physics applies to F, for any perceptions F may have of the electron spin measurement before F's record is observed by W ([3]). In contrast, W has no privileged role in the reinterpretation suggested in this paper.

1. – Definitions and postulates.

« Systems », « quantities » and « runs » are three basic terms which, logically, we leave undefined. Intuitively, a ***system*** is a set of possibilities, typically characterized by certain translational and rotational degrees of freedom, for a small isolated part of the physical universe. A *quantity* is an observable which can be measured at any time, on each occurrence of a system and to arbitrary accuracy by operationally defined procedures. A *run* is the development over space-time of just one occurrence of some closed system; although actual runs are subject to uncontrolled external perturbations, these presumably can be made arbitrarily small over long times so as to approximate idealized runs.

Our basic physical assumption is that each run is fully determined by a combination of initial and final conditions, though not quite by initial or final conditions alone, because of the quantum uncertainty relations which apply to both ([5]). Now we make four mathematical postulates to make this precise, limiting ourselves for simplicity to the Schrödinger picture for the time development of systems whose Hamiltonian is constant.

Postulate 1. For each system, there is a Hilbert space $\mathscr{H}$.

([5]) A. Einstein, R. Tolman and B. Podolsky: *Phys. Rev.*, **37**, 780 (1931).

Postulate 2. For each quantity of a system, there is a self-adjoint operator on at least a dense subspace in $\mathscr{H}$; among these there is the Hamiltonian H of the system, which represents its total energy.

Postulate 3. For each run of a system and time t, there is an operator $\varrho(t)$ on $\mathscr{H}$ which is

a) idempotent, $\varrho^2(t) = \varrho(t)$;

b) of rank one, *i.e.* the set of all $\varrho(t)\psi$, for $\psi \in \mathscr{H}$, is a one-dimensional subspace in $\mathscr{H}$, which may depend on t;

c) a differentiable function of t, satisfying

$$i\hbar\varrho'(t) = H\varrho(t) - \varrho(t)H\,,$$

where ϱ' is the time derivative of ϱ and $2\pi\hbar$ is Planck's constant.

Postulate 4. The value x of a quantity X is uniquely determined at time t by the initial conditions of a run if and only if

$$X\varrho(t) = x\varrho(t)\,.$$

The value x of a quantity X is uniquely determined at time t by the final conditions of a run if and only if

$$\varrho(t)X = x\varrho(t)\,.$$

The only values uniquely determined in a run are those determined by initial conditions, final conditions or both.

2. – Theorems.

Theorem 1. For each run of a physical system there are two time-dependent state vectors $\psi(t)$ and $\varphi(t)$ of $\mathscr{H}$ with the properties

a) $\big(\varphi(t), \psi(t)\big) = 1$,

b) $i\hbar\psi'(t) = H\psi(t)$ and $i\hbar\varphi'(t) = H\varphi(t)$,

c) $X\psi(t) = x\psi(t)$,

if and only if the value x of the quantity X is uniquely determined at time t by initial conditions, and

d) $X\varphi(t) = x\varphi(t)$,

if and only if the value x of the quantity X is uniquely determined at time

t by final conditions. If both initial and final conditions determine a value of a quantity at some time, then they must determine the same values.

Proof. For any rank-one operator ϱ, there are nonzero vectors ψ and φ for which $\varrho\zeta = (\varphi, \zeta)\,\psi$ for all vectors ζ of $\mathscr{H}$. These ψ and φ are unique to within reciprocal complex factors, and ϱ is indempotent if and only if $(\varphi, \psi) = 1$, thus proving part *a*) of the theorem. Part *b*) follows from postulate 3*c*), and, for part *c*), use postulate 3*d*) and the fact that $X = X^\dagger$, $X\psi = x\psi$, $X\varphi = y\varphi$ and $(\varphi, \psi) \neq 0$ implies $x = y$.

In other words, for each run of a system there are two solutions to the time-dependent Schrödinger equation, *i.e.* a retarded solution $\psi(t)$ determined by initial conditions and an advanced solution $\varphi(t)$ determined by final conditions. The postulate that ϱ is idempotent ensures that these two state vectors are not orthogonal at any time and so they do not determine conflicting values for any quantity. However, since we do *not* postulate that ϱ is self-adjoint, the state vectors ψ and φ may be linearly independent, and in this case there are some self-adjoint operators for which just ψ or φ is an eigenstate but not both. In this sense, ψ and φ specify complementary but not contradictory aspects of a run.

In an Einstein-Podolsky-Rosen situation, the retarded state ψ_{AB} of a composite system is not a tensor product $\psi_A \otimes \psi_B$ of any two states ψ_A and ψ_B of its remote parts A and B. Nevertheless, the advanced state φ_{AB} may still be a tensor product $\varphi_A \otimes \varphi_B$ of advanced states φ_A and φ_B of each part. In this formulation of quantum physics, but not in the standard one, there are then retarded as well as advanced states for each one of the remote parts which determine the same values for all local quantities as the composite states do.

Theorem 2. For each vector φ_A of a Hilbert space $\mathscr{H}_A$, φ_B of a Hilbert space $\mathscr{H}_B$, and ψ_{AB} of the tensor product space $\mathscr{H}_A \otimes \mathscr{H}_B$, there are unique vectors ψ_A of $\mathscr{H}_A$ and ψ_B of $\mathscr{H}_B$ satisfying

$$(\varphi_A, X_A \psi_A) = \big(\varphi_A \otimes \varphi_B, (X_A \otimes 1_B)\,\psi_{AB}\big)$$

and

$$(\varphi_B, Y_B \psi_B) = \big(\varphi_A \otimes \varphi_B, (1_A \otimes Y_B)\,\psi_{AB}\big)$$

for all operators X_A on $\mathscr{H}_A$ and Y_B on $\mathscr{H}_B$, where 1_A and 1_B are the identity operators on $\mathscr{H}_A$ and $\mathscr{H}_B$, respectively. These ψ_A and ψ_B are also uniquely determined by

$$(\zeta_A, \psi_A) = (\zeta_A \otimes \varphi_B, \psi_{AB})$$

and

$$(\zeta_B, \psi_B) = (\varphi_A \otimes \zeta_B, \psi_{AB})$$

for all vectors ζ_A of $\mathscr{H}_A$ and ζ_B of $\mathscr{H}_B$.

Proof. Use the identities

$$(\varphi_A, X_A \psi_A) = (X_A^\dagger \varphi_A, \psi_A) = (X_A^\dagger \varphi_A \otimes \varphi_B, \psi_{AB}) =$$
$$= \big((X_A^\dagger \otimes 1_B)(\varphi_A \otimes \varphi_B), \psi_{AB}\big) = \big(\varphi_A \otimes \varphi_B, (X_A \otimes 1_B)\psi_{AB}\big).$$

3. – Wigner's friend in an EPR situation.

To apply this formalism to the thought experiment sketched in the introduction, let $\mathscr{H}_p$, $\mathscr{H}_e$ and $\mathscr{H}_F$ be two-dimensional Hilbert spaces representing the spin states of the proton, the spin states of the electron and the two orthogonal states used by F to record the result of the electron spin measurement.

Since spin interactions are assumed to be negligible during the ionization of the hydrogen atom and the storage of the proton and of the electron, a retarded spin vector for the proton-electron system is $\psi_{pe} = \downarrow\uparrow - \uparrow\downarrow$ of $\mathscr{H}_p \otimes \mathscr{H}_e$. F's measurement of the z-component of the electron spin determines the electron advanced spin vector $\varphi_e = \uparrow$, W's measurement of the x-component of proton spin determines the proton advanced spin vector $\varphi_p = \uparrow + \downarrow$, and W's observation of F's record determines its advanced vector $\varphi_F = \uparrow$. Then these data and theorem 2 uniquely determine the retarded spin vectors of both proton and electron, $\psi_p = \downarrow$ and $\psi_e = \uparrow - \downarrow$. The interaction between electron and system F determines F's retarded vector $\psi_F = \uparrow - \downarrow$, as in the standard interpretation of quantum physics.

The retarded and advanced vectors for the proton and the electron determine the runs of these parts of the composite system, and these runs are represented by the rank-one, idempotent, but not self-adjoint operators $\varrho_p = \frac{1}{2}(1 - \sigma_z)(1 + \sigma_x)$ on $\mathscr{H}_p$, and $\varrho_e = \frac{1}{2}(1 - \sigma_x)(1 + \sigma_z)$ on $\mathscr{H}_e$. Since the proton-electron advanced vector is the tensor product $\varphi_{pe} = \uparrow\uparrow + \downarrow\uparrow$ of the advanced vectors $\varphi_p = \uparrow + \downarrow$ and $\varphi_e = \uparrow$, the operator on $\mathscr{H}_p \otimes \mathscr{H}_e$ representing the composite proton-electron run is similarly determined.

4. – Conclusions.

This formulation of quantum physics has the following features in common with macroscopic theories:

1) In the formalism there is a representation for each run of a system, independent of any person's knowledge of it.

2) Each noninteracting component of a composite system, as in an EPR situation, has its own representation with an independent time development, in addition to the representation of the composite run.

3) Our postulates concern only those values of quantities which are uniquely determined in a single run. Single measurements can refute assertions about these; measurements on ensembles are necessary to refute statistical assertions. The statistical features of an ensemble can be derived by considering it as a large composite system, as has been done by HARTLE in the standard formulation of quantum physics [6].

4) Interactions between a physical system and a conscious being are not considered essentially different from other interactions among physical systems.

In this formulation of quantum physics, effects of interactions with a previously closed system « propagate backward in time », not changing the value of any quantity which had been determined by the initial conditions of this run, but rather determining additional quantities. These effects have current significance only if « echoed » forward by a previously established coherence of the EPR type between this run and another with which it once interacted. Then additional quantities may be determined in some current but distant run. This approach is akin to the space-time view of electromagnetism and quantum physics of Stückelberg [7], Wheeler [8], Feynman [9] and de Beauregard [10]. However, here we use a tensor product of advanced and retarded waves instead of a superposition of them. One consequence of this aspect of the formalism is that symmetry under space-time inversion is maintained for measurement processes [11]. Another is that Bell's theorem excluding a local resolution of the EPR paradox is not applicable, since it assumes all effects to propagate forward in time [12].

In the example considered, F's record shows the value of a quantity determined by final and not initial conditions. Since F may be a person aware of making this record, we conclude that, even though we assume that our perceptions are fully determined by the values of physical quantities, they are not fully determined by our pasts.

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● RIASSUNTO (*)

Rappresentando ogni verificarsi di un sistema quantico chiuso mediante un operatore che si scompone in un prodotto tensoriale di una soluzione ritardata e di una avanzata dell'equazione di Schrödinger dipendente dal tempo, si ottiene una descrizione locale e obiettiva per ciascuna delle parti remote di una situazione del tipo di quella di Einstein-Podolsky-Rosen.

(*) *Traduzione a cura della Redazione.*

Квантовая физика изолированных систем.

Резюме (*). — Представляя каждое существование замкнутой квантовой системы с помощью оператора, который распадается на тензорное произведение запаздывающего и опережающего решений уравнения Шредингера, зависящего от времени, получается локальное и объективное описание для каждой из отдаленных частей для ситуации типа Эйнштейна-Подольского-Розена.

(*) *Переведено редакцией.*