Causal Superluminal Interpretation of the Einstein-Podolsky-Rosen Paradox.

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In three recent papers (1) a possible causal interpretation of the EPR paradox has been suggested in terms of the superluminal propagation of de Broglie's (2) and Bohm's (3) quantum potential in the causal interpretation of quantum mechanics. Indeed non-local interactions now seem unavoidable in quantum theory if aspect's forthcoming experiment confirms (as believed by the authors) the experimental predictions of quantum mechanics and disproves the validity of Bell's inequalities.

The aim of the present letter is double. We first want to present a quantitative description of this model in some detail for the simple case of two identical, correlated, scalar particles and thus interpret in this context the first form of the EPR paradox (*i.e.* the simultaneous measurement of their positions and momenta) discussed within the frame of the causal interpretation, by Bohm and Hiley (4). We then want to analyse briefly the evident connection (and discrepancies) between our point of view and the well-known superluminal tachyonic interactions introduced in the literature by Sudarshan, Feinberg, Recami *et al.* (5). We show in particular that superluminal, phaselike, phononlike, collective motions of the quantum potential in Dirac's «ether» do not induce the well-known causal paradoxes (5) of tachyon theory.

Our starting point is just the two-particle generalization in configuration space of our one-particle model (1). Indeed let us assume two identical scalar particles labelled 1

(1) J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 258, 265 (1979); N. CUFARO PETRONI and J. P. VIGIE: *Lett. Nuovo Cimento*, **25**, 151 (1979).
(2) L DE BROGLIE: *La physique quantique restera-t-elle indeterministe?* (Paris, 1953).
(3) D. BOHM: *Phys. Rev.*, **85**, 166, 180 (1952).
(4) D. BOHM and B. HILEY: in *Quantum Mechanics a Half Century Later*, edited by J. LEITE-LOPES and M PATY (1975).
(5) O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. J. Phys.*, **30**, 718 (1962); G. FEINBERG: *Phys. Rev.*, **159**, 1089 (1967); E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974); E. RECAMI: *Found. Phys.*, **8**, 329 (1978).

and 2 imbedded in Dirac's stochastic « ether » ([6]). The pair's motions along any world line, in configuration space-time, build a fluid in this space-time. These motions are not independent (since the presence of particle 1 disturbs the « ether » *i.e.* the motion of particle 2 and vice-versa) and one assumes that we are dealing (as in the one-particle case) with stochastic jumps at the velocity of light (in physical space-time) which pass the pair 1, 2 from one drift line of flow (in configuration space) to another. Physically this amounts (in the hydrodynamical model of Bohm and Vigier) to the superposition in space-time of two interacting fluids 1 and 2 which undergo lightlike internal stochastic motions, particle-antiparticle transitions and possible number-preserving transfers from one fluid to another ... so that we have a conserved scalar fluid particle density in configuration space.

Mathematically this model can thus be described by in an eight-dimensional configuration space where a pair position is defined by an eight-component vector X^i ($i = 1, ..., 8$) where

$$\{X^i\}_{i=1,...,8} = \{x_1^\mu; x_2^\nu\}_{\mu,\nu=0,...,3} \tag{1}$$

with x_1^μ, x_2^ν four-vectors of the position of each body. The metric is defined by

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

so that

$$X^2 = X_i X^i = g_{ij} X^i X^j = (x_1)^2 + (x_2)^2 . \tag{3}$$

If $x_1^\mu(\tau_1)$, $x_2^\nu(\tau_2)$ are the trajectories for the two particles, the trajectory in configuration space will be an $X^i(\tau_1, \tau_2)$. As a consequence of Nelson's equations ([1]) we can now generalize the differential operators defined by GUERRA and RUGGIERO ([7]) for the single-particle case to a system of two identical particles

$$\begin{cases} D = \dfrac{\partial}{\partial\tau_1} + \dfrac{\partial}{\partial\tau_2} + b^i\partial_i , \qquad \delta D = \delta b_i \partial^i - \dfrac{h}{2m}\square , \\[2ex] \partial_i = \dfrac{\partial}{\partial X^i} , \qquad \square = \partial_i\partial^i = \square_1 + \square_2 , \qquad\qquad i = 1, ..., 8 , \\[2ex] b_i = DX_i ; \qquad \delta b_i = \delta D X_i . \end{cases} \tag{4}$$

Now a direct extension of Guerra and Ruggiero ([7]) formulae gives the following de-

([6]) P. M. A. DIRAC: *Nature (London)*, **168**, 906 (1951).
([7]) F. GUERRA and P. RUGGIERO: *Lett. Nuovo Cimento*, **23**, 529 (1978).

pendence of δb_i on a density $\varrho(X^i, \tau_1, \tau_2)$:

$$\delta b_i = -\frac{\hbar}{m}\,\partial_i \log \varrho^{\frac{1}{2}}\,, \tag{5}$$

where for the density we have as continuity equations

$$\frac{\partial \varrho}{\partial \tau_1} = -\,\partial_{1\mu}(\varrho b_1^\mu) \quad \text{and} \quad \frac{\partial \varrho}{\partial \tau_2} = -\,\partial_{2\mu}(\varrho b_2^\mu)\,, \tag{6}$$

with

$$b_1^\mu = Dx_1^\mu\,, \quad b_2^\mu = Dx_2^\mu\,, \quad \partial_1^\mu = \frac{\partial}{\partial x_{1\mu}}\,, \quad \partial_2^\mu = \frac{\partial}{\partial x_{2\mu}}\,. \tag{7}$$

In our model, as a generalization of the assumption (7) that ϱ is independent of the proper time in the one-body case, we make the physical hypothesis that the total number of particles (*i.e.* pair in the real space-time) is conserved, and thus we write

$$\frac{\partial \varrho}{\partial \tau_1} + \frac{\partial \varrho}{\partial \tau_2} = 0\,, \tag{8}$$

so that our continuity equations in configuration space is

$$\partial_i(\varrho b^i) = 0\,. \tag{9}$$

We assume as before (7) that our fluid motion is irrational, so that

$$b^i = \frac{1}{m}\,\partial^i \Phi\,, \tag{10}$$

where $\Phi(X^i, \tau_1, \tau_2)$ is a phase function, and, if we look for a steady state (*i.e.* proper time independent) equation,

$$\Phi(X^i, \tau_1, \tau_2) = \frac{mc^2}{2}\,(\tau_1 + \tau_2) + S(X^i)\,. \tag{11}$$

Now it is clear that (as generally assumed and later demonstrated by CUFARO PETRONI and VIGIER (1,8)) Newton's equations for the two free particles can be written in the compact form

$$(DD - \delta D\,\delta D)\,X^i = 0\,. \tag{12}$$

Starting from (9), (12) and using (5), (10), (11) we obtain an Hamiltonian-Jacobi–type equation ($R = \varrho^{\frac{1}{2}}$) for our two-body system *i.e.*:

$$\left(\partial_i \partial^i - \frac{\partial_i S\,\partial^i S}{\hbar} - 2\,\frac{m^2c^2}{\hbar^2}\right) R = 0 \tag{13}$$

(8) N. CUFARO PETRONI and J. P. VIGIER: *A Markov process at the velocity of light: the Klein-Gordon statistic*, preprint Inst. H. Poincaré, Paris (June 1979).

which yields for the continuity equation the form

$$2\partial_i R\,\partial^i S + R\,\partial_i\,\partial^i S = 0\,. \tag{14}$$

Finally if we consider (13) as the real part and (14) as the imaginary part the total equation for $\psi = R\exp[iS/\hbar]$ is

$$\left(\Box - 2\,\frac{m^2c^2}{\hbar^2}\right)\psi = 0\,. \tag{15}$$

From relation (15) one evidently deduces, in the nonrelativistic limit, the usual two-particle Schrödinger equation which (writing $\psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = R(\boldsymbol{x}_1, \boldsymbol{x}_2, t)\exp[iS/\hbar]$) splits into real and imaginary parts *i.e.*:

$$\frac{\partial P}{\partial t} + \boldsymbol{\nabla}_1\left(P\,\frac{\boldsymbol{\nabla}_1 S}{m}\right) + \boldsymbol{\nabla}_2\left(P\,\frac{\boldsymbol{\nabla}_2 S}{m}\right) = 0 \tag{16}$$

with $P = R^2 = \psi^*\psi$ and

$$\frac{\partial S}{\partial t} + \frac{(\boldsymbol{\nabla}_1 S)^2}{m} + \frac{(\boldsymbol{\nabla}_2 S)^2}{m} + Q = 0\,; \tag{17}$$

with $Q = -(\hbar^2/2m)[(\nabla_1^2 R/R) + (\nabla_2^2 R/R)]$. Clearly relation (16) represents the conservation of the probability $P = \psi^*\psi$ in configuration space $(\boldsymbol{x}_1, \boldsymbol{x}_2)$ while relation (17) as discussed by BOHM and HILEY (4) corresponds to a Hamilton-Jacobi equation for two particles which interact through a nonlocal quantum potential Q with which they have interpreted the first form of the EPR paradox in its original position-momentum formulation. In the causal interpretation of this situation one adds of course that our particle momenta are described in real space by $\boldsymbol{V}_1 = \boldsymbol{\nabla}_1 S/m$ on $\boldsymbol{V}_2 = \boldsymbol{\nabla}_2 S/m$ as the mapping of configuration space into real space suggests (9).

According to plan we conclude this letter with a brief discussion of the physical implications of nonlocality, since this question is now strongly reproposed by recent developments of the analysis of measurement process for correlated systems (10). The first implication is the possibility of time inversions of such events under specific Lorentz transformations. As one knows the question of the time exchange of two causally correlated events has already been discussed (for tachyons) by several authors on the basis of the reinterpretation principle (5) and rests on the remark that a Lorentz transformation which exchanges time co-ordinates of two spacelike events also exchanges energy signs and hence (on the basis of the particle-antiparticle symmetry (11)) also exchanges the cause-effect role: so that the cause always precedes the effect. Finally we can preserve the right time succession of causes and effects if we abandon the independence from the observer of what is cause and what is effect. (For a detailed discussion see ref. (5).)

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(9) J. ANDRADE E SILVA: *La théorie des systémes de particles dans l'interprétation causale de la mécanique ondulatoire* (Paris, 1960).
(10) A. GARUCCIO and F. SELLERI: *Action at distance in quantum mechanics*, in *Communication at Einstein's Centenary Commemoration* (Paris, June 1979); N. CUFARO PETRONI, A. GARUCCIO, F. SELLERI and J. P. VIGIER: *Sur la contradiction d Einstein-Bell entre les théories de la mésure quantique et la théorie locale de la rélativité restreinte*, preprint Inst. H. Poincaré (June 1979).
(11) R. P. FEYNMAN: *Phys. Rev.*, **76**, 749, 769 (1949).

More care must be used to solve the second implication *i.e.* the so-called « causal anomalies » which can be condensed in the following paradox (see fig. 1):

Let us consider two relatively moving observers O_1, O_2 with respective rest frame S, S'. At the event ε_1, O_1 sends (in its relative future) a superluminal signal to O_2 which absorbs it at ε_2; after some time, at the event ε_3, O_2 sends (in its relative future) another superluminal signal to O_1 which absorbs it at ε_4. It is easy to verify ([5]) that we can always arrange this experiment so that ε_4 precedes ε_1 ... so that we can use superluminal signals in order to modify the absolute past of O_1!

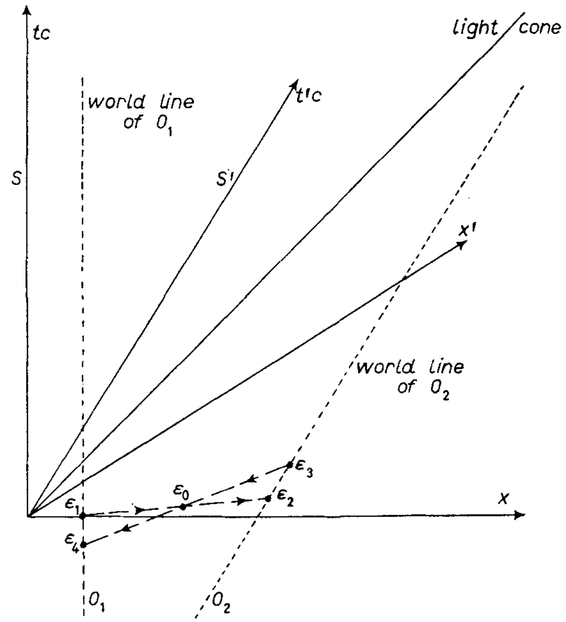

Fig. 1. – Causal anomaly.

Of course possible solutions of this causal paradox have already been proposed ([5]) for the case of signals carried by tachyons: but it is significant to note that the problem does not exist in our model where superluminal signals are not tachyons, since their propagation is now carried by a collective motion of the extended particles of Dirac's vacuum ([6]). This collective phaselike motion behaves like an heat flux ([1]) having, as it is known ([12]) a superluminal diffusion velocity. In these models the causality is first preserved in the sense that, although heat diffusion velocity is infinite, the carrier particles always remain within the light cone ([1]). However, as in the preceding example, the possibility apparently remains in principle to send signals which can modify the absolute past of any physical system. To avoid this causal anomaly, one must forbid the closed paths of fig. 1 which are evidently responsible of all causal paradoxes. This is true, as we shall show, if superluminal signals are real collective motions carried by extended vacuum particles. In that case we can require, indeed, that each particle has an intrinsic absolute flux of time (its own « proper » time) so that, with respect to this time and for this particle, no causal effect can precede its cause. Since our vacuum

([12]) R. HAKIM: *Lett. Nuovo Cimento*, **25**, 108 (1979).

particles are extended ([1,13]) the superluminal signal must always « cross » the world-tube of these « carrier » particles in the positive sense of their own time flux (as in fig. 2). So that the propagation of a signal from ε_1 to ε_2 is always possible provided that $\tau_1 < \tau_2$.

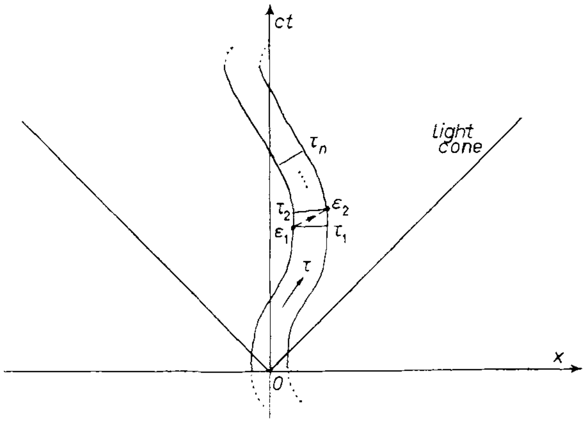

Fig. 2. – World tube of a vacuum particle with proper time τ defined as proper time of the centre of matter ([13]). A signal between $\varepsilon_1, \varepsilon_2$ always travels in the positive τ direction.

Of course for superluminal signals the time succession of ε_1 and ε_2 can be reversed for another Lorentz frame, but we can now show that this feature is irrelevant in order to avoid causal anomalies. Indeed the condition $\tau_1 < \tau_2$ (locally verified for each signal which crosses a world tube) is sufficient to forbid a path like that of fig. 1 because in the event ε_0 there are at least two criss crossing superluminal signals, so that at least one of these two signals cannot satisfy the condition $\tau_1 < \tau_2$ ... provided that vacuum particles always move with infraluminal velocity. An analog analysis can be made in the four-dimensional case if we consider superluminal signals to be « acoustical » waves with associated quantum potential propagating in the vacuum in all space directions. It is interesting to note that this elimination of causal paradoxes is only possible in a subquantum model built on a Dirac's vacuum and cannot be applied to theories where superluminal signals are carried by tachyonic particles, and to theories of the Costa de Beauregard ([14]) type where the causal connection between two spacelike events is always possible in principle through time travel into the absolute past of any physical system.

([13]) D. Bohm and J. P. Vigier: *Phys. Rev.*, **109**, 1882 (1958); D. Gutkowski, M. Moles and J. P. Vigier: *Nuovo Cimento B*, **39**, 193 (1977); F. Halbwachs: *Théorie relativiste des fluides γ spin* (Paris, 1960).

([14]) O. Costa de Beauregard: *Phys. Lett. A*, **67**, 171 (1978); *Ann. Fond. de Broglie*, **2**, 231 (1977)