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Spin Entropy of a Rotating Black Hole.

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Summary. — An interpretation as « spin entropy » of the area of the inner horizon of a Kerr black hole is proposed. The black hole is considered as a spin system with a spin temperature defined on the inner horizon. Those considerations enable us to obtain a mass formula similar to Smarr's one which contains also the new parameters defined on the inner horizon.

The definition of the temperature and the angular velocity associated with the event horizon of a Kerr black hole is mainly founded on the Killing-horizon nature of the event horizon [1]. Also the inner horizon of a Kerr black hole is a Killing horizon. Therefore, it is possible to define in the same way two analogous parameters for it, namely

the angular velocity $\quad \Omega_- = \dfrac{4\pi L}{MA_-}$

and the «temperature» $\quad T_- = \dfrac{A_- - A_+}{32\pi MA_-}$,

where M and L are the mass and the angular momentum, while A_+ and A_- are, respectively, the area of the event horizon and the area of the inner horizon of the Kerr black hole.

The existence of the inner horizon and the variability of its area have been recognized to be related to the existence and to the variability of the rotational

(1) B. CARTER: in *Les astres occlus* (New York, N. Y., 1973).

energy of the Kerr black hole ([2-4]). Hence it is natural to look for an interpretation of the parameters A_- and T_- as an « entropy » and a « temperature » (respectively), suitably related to the rotational properties of a Kerr black hole.

To this purpose, we shall follow a procedure similar to Bekenstein's one ([5]) to define the entropy A_+.

Let us consider a pure spin interaction of the hole with a particle; *i.e.* a transformation on the event horizon which changes the angular momentum of the black hole keeping the entropy A_+ constant.

It is known ([6]) that the energy E of such a particle is $E = ap_\varphi/(r_+^2 + a^2)$, where $a = L/M$, r_+ is the radius of the event-horizon and p_φ is the angular momentum of the particle.

The change in the total energy is given by

$$\delta M = \Omega_+ \delta L\,, \qquad \delta L = p_\varphi\,. \tag{1}$$

The above relation can be written as follows:

$$\delta M = \Omega_- \delta L + T_- \delta A_-\,, \tag{2}$$

where δA_- is the corresponding variation of the area A_-:

$$\delta A_- = 128\pi^2 Map_\varphi/A_+ \quad (\text{we recall that } A_+A_- = 64\pi^2 L^2).$$

In other words, during a spin transformation, there is a variation of the area A_- which is positive (respectively negative) if the particle is co-rotating with the hole, *i.e.* a and p_φ have the same sign (or, respectively, counterrotating, *i.e.* a and p_φ have opposite sign).

This argument enables us to consider A_- as a « spin entropy » and T_- as a « spin temperature » of the black hole.

In analogy with ([5]) (where it was suggested to consider the black-hole entropy A_+ as a measure of the number of the possible distinct internal configurations of a black hole), we can consider the spin entropy as a measure of the number of the spin internal configurations compatible with the macrostate. With « spin internal configuration » we mean the set of orientations of the angular momenta of the particles which have formed the hole. When such orientations (as regards the orientation of the angular momentum of the hole itself) are fairly equally distributed, there would be more « spin disorder » (more spin entropy) than in the case of a population of particles mostly co-rotating (or counterrotating) with the hole.

([2]) L. Smarr: *Phys. Rev. Lett.*, **30**, 71 (1973).
([3]) A. Curir and M. Francaviglia: *Rend. Acc. Naz. Lincei*, **61**, 448 (1976); *Acta Phys. Polon.*, **9** B, 3 (1978).
([4]) M. Calvani and M. Francaviglia: *Acta Phys. Polon.*, **9** B, 11 (1978).
([5]) J. Bekenstein: *Phys. Rev. D*, **7**, 2333 (1973).
([6]) D. Christodoulou: *Phys. Rev. Lett.*, **25**, 1596 (1970).

The spin entropy of a rotating black hole with given M and L can be also considered (from a point of view of the information theory) as a measure of the inaccessibility of information about the orientations of the angular momenta of the particles which have formed the hole.

It is known [7] that a spin temperature can be detected by means of an external field (for example, a magnetic field). Analogously, let us assume that the rotation of the hole is a kind of external field. With this in mind, there is no spin temperature in a nonrotating black hole, because there is no « external spin field » and, as a consequence, in the absence of inner horizon, the parameters T_- and A_- are meaningless.

We observe that the capture of a particle with spin opposite to the spin of the hole produces a decrease of the spin entropy. This would suggest that most of the interior configurations are counterrotating, and this is consistent with the fact that the spin temperature T_- defined by the mass formula (2) is negative definite.

In fact, a spin system has an energetic upper limit to its allowed states and, therefore, it can reach negative temperatures with a finite energy. A negative spin temperature means that the high-energy states are occupied more than the low-energy ones (this implies that an increase of the total energy M with $L =$ constant produces a decrease of the spin entropy A_-).

Therefore, we can consider a Kerr black hole as double system: a rigidly rotating body with a temperature T_+ or a spin system with a spin temperature T_- associated with the rotation of the black hole.

The interaction with a particle having energy $E = \delta M$ and spin $p_\varphi = \delta L$ can be evaluated for the rotating body by the formula

$$\delta M = T_+ \delta A_+ + \Omega_+ \delta L \,, \tag{3}$$

while for the spin system it can be evaluated by (2).

We can call $T_- \delta A_-$ the « spin heat » δQ_{spin}. As a consequence $\Omega_- \delta L$ will be the work done, evaluated on the inner horizon, to increase or decrease the angular momentum of the black hole (cf. [8]).

Let us now consider the two minima of energy on the horizons for a particle with spin p_φ.
They are

$$E^+_{\min} = a p_\varphi/(r_+^2 + a^2) = \Omega_+ \delta L \qquad \text{on the event horizon } r = r_+ \,,$$

$$E^-_{\min} = a p_\varphi/(r_-^2 + a^2) = \Omega_- \delta L \qquad \text{on the inner horizon } r = r_- \,.$$

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[7] A. Abragam and W. G. Proctor: *Phys. Rev.*, **109**, 1441 (1958).
[8] N. Ramsey: *Phys. Rev.*, **103**, 20 (1956).

Therefore, it is clear that all isentropic transformations (*i.e.* the isoareal transformations) are possible only on the two horizons.

We can express the variation of the spin heat during an external isentropic transformation as the difference between the two minima of the energy of the interacting particle:

$$\delta Q_{\text{spin}} = E^{+}_{\min} - E^{-}_{\min}\,.$$

Since $E^{+}_{\min}$ represents the energy given to the hole by the particle, we see that the variation of the spin heat is the fraction of this energy which does not produce any increase in the rotation of the inner horizon.

The heat produced in an inner isentropic transformation is

$$\delta Q = E^{-}_{\min} - E^{+}_{\min}$$

and we see that the increase of thermal heat of the black hole is the fraction of given energy wich has not increased the rotation of the event horizon.

If we consider the rotating hole as a double system, we can modify as follows the Smarr formula [2] for Kerr black holes:

$$M = T_+ A_+ + T_- A_- + \Omega_+ L + \Omega_- L\,, \tag{4}$$

or, equivalently,

$$M = \Omega_+ L + \Omega_- L\,. \tag{5}$$

We can consider $\Omega_+ L$ as the rotational energy of the rigid system and $\Omega_- L$ as the energy of the « external spin field » for the spin system.

Relation (5) can be considered as the dual of

$$M^2 = A_+/16\pi + A_-/16\pi\,,$$

where the square of the mass is expressed as the sum of the entropies only [3].

We can also notice that, in the limit of Kerr extreme black holes, we have $T_- = 0^-$, according to the fact that $T_+ = 0^+$ when $L = M^2$. Therefore, our interpretation of T_- as a spin temperature could suggest the following ideal situation: an extreme Kerr black hole is a spin system in which the spins of the particles are all opposite to the spin of the hole (while co-rotating spins have all contributed to the spin of the hole).

We remark that this is inconsistent with the fact that, for $T_- = 0$, we do not have corresponding zero spin entropy. However, we stress that the same paradox arises in the interpretation of A_+ as a thermal entropy for the extreme Kerr case.

We are presently investigating an analogous interpretation for the area of the inner horizon in the case of Reissner-Nordstrom and Kerr-Newman black holes.

It could be also interesting to study the possible applications to a rotating black hole of the properties of negative temperature (like, *e.g.*, the possible violation of Kelvin's postulate [8]).

In a forthcoming paper, we shall study the isotherms $T_+ =$ constant, $T_- =$ constant and the behaviour of the spin heat capacity $C = T_-(\partial A_-/\partial T_-)_L$.

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● RIASSUNTO

Si propone un'interpretazione dell'area dell'orizzonte interno di un buco nero di Kerr in termini di « entropia di spin ». Il buco nero é in tal modo interpretato come un sistema di spin dotato di una temperatura di spin definita sull'orizzonte interno. Queste considerazioni permettono di scrivere una formula di massa simile a quella di Smarr, ma coinvolgente anche i parametri definiti sull'orizzonte interno.

Спиновая энтропия вращающейся черной дыры.

Резюме (*). — Предлагается интерпретация области внутреннего горизонта черной дыры Керра как « спиновой энтропии ». Черная дыра рассматривается, как спиновая система со спиновой температурой, определенной на внутреннем горизонте. Такое рассмотрение позволяет нам получить массовую формулу, аналогичную формуле Смарра, которая содержит новые параметры, определенные на внутреннем горизонте.

(*) *Переведено редакцией.*