De Broglie Waves on Dirac Aether: A Testable Experimental Assumption.

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Since Dirac's pioneer work (1) it has been known that Einstein's relativity theory (and Michelson's experiment) are perfectly compatible with an underlying relativistic stochastic « aether » model. Inherent to this model is Einstein's idea that quantum statistics reflects a real subquantal physical vacuum alive with fluctuations and randomness. This concept of a nonempty vacuum has been recently revived not only to yield a foundation to the stochastic interpretation of quantum mechanics (2-4), but also to explain causally possible nonlocal superluminal interactions resulting from the Einstein-Podolsky-Rosen paradox (5,6). Indeed, if a forthcoming experiment of Aspect (7) confirms their existence, the only way out of the resulting contradiction between relativity and the quantum theory of measurement (7) seems to lie in the direction of an extension of the causal stochastic interpretation of quantum mechanics (8). This assumes the existence of causal subquantal random fluctuations induced by a stochastic « hidden » thermostat proposed by BOHM, VIGIER and DE BROGLIE (9,10). By « causal » we imply

a) that no individual particle can leave the light-cone;

b) that the sequence of causes and effects on any timelike particle motion is observer independent;

c) that one can deduce the form of de Broglie « pilot » quantum waves (and the corresponding particle motions) from the assumption that they represent real col-

(1) P. A. M. DIRAC: *Nature (London)*, **168**, 906 (1951).
(2) J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 258, 265 (1979).
(3) N. CUFARO PETRONI and J. P. VIGIER: *A Markov process at the velocity of light: the Klein-Gordon statistics*, in press to *Int. J. Theor. Phys.*
(4) A. GARUCCIO and J. P. VIGIER: *Possible experimental test of the causal stochastic interpretation of quantum mechanics. Found of physics*, in press.
(5) N. CUFARO PETRONI and J. P. VIGIER: *Lett. Nuovo Cimento*, **25**, 151 (1979).
(6) N. CUFARO PETRONI and J. P. VIGIER: *Lett. Nuovo Cimento*, **26**, 149 (1979).
(7) A. ASPECT: *Phys. Rev. D*, **14**, 1944 (1976).
(8) N. CUFARO PETRONI, A. GARUCCIO, F. SELLERI and J. P. VIGIER: *C. R. Acad. Sci. Ser. B*, **290**, 111 (1980).
(9) D. BOHM and J. P. VIGIER: *Phys. Rev.*, **96**, 208 (1954).
(10) L. DE BROGLIE: *Thermodynamique cachée des particules* (Paris, 1964).

lective drift and random motions on the top of Dirac « aether » in which the quantum jumps occur at the velocity of light [2,11];

d) that possible superluminal interactions can be interpreted in terms of a quantum potential Q (reflecting the particle interaction with the thermostat) which also carries their associated de Broglie waves; this potential's motion is not carried by individual particles, but results from the superluminal phaselike collective motion carried by the vacuum.

The aim of the present letter is

A) To extend Dirac « aether » to include the concept of spin.

B) To demonstrate that the corresponding wave equations and quantum-mechanical diffusion coefficient $D = \hbar/2m$ (common to all present stochastic interpretations of quantum mechanics) can now be theoretically deduced from Dirac-aether zero-point fluctuations at the velocity of light.

C) To discuss Dirac's aether (with spin) as a common realistic basis for various stochastic models successively proposed in the literature: such as Bohm and Vigier's initial subquantum thermostat [9], the zero-point electromagnetic model of stochastic electrodynamics (SED) of de la Peña, Marschall *et al.* [12,13] and Sudarshan *et al.* [14]. All are subvarieties of Dirac covariant mixture of $J = 0$, $J = \frac{1}{2}$ and $J = 1$ particle-antiparticle pairs: the total defining a background sea, at absolute zero temperature, on which the de Broglie waves of quantum mechanics travel.

Before discussing *A*) let us recall that Dirac « aether » rests on the idea that through any point 0 there passes a flow of stochastic particles and antiparticles (described in fig. 1 as particle moving backwards in time), whose momenta have the extremities of their four-vectors G_μ (with $G_\mu G^\mu = -m^2 c^2$) distributed with a uniform surface density on the two three-dimensional surfaces of the hyperboloids H^+ and H^-. They will thus remain invariant under all Lorentz transformations.

This stochastic relativistic distribution constitutes the only possible model for a physical undetectable thermostat for spin-zero particles into which we can study the relativistic analog of the classical nonrelativistic Brownian motion. DIRAC has derived this from the indeterminacy principle. However, it differs from it by two new physical properties.

a) Since the light-cone behaves like an asymptotic accumulation manifold of Dirac's stochastic distribution, we can assume that the corresponding stochastic jumps of a Brownian particle, submitted to its random action, occur practically at the velocity of light. Indeed, any given exchanged energy is statistically superseded by more energetic interactions.

b) This ultrarelativistic Brownian motion includes the possibility of pair creation and/or annihilation. This is important, since the mixture of particles and antiparticles has been shown to provide a realistic interpretation [15] of possible negative probability distributions.

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(11) W. LEHR and J. PARK: *J. Math. Phys.* (*N. Y.*), **18**, 1235 (1977).
(12) L. DE LA PEÑA-AUERBACH and A. M. CETTO: *Phys. Rev. D*, **3**, 795 (1971).
(13) T. MARSHALL: *Proc. R. Soc. London Ser. A*, **276**, 475 (1963).
(14) E. C. G. SUDARSHAN, K. P. SINHA and C. SIVARAM: *Found Phys.*, **6**, 65, 717 (1976); **8**, 823 (1978).
(15) YA. P. TERLETZKI and J. P. VIGIER: *Ž. Èksp. Teor. Fiz.*, **13**, 356 (1961).

The preceding model can evidently be extended to include spin if (following BOHM *et al.*([16]) and SOURIAU *et al.* ([17])) we endow our extended Dirac-« aether » elements, wave elements and concentrated-particle elements with internal rotations which represent spin classical counterpart ([18]).

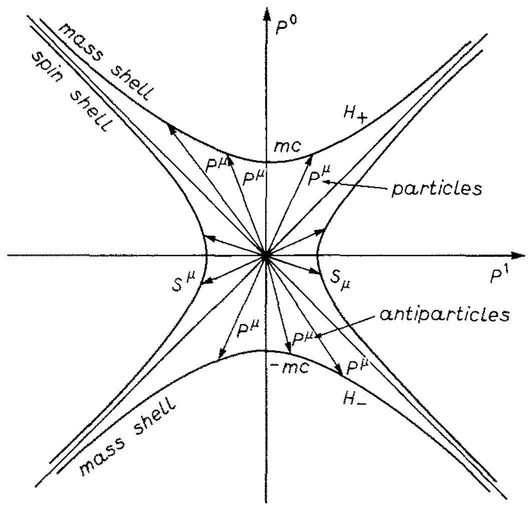

Fig. 1. – The shells correspond to the two Casimir invariants of the Poincaré group, *i.e.* $G_\mu G^\mu = -m^2c^2$ and $S_\mu S^\mu =$ constant.

In this model all extended elements are characterized by an internal angular momentum $S_{\alpha\beta}$ added to their four-momentum G_μ and unitary four-velocity $\dot{x}_\mu = \mathrm{d}x_\mu/\mathrm{d}\tau$ (*i.e.* $\dot{x}_\mu \dot{x}^\mu = -c^2$), which are no longer co-linear. The conservation equations yield $\dot{G}_\mu = 0$ (*i.e.* $G_\mu G^\mu = -m^2c^2$) and $G_\mu \dot{x}_\nu - G_\nu \dot{x}_\mu = \dot{S}_{\mu\nu}$ with $S_{\mu\nu} S_{\mu\nu} = -\sigma_0^2$.

On this basis one can introduce three classical extended models corresponding to the three spin states (*i.e.* $J = 0$, $J = \frac{1}{2}$ and $J = 1$) by the three constraints $\tilde{S}_{\mu\nu}\dot{x}^\nu = 0$, $S_{\mu\nu}\dot{x}^\nu = \sigma_0 \ddot{x}_\mu$ and $S_{\mu\nu}\dot{x}^\nu = 0$. One then obtains $S_\mu S^\mu =$ const and $s_\mu s^\mu =$ const with $S_\mu = \tilde{S}_{\mu\nu} G^\nu$ and $s_\mu = \tilde{S}_{\mu\nu}\dot{x}^\nu$. In particles, internal motions then include circular rotations of a centre-of-matter density (denoted by (c, c), $x_\mu(\tau)$) around a centre-of-mass (denoted by (C, M) and $y_\mu(\theta)$) moving parallel to G_μ with a constant radius R_μ (defined by $m^2c^2 R_\mu = S_{\mu\nu} G^\nu$) and de Broglie frequency $\hbar\omega = mc^2$ with respect to its proper time. This corresponds to a classical counterpart of the quantum « zitterbewegung ». The (c, c)'s internal Darboux-Frenet frames which now represent internal spin do not rotate $(J = 0)$, rotate twice $(J = \frac{1}{2})$ or once $(J = 1)$, while the (c, c) rotates once around the (C, M). Antiparticles have opposite (mirror) internal motions. Along the (C, M) motion one thus recovers classically Planck formula $A = mc\theta$ (where A represents the relativistic action and θ the path interval) which can also be written as $A = mc\theta = \hbar\alpha$; α denoting our new internal de Broglie phase and $\hbar$ its conjugate momenta. The corresponding internal rotations (oscillations) also yield $\theta = (\hbar/mc) = \lambda\alpha$, where $\lambda = (\hbar/mc)$ is Compton classical radius, *i.e.* the (C, M) internal motion associated with one (2π) internal rotation.

([16]) D. BOHM and J. P. VIGIER: *Phys. Rev.*, **109**, 882 (1958).
([17]) F. HALBWACHS, J. M. SOURIAU and J. P. VIGIER: *J. Phys. Radium*, **22**, 26 (1961).
([18]) C. FENECH, M. MOLES and J. P. VIGIER: *Lett. Nuovo Cimento*, **24**, 56 (1979).

On the preceding basis we can clearly introduce spin into Dirac-aether model. To every particle or antiparticle's G_μ in fig. 1 is associated the orthogonal spacelike spin vector S_μ, whose extremities will have a constant surface density on the « spin shell », *i.e.* timelike continuous « spin shell » surface $S_\mu S^\mu = \text{const}$. Such a stochastic distribution complements the constant surface distribution of the G_μ's on the « mass shell » $G_\mu G^\mu = -m^2c^2$ and is evidently invariant under any Lorentz transformation: so that the stochastic distribution of the (G_μ, S_μ) pairs now represent a $J=0$, $J=\frac{1}{2}$ or $J=1$ Dirac aether. The same result of course is obtained if we replace in fig. 1 the G_μ's by $\dot{x}_\mu$ (since $\dot{x}_\mu\dot{x}^\mu = -c^2$) and the S_μ's by s_μ.

Dirac's aether is thus now assumed to consist of a covariant mixture of $J=0$, $J=\frac{1}{2}$ and $J=1$ particle-antiparticle pairs: the totality clearly defining a background sea which encompasses (as we shall later discuss) Bohm and Vigier subquantum level [9] de Broglie subquantum thermostat [10] Sudarsham *et al.* aether [14] or (in the $J=1$ limit for zero mass) the zero-point electromagnetic model of stochastic electrodynamics (SED) of de la Peña [12] and Marshall [13].

Before calculating D, let us briefly recall the connection between de Broglie waves and Dirac aether. As already shown in the literature [2,19,20], Dirac-aether model with spin implies the correct form of the $J=0$, $J=\frac{1}{2}$ and $J=1$ wave equations if they are considered as describing the behaviour of organized collective excitations (spin waves) on the top of our covariant subquantal vacuum. This results from the preceding hypothesis *a*) and *b*) combined with the assumption that all our stochastic motions have only short range (*i.e.* contact particle-particle collision-type interactions), so that random velocities and accelerations only result from direct short-range averaging over limited four-dimensional volume elements [2].

The wave equations can then be deduced along the two lines of demonstration utilized in nonrelativistic stochastic theory, *i.e.* Einstein and Smoluchowski Brownian-motion theory [2] and Chandrasekhar random walk (at the velocity of light) on discrete covariant four-dimensional lattices [3]. For example in the first line, assuming that our extended particles associated with our collective perturbation 1) are carried along the lines of flow of a regular drift motion $\boldsymbol{v}$ associated with a collective motion on the top of Dirac thermostat (characters in bold-face type denoting four-vectors), 2) jump stochastically at the velocity of light from one average drift line of flow to another and thus (for a conserved set of identical particles with arbitrary initial positions) reach an average mean conserved distribution $P(x) = \psi^*\psi$, one can immediately demonstrate Nelson law [2], *i.e.*

$$m(D_d\boldsymbol{v} - D_s\boldsymbol{u}) = F^+ , \tag{1}$$

where the stochastic four-velocities D_d and D_s represent the covariant derivatives along the drift and stochastic lines of flow (*i.e.* $D_d f = \partial f/d\tau + (\boldsymbol{v}\cdot\boldsymbol{\nabla})F$ and $D_s f = (\boldsymbol{u}\cdot\boldsymbol{\nabla})f + D(\boldsymbol{\nabla}\cdot\boldsymbol{\nabla})f)F^+$ and F^+ the external force: the constant D now denoting the diffusion coefficient

$$\langle \delta r_i, \delta r_j/2\,\Delta\tau\rangle = D\delta_{ij} = (\langle(\delta r)^2\rangle/2\,\Delta\tau)\,\delta_{ij} , \tag{2}$$

where $\langle(\delta r)^2\rangle = \langle\sum_i \delta r_i\,\delta r_i\rangle$ denotes the average square spacelike distance of our random jumps and τ the proper time along the drift lines of flow. From (1) which has thus been shown to describe particular Markov processes at the velocity of light one deduces the Klein-Gordon [2,21], Feynman–Gell-Mann [20] and Proca equations [19]. Their corresponding (de Broglie « pilot ») wavelike solutions all satisfy de Broglie « phase correlation principle »: a principle characterized by the fact that all internal (zitterbewegung plus spin) rotations of the associated extended particle inhomogeneities (or nonlinear solitons) which follow the drift lines of their surrounding « pilot » wave rotate in phase with the fluid elements which surround them. One can compare this process with a plane flying at Mach 1 within its own sound wave: the waves themselves (thus very different in nature from ordinary sound waves) can be best described as organized « spin waves » propagating on a chaos of the spinning classical $J = 0$, $J = \frac{1}{2}$ and $J = 1$ tops which now constitute Dirac aether with spin. As one knows [22], all wave equations can be mapped into relativistic hydrodynamical representations where particles move (on the average) from one line of flow to another and jump (at the velocity of light) from one drift line to another. The quantum potential (which now also includes quantum torques) represent stochastic interactions (action and reaction) between our « particles » and the waves, both carried by Dirac aether. De Broglie phase correlation clearly expresses the well-known property [23] that oscillators can only transfer energy by contact to their immediate neighbours, provided they oscillate in phase. This property is true for example for neighbouring closed vortex tubes in classical fluids — or for connected spring systems [23]. It implies that the stochastic jumps do not disturb the regular ordering of de Broglie clocks in de Broglie waves and justifies de Broglie's idea [10] that the subquantum thermostat is built with a superposition of independent $J = 0$, $J = \frac{1}{2}$ and $J = 1$ co-variant thermostats. This last point (plus the results summarized at the beginning of this letter) can now be used to clarify a long-standing problem common to all stochastic interpretations of quantum mechanics (including stochastic electrodynamics), *i.e.* what is the connection between h, which is physically associated with internal frequency and/or spin, and D which represents random jumps?

To do this, we will limit the demonstration to the Klein-Gordon case, since spin (as can easily be shown) complicates, but does not modify substantially, our demonstration.

To calculate D we now assume

a) That in our stochastic jumps D describes (on the average) one jump only.

b) That in each individual jump the particles' (c, c) undergoes one rotation only around the (C, M). This is natural since jumps do not cancel rotation.

Assumptions *a*) and *b*) evidently represent the simplest possible Brownian-motion scheme which preserves both the average drift motion *and* the local phase correlation *and* average ordering of the de Broglie-clock zitterbewegung. If no interaction occurs during one stochastic jump, we can consider it as the $v \to c$ limit of a free-particle motion described by the Lagrangian $\delta\mathscr{L} = -mc\sqrt{-\dot{x}_\mu\dot{x}_\mu}\,d\tau$, where $d\tau$ represents the corresponding drift interval. As stated above, during the corresponding free jump the (c, c) rotates of a quantity 2π, so that the corresponding distance $\langle(\delta x)^2\rangle^{\frac{1}{2}}$ *must* be equal to Compton wave-length $\lambda = (\hbar/mc)$. When going to the $v = c$ limit we thus write $\langle(\delta x)^2\rangle = c^2\,\Delta\tau^2$, which yields (since $D = \langle(\delta x)^2\rangle/2\,\Delta\tau$) the diffusion constant $D = \hbar/2m$.

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[19] N. CUFARO PETRONI and J. P. VIGIER: *Phys. Lett. A*, **73**, 289 (1979).
[20] L. DE LA PEÑA-AUERBACH: *J. Math. Phys.* (*N. Y.*), **12**, 453 (1971).
[21] F. GUERRA and P. RUGGIERO: *Lett. Nuovo Cimento*, **23**, 529 (1978).
[22] F. HALBWACHS: *Théorie relativiste des fluides à spin* (Paris, 1960).
[23] Cf. P. HILLION: *Interprétation causale de la limite non relativiste pour l'atome d'hydrogène de la representation hydrodynamique de l'equation de Dirac*, Thèse, Paris (1957).

These results shed interesting light on the problem of the behaviour of Dirac vacuum. We limit our observations to four points.

I) The preceding model of Dirac aether evidently recovers the main results of SED with one difference: the vacuum now always contains a mixture of waves *and* particles so that one cannot reduce SED electromagnetic vacuum to a superposition of classical Maxwell waves. Indeed one sees it is now built with a distribution of plane Proca waves with Poynting vectors uniformly distributed on the photon mass shell. Indeed if we go to a negligible mass limit of Proca equations we see, following MARSHALL [24], that Dirac isotropic covariant distribution goes into $I(\nu) = (4\pi h/c^3)\nu^3$,—but that we cannot suppress discrete photons—a fact confirmed by experiment. Of course electron waves in Dirac $J=\frac{1}{2}$ aether are also submitted to the Braffort Tzara zero-point e.m. vacuum as shown directly by SUDARSHAM *et al.* [14].

II) One can utilize the real internal rotations in our vacuum and wave elements to complete (and justify) some essential properties of de Broglie « hidden thermostat ». Indeed we first see that Dirac-aether absolute temperature is necessarily zero. This clearly results from the Langevin equation associated with our Brownian motion, *i.e.*

$$m\dot{\boldsymbol{v}} = \beta \boldsymbol{v} + \langle F_{\mathrm{Q}} \rangle + \langle F_{\mathrm{E}} \rangle \,, \tag{3}$$

where β is the friction [25] coefficient of the Brownian particle: $\langle F_{\mathrm{Q}} \rangle$ and $\langle F_{\mathrm{E}} \rangle$ representing averages of the quantum (stochastic) and external force. Since we can define the vacuum temperature T^0 by $D = (kT^0)\cdot\tau/m$, where τ represents the usual relaxation time, *i.e.* $\tau = m/\beta$, we get $D = \hbar/2m = k(T^0/\beta)$, so that only $T^0 = 0$ yields $\beta = 0$—*i.e.* a frictionless vacuum. On this vacuum our fluid elements and/or particles undergo a Brownian Markov process at the velocity of light depicted by the relativistic Fokker-Planck equation.

$$\frac{\partial \varrho}{\partial \tau} + \boldsymbol{\nabla}(\boldsymbol{w}_{\pm}\varrho) \mp \Box\varrho = 0 \,, \tag{4}$$

where $\boldsymbol{w}_{\pm} = \boldsymbol{v} \pm \boldsymbol{u}$.

III) The conservation of the average number ϱ of extended oscillators within our drift tubes of flow (an essential point in our demonstration of Nelson's equations [2]) also allows an immediate definition of the temperature, entropy and heat carried by de Broglie's pilot Klein-Gordon wave. To that effect, we first recall Boltzmann's [26] famous analysis of the stochastic behaviour of distributions of periodic mechanical systems. Let us consider a mechanical system characterized 1) by microscopic variables q_i ($i = 1, \ldots$) which oscillates rapidly with a period $\tau = 1/\nu$; 2) by macroscopic co-ordinates a_i which remain constant when the system is in a given stable state, but changes slowly when it passes from a state to another in a reversible way. One sees immediately that this is exactly the situation in the (average) rest frame of a three-dimensional slice v_0 of a drift tube (see fig. 1 of ref. [2]): the variables q_i corresponds to the vectors R_μ: the reversible character in time resulting from the particle-antiparticle mixture. The a_i now represent the variable surface conditions on the tube surface which describe its external conditions (surface tension etc. [22]) and their variation corresponds to the work performed on the tube from the outside [22]. Boltzmann's result can be summarized as follows [26].

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[24] T. W. MARSHAL: *Proc. Cambridge Philos. Soc.*, **61**, 537 (1965).
[25] V. J. LEE: *Found. Phys.*, **10**, 77 (1980).
[26] Summarized in R. DUGAS: *La théorie physique au sens de Boltzmann* (Neuchâtel, 1959).

If $\mathscr{A}$ denotes the cyclic integral of the Manpertuisian action on one period of the internal motion, the quantity of heat, δQ, given to v_0 (when it undergoes a reversible change of state in its rest frame) is defined by $-\delta Q_0 = v_0\, \delta\mathscr{A}_0 = T_0\, \delta s$, where T_0 represents the temperature in its rest frame and s the system entropy. Since T and Q must Lorentz transform in the same way, two equivalent possibilities can be used, *i.e.* *a*) Tolman's [27,28] and Eckart's [29] definitions who assume that s, T_0 and Q_0 are scalars or *b*) Planck's and de Broglie's [30] (denoted by $'$) which connect them with the drift motion, *i.e.* $T_0 = T'_0 u_0$ and $Q_0 = Q'_0 u_0$ ($u_0 = -1$ in the rest frame), so that T'_0 and Q'_0 transform like $T'_0\sqrt{1-\beta^2}$ and $Q'_0\sqrt{1-\beta^2}$ (*i.e.* exactly like $\nu = \nu_0\sqrt{1-\beta^2}$), s remaining a scalar. As shown by BOHM and VIGIER in the stochastic model of the Klein-Gordon equation (with a wave $\psi(x, t) = R \exp[iS/\hbar]$) we know [9] that the rest frame's particle moving equilibrium distribution is given by $R^2 = \psi^* \psi$. Since we also know that the action $\mathscr{A} = S$ satisfies $\dot{S} = -Mc^2$ $\left(\text{since } \mathscr{A} = \int_{\tau_1}^{\tau_2} Mc^2\, d\tau\right)$, we see that the maximum of the entropy s corresponds to a minimum of the action $\mathscr{A}$, defined from the motion of our particles, if $M^2 = m^2 - (\hbar^2/c^2)[\Box R/R)$. Any isolated particle thus minimizes the value of the action integral $S = \int -Mc^2\, d\tau$, and we can write as a consequence of Boltzmann's analysis $h\nu_0 = mc^2 = kT$, so that $-s/k = \mathscr{A}/h$ with $\delta Q_0 = -Mc^2$ and $\mathscr{A} = -hM/m$, where k represents Boltzmann's constant.

IV) As a consequence

a) The particle internal-oscillation energy $E_0 = h\nu_0 = Mc^2$ along the drift lines is in equilibrium with the wave local temperature T.

b) The various random paths are weighted by $\exp[i\int \mathscr{L}\, d\tau]$, which means that Feynman graphs really correspond to possible random motions within Dirac aether, the i factor resulting [31] from the $-$ sign in Nelson's equation (31).

c) No information can be carried by the quantum potential from one particle to another, despite the fact of the superluminal propagation of the stochastic quantum potential.

d) The Klein-Gordon fluid satisfies the first two laws of thermodynamics. This can be shown directly from the decomposition of Klein-Gordon energy-momentum tensor explicitly performed by HALBWACHS [22]. Indeed he has deduced from the Lagrangian $\mathscr{L} = \partial_\mu \psi^* \partial_\mu \psi - (m^2 c^2/\hbar^2)\psi^* \psi$ the expression

$$t_{\mu\nu} = \varrho v_\mu v_\nu + q_\mu v_\nu + q_\nu v_\mu + \theta_{\mu\nu}\,,$$

where q_μ denotes a heat current (orthogonal to the conserved current $j_\mu = \varrho v_\mu$) with $v_\mu v_\mu = -c^2$. From $\partial_\mu t_{\mu\nu} = 0$ one immediately obtains introducing the scalar $\omega c^4 = t_{\alpha\beta} v^\alpha v^\beta$ and writing $\omega = \varrho(\varepsilon + mc^2)$ (where ε represents the integral energy) exactly Eckart's form [29] of the first law, *i.e.*

$$mD_d\,\varepsilon + (1/c)[\partial_\alpha q_\alpha + q_\alpha \dot{v}_\alpha] + \omega_{\alpha\beta}\partial_\alpha v_\beta = 0\,, \tag{5}$$

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(27) R. C. TOLMAN: *Theory of the Relativity of Motion*, Chap. XI (Berkeley, Cal., 1918).
(28) R. C. TOLMAN: *Relativity Thermodynamics and Cosmology*, Chap. III (1934).
(29) C. ECKART: *Phys. Rev.*, **58**, 919 (1940).
(30) L. DE BROGLIE: *C. R. Acad. Sci.*, **253**, 1078 (1961).
(31) M. BERRONDO: *Nuovo Cimento B*, **18**, 95 (1973).

where $\omega_{\alpha\beta} = t_{\gamma\delta}\eta_{\gamma\alpha}\eta_{\delta\beta}$ with $\eta_{\mu\nu} = \delta_{\mu\nu} + (v_\mu v_\nu/c^2)$. As noted by DE BROGLIE the second law is a straightforward consequence of $-s/k = \mathscr{A}/h$, since entropy is thus maximized along the fluid drift lines.

e) The heat current q_α takes the form $q_\alpha = -G\eta_{\alpha\beta}\partial^\beta\Theta$, where G represents the local thermal conductivity of the Klein-Gordon fluid and Θ the local (probabilistic) temperature density $\Theta = T\cdot R^2 = (mc^2/k)R^2$. Indeed its calculated form [22]

$$q_\mu = \varrho(\hbar^2/M^2)u^\lambda\partial_\lambda \log R\eta_{\mu\nu}\partial^\nu \log R$$

can be transformed into

$$q_\mu = (k\hbar^2/2M\cdot mc^2)\, v^\lambda\partial_\lambda \log R\eta_{\mu\nu}\partial^\nu\Theta = G\eta_{\mu\nu}\partial^\nu\,\Theta\,.$$

V) One can recover Sudarshan *et al.*'s description of the aether as a superfluid state of particle-antiparticle pairs [14]. Indeed if we start from our covariant stochastic distribution of fermion-antifermion pairs in the $L = 0$ orbital states with zero values of the total spin $S = \frac{1}{2}(\sigma_+ + \sigma_-)$ (where σ_+ and σ_- are the spin operators of the particles in question for example the positron and the electron, respectively) the Hamiltonian can be written in the momentum representation

$$H = \sum \varepsilon_k c^\dagger_{k,\sigma_-} c_{k,\sigma_-} + \sum \varepsilon_k d^\dagger_{k,\sigma_+} d_{k,\sigma_+} - \sum V_r(k, k')\, c^\dagger_{k',\sigma_-} d^\dagger_{-k',-\sigma_-} d_{-k,-\sigma_-} c_{k,\sigma_-}\,, \tag{6}$$

where $(c^\dagger_{k,\sigma_-}, c^\dagger_{k,\sigma_-})$ and $(d_{k,\sigma_+}, d_{k,\sigma_+})$ are the particle and antiparticle (creation and annihilation) operators, respectively, for the momentum state k. With $S = 0^-$ we have $\sigma_+ = -\sigma_-$ for the pair and the interaction (third term of (5)) satisfies condition *b*). H has the same form as that of Bardeen-Cooper and Schrieffer for superconductivity in metals except that each pair is now electrically neutral; an idea which can be generalized to any quantum number and any particle-antiparticle pair. Denoting by ε_k (with $V_r = V_0 =$ const for $|\varepsilon_k| < \varepsilon_0$ and zero otherwise) the Hartree-Fock energies of the fermion and of the antifermion we see that the ground-state energy of our vacuum sea will be lowered by $\Delta E = \frac{1}{2}N(0)\Delta^2$ where Δ is the gap parameter

$$\Delta = \varepsilon_0 \exp\left[-1/N(0)\cdot V_0\right]\,,$$

N_0 being the density of states on the Fermi surface. Our aether will now display long-range correlations. Particles or collective excitations (de Broglie waves) will not exchange momentum (up to a particle momentum) and there will be negligible viscosity. However, when the energies involved exceed Δ new phenomena might arise. The energy of an elementary quasi-particle excitation E_k is given by $E_k = (\varepsilon_k^2 + \Delta^2)^{\frac{1}{2}}$. For a pair it is $2E_k$ and, if we assume that masses arise predominantly from interactions, we get $|\Delta| = mc^2$. If $\Delta \gg \varepsilon_k = pc$ (p representing the particle linear momentum magnitude) the excitation of a real pair will cost $2|\Delta|$ and we see that the gap will depend on the strength of the interactions in which they take part.

This model opens many possibilities. The most remarkable is that superfluid behaviour occurs when fluid velocity is less than the velocity of the elementary excitation. The critical velocity of the fluid above which the superfluidity condition will not be satisfied for the aether can be calculated from $w_c = |\Delta|/\hbar k$ when $\hbar k$ is the momentum of the excitation. If we choose as before k to be the inverse Compton wave-length of our excitation we get $v_c = mc^2/mc = c$, so that our vacuum implies a limiting velocity c which justifies Einstein's assumption.

To conclude we want to stress that the stochastic interpretation of quantum mechanics (SIQM) is (despite many unsolved and/or open problems) much more than an intellectual curiosity, since it has now reached a testable stage. To that effect we only mention the following points.

1) SIQM presents one of the two possible interpretations of superluminal interactions predicted in the EPR paradox ([5])—if they are confirmed by Aspect's and Rapisarda's experiments.

2) SIQM predicts as shown by FITCHARD ([32]) a possible discrepancy with the Copenhaghen interpretation of quantum mechanics (CIQM) in a perfectly feasible test of wave packet reduction and measurement of the uncertainty principle.

3) SIQM implies that one can test direct consequences of the real existence of de Broglie waves which are not predicted by CIQM. This is the case in induced superfluorescence (by SELLERI and VIGIER ([33])) and in a modified form of the Pfleegor-Mandel experiment ([4]).

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