Squeezed States in the Presence of a Time-Dependent Magnetic Field.

A. JANNUSSIS

Department of Physics, University of Patras - Patras 26110, Greece
Institute for Basic Research - 96 Prescott Str., Cambridge, Mass., 02138

E. VLAHOS, D. SKALTSAS, G. KLIROS and V. BARTZIS

Department of Physics, University of Patras - Patras 26110, Greece

(ricevuto il 6 Marzo 1989)

Summary. — In this paper we study various properties of the prototype squeeze operator in a uniform magnetic field in two dimensions. The corresponding creation and annihilation operators are of the Yuen form and describe squeezed states. The case of the free electron in a time-varying magnetic field leads also to the squeezed states.

PACS 03.65 – Quantum theory; quantum mechanics.
PACS 42.50.Dv – Nonclassical photon states (including antibunched, squeezed, sub-Poissonian).

1. – Introduction.

A prototype for the generation of the so-called squeezed states is the Hamiltonian [1-3]

$$H = \hbar\omega a^{\dagger} a - i\hbar\chi(\varepsilon a^2 - \varepsilon^* a^{\dagger 2}), \tag{1.1}$$

(1) D. WALLS: *Squeezed states of the electromagnetic field*, in *Coherence and Quantum Optics V*, edited by L. MANDEL and E. WOLF (Plenum Press, New York, N.Y., 1984), p. 609; *Nature* (*London*) **306**, 141 (1983).
(2) H. YUEN: *Phys. Rev. A*, **13**, 2226 (1976).
(3) D. STOLER: *Phys. Rev. D*, **1**, 3217 (1970).

where χ is a nonlinear optical susceptibility, ε is the amplitude of a classical driving field and a, $a^\dagger$ are the usual annihilation and creation operators.

Recently Jannussis and Skaltsas [4] have transformed the prototype operator (1.1) into an equivalent operator of the following form:

$$H = \frac{1}{2m} p^2 + \frac{1}{2} m\omega^2 q^2 + \frac{\gamma}{2}(pq + qp)\,, \tag{1.2}$$

where $\gamma = 2\chi|\varepsilon|$ is a real parameter or the friction coefficient.

In ref. [5,6] we have shown that the operator $\exp[i\gamma(pq + qp)/2]$ is exactly the squeeze operator.

In the present paper we extend the prototype operator (1.2) for the case of a uniform magnetic field in two dimensions, as follows:

$$H = \frac{1}{2m}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{q})\right)^2 + \frac{\gamma}{4}(\boldsymbol{qp} + \boldsymbol{pq})\,. \tag{1.3}$$

In the next sections we study various properties of the above operator and we determine the corresponding annihilation and creation operators which are of the Yuen [2] form and describe squeezed states.

The new Yuen operators are exactly of the same form in the phase space. In fact, this has been expected because the operator of the free electron in a uniform magnetic field is equivalent to the operator of the harmonic oscillator in the phase space.

Lately, the squeezed states are suited in the phase space [9-11] with the determination of the corresponding Wigner distribution of the squeezed functions.

The corresponding prototype Wigner operator which results from operator

---

(4) A. JANNUSSIS and D. SKALTSAS: *Squeezed states in the Caldirola-Montaldi procedure*, Preprint 1988, Dept. of Physics, University of Patras (Greece).

(5) A. JANNUSSIS and V. BARTZIS: *Nuovo Cimento B*, **102**, 33 (1988) and references therein.

(6) A. JANNUSSIS and V. BARTZIS: *Phys. Lett. A*, **132**, 324 (1988).

(7) W. SCHLEID and J. WHEELER: *Nature*, **326**, 574 (1987); *The physics of phase space*, in *Lectures Notes in Physics*, edited by Y. KIM and W. ZACCHARY vol. 278 (Springer Verlag, 1986), p. 200; W. SCHLEICH, D. WALLS and J. WHEELER: *Phys. Rev. A*, **38**, 1177 (1988).

(8) Y. KIM and E. WIGNER: *Phys. Rev. A*, **38**, 1159 (1988).

(9) A. JANNUSSIS, V. BARTZIS and E. VLACHOS: *Coherent and squeezed states in phase space*, presented in the *Fourth Workshop on Hadronic Mechanics, Skopje, Yugoslavia, August 22-27, 1988*, to appear in the proceedings of the workshop.

(10) A. JANNUSSIS and N. PATARGIAS: *Phys. Lett. A*, **53**, 357 (1975); A. JANNUSSIS, P. FILIPAKIS and TH. FILIPAKIS: *Physica A*, **102**, 561 (1980).

(11) A. JANNUSSIS, N. PATARGIAS and G. BRODIMAS: *J. Phys. Soc. Jpn.*, **45**, 336 (1978).

(1.2) with the help of transformation

$$(1.4) \qquad \begin{cases} q \to q + \dfrac{i\hbar}{2}\dfrac{\partial}{\partial p}, & p \to p - \dfrac{i\hbar}{2}\dfrac{\partial}{\partial q}, \\ q^* \to q - \dfrac{i\hbar}{2}\dfrac{\partial}{\partial p}, & p^* \to p + \dfrac{i\hbar}{2}\dfrac{\partial}{\partial q} \end{cases}$$

is of the form

$$(1.5) \qquad W\left(q, p, \frac{\partial}{\partial q}, \frac{\partial}{\partial p}\right) = H(q,p) - H(q^*, p^*) =$$

$$= -\frac{i\hbar}{m} p \frac{\partial}{\partial q} + i\hbar m\,\omega^2 q \frac{\partial}{\partial p} + \frac{i\hbar\gamma}{2}\left(\frac{\partial}{\partial p} p + p\frac{\partial}{\partial p} - \frac{\partial}{\partial q} q - q \frac{\partial}{\partial q}\right) =$$

$$= -i\hbar\omega\left[p\frac{\partial}{\partial(m\omega q)} - m\omega q\frac{\partial}{\partial p} - \frac{\gamma}{2\omega}\left(\frac{\partial}{\partial p}p + p\frac{\partial}{\partial p} - \frac{\partial}{\partial q}q - q\frac{\partial}{\partial q}\right)\right].$$

For $m\omega q = p'$ the above operator takes the form

$$(1.6) \qquad W = -i\hbar\omega\left[p\frac{\partial}{\partial p'} - p'\frac{\partial}{\partial p} - \frac{\gamma}{2\omega}\left(\frac{\partial}{\partial p}p + p\frac{\partial}{\partial p} - \frac{\partial}{\partial p'}p' - p'\frac{\partial}{\partial p'}\right)\right]$$

and for

$$(1.7) \qquad p' = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger), \qquad \frac{\partial}{\partial p'} = \sqrt{\frac{m\omega}{2\hbar}}(a - a^\dagger),$$

$$(1.8) \qquad p = \sqrt{\frac{\hbar}{2m\omega}}(b + b^\dagger), \qquad \frac{\partial}{\partial p} = \sqrt{\frac{m\omega}{2\hbar}}(b - b^\dagger),$$

the Wigner operator can be written as

$$(1.9) \qquad W = -i\hbar\omega(ab^\dagger - a^\dagger b) + \frac{i\hbar\gamma}{2}(b^2 - b^{\dagger 2} - a^2 + a^{\dagger 2}).$$

The above operator is exactly a prototype two-mode operator and describes squeezed states in the phase space.

In the following we study the equations of motion of operator (1.9) in the Heisenberg picture.

2. – Squeezed states in phase space.

The Heisenberg equations of motion for operator (1.9) are the following:

$$\begin{cases} \mathrm{d}a/\mathrm{d}t = \omega b + \gamma a^\dagger, & \mathrm{d}a^\dagger/\mathrm{d}t = \omega b^\dagger + \gamma a, \\ \mathrm{d}b/\mathrm{d}t = -\omega a - \gamma b^\dagger, & \mathrm{d}b^\dagger/\mathrm{d}t = -\omega a^\dagger - \gamma b. \end{cases} \tag{2.1}$$

The time evolution of the operators $a(t)$, $a^\dagger(t)$, $b(t)$ and $b^\dagger(t)$ are given after some algebra by the expressions

$$a(t) = a(0)\cos\Omega t + \frac{1}{\Omega}[\gamma a^\dagger(0) + \omega b(0)]\sin\Omega t, \tag{2.2}$$

$$a^\dagger(t) = a^\dagger(0)\cos\Omega t + \frac{1}{\Omega}[\gamma a(0) + \omega b^\dagger(0)]\sin\Omega t, \tag{2.3}$$

$$b(t) = b(0)\cos\Omega t - \frac{1}{\Omega}[\omega a(0) + \gamma b^\dagger(0)]\sin\Omega t, \tag{2.4}$$

$$b^\dagger(t) = b^\dagger(0)\cos\Omega t - \frac{1}{\Omega}[\omega a^\dagger(0) + \gamma b(0)]\sin\Omega t, \tag{2.5}$$

where

$$\Omega^2 = \omega^2 - \gamma^2. \tag{2.6}$$

The above operators satisfy the commutation relations

$$\begin{cases} [a(t), a^\dagger(t)] = 1, & [a(t), b(t)] = 0 & [a^\dagger(t), b(t)] = 0, \\ [a(t), b^\dagger(t)] = 0, & [a^\dagger(t), b^\dagger(t)] = 0, & [b(t), b^\dagger(t)] = 1 \end{cases} \tag{2.7}$$

and describe squeezed states.

According to ref. ([4]), for the case $\Omega > 0$, the eigenfunctions of the prototype operator (1.2), in the q-representation, have the following form:

$$\psi_n(q) = \sqrt[4]{\frac{m\Omega}{\pi\hbar}}\,\frac{1}{\sqrt{2^n n!}}\exp\left[-\frac{m}{2\hbar}(\Omega + i\gamma)q^2\right]\cdot H_n\left(\sqrt{\frac{m\Omega}{\hbar}}\,q\right), \tag{2.8}$$

where $H_n(x)$ are the Hermite polynomials.

The corresponding eigenvalues are

$$E_n = \hbar\Omega(n + 1/2), \qquad n = 0, 1, 2, \ldots. \tag{2.9}$$

It is known [10] that the operator which describes the phase space, in a similar way to the Hamiltonian operator in the Schrödinger theory, is the Wigner operator (1.5) and the eigenfunctions and eigenvalues are, respectively,

$$f_{n,l}(q,p)=\frac{1}{\pi\hbar}\int \mathrm{d}y\,\psi_n^*(q+y)\,\psi_l(q-y)\,\exp\left[\frac{2ip}{\hbar}y\right], \tag{2.10}$$

$$E_N=\hbar\Omega N\,, \qquad N=n-l=0,1,2,\dots\,. \tag{2.11}$$

The eigenfunctions $f_{N+l,l}(q,p)$ are l-fold degenerate.

By the same way, if we know the squeezed states in the q-representation, the corresponding squeezed states in the phase space are given from the Wigner distribution function [5,6,9].

As has been referred in the introduction, the case of the free electron in a uniform magnetic field is equivalent to harmonic oscillator of Bopp [11] form in phase space.

In the following we study the operator (1.3) with symmetrical vector potential

$$\boldsymbol{A}(\boldsymbol{q})=\left(-\frac{1}{2}Hq_2,\frac{1}{2}Hq_1,0\right) \tag{2.12}$$

in the Heisenberg picture.

3. – Squeezed states in uniform magnetic field.

The Heisenberg equations of motion for the operator (1.3) with the vector potential (2.12) are the following:

$$\mathrm{d}q_1/\mathrm{d}t=\frac{1}{m}\left(p_1+\frac{eH}{2c}q_2\right)+\frac{\gamma}{2}q_1\,, \tag{3.1}$$

$$\mathrm{d}q_2/\mathrm{d}t=\frac{1}{m}\left(p_2-\frac{eH}{2c}q_1\right)+\frac{\gamma}{2}q_2\,, \tag{3.2}$$

$$\mathrm{d}p_1/\mathrm{d}t=\frac{\omega_L}{2}\left(p_2-\frac{eH}{2c}q_1\right)-\frac{\gamma}{2}p_1\,, \tag{3.3}$$

$$\mathrm{d}p_2/\mathrm{d}t=-\frac{\omega_L}{2}\left(p_1+\frac{eH}{2c}q_2\right)-\frac{\gamma}{2}p_2\,, \tag{3.4}$$

with $\omega_L=eH/mc$.

We use the corresponding annihilation and creation operators for the pair of the operators

$$(q_1,p_1)\rightarrow(a_1,a_1^\dagger) \quad\text{and}\quad (q_2,p_2)\rightarrow(a_2,a_2^\dagger)\,.$$

For $\omega = \omega_L$ and after some algebra we obtain the expressions

$$a_1(t) = f_1(t)\,a_1(0) + f_2(t)\,a_1^\dagger(0) + f_3(t)\,a_2(0) + f_4(t)\,a_2^\dagger(0)\,, \tag{3.5}$$

$$a_1^\dagger(t) = f_1^*(t)\,a_1^\dagger(0) + f_2(t)\,a_1(0) + f_3^*(t)\,a_2^\dagger(0) + f_4(t)\,a_2(0)\,, \tag{3.6}$$

$$a_2(t) = -f_3(t)\,a_1(0) - f_4(t)\,a_1^\dagger(0) + f_1(t)\,a_2(0) + f_2(t)\,a_2^\dagger(0)\,, \tag{3.7}$$

$$a_2^\dagger(t) = -f_3^*(t)\,a_1^\dagger(0) - f_4(t)\,a_1(0) + f_1^*(t)\,a_2^\dagger(0) + f_2(t)\,a_2(0)\,, \tag{3.8}$$

where

$$f_1(t) = (\cos\Omega_1 t + \cos\Omega_2 t)/2 - \frac{i\omega}{2\Omega}(\sin\Omega_1 t - \sin\Omega_2 t)\,, \tag{3.9}$$

$$f_2(t) = \frac{\gamma}{2\Omega}(\sin\Omega_1 t - \sin\Omega_2 t)\,, \tag{3.10}$$

$$f_3(t) = (\sin\Omega_1 t + \sin\Omega_2 t)/2 + \frac{i\omega}{2\Omega}(\cos\Omega_1 t - \cos\Omega_2 t)\,, \tag{3.11}$$

$$f_4(t) = -\frac{\gamma}{2\Omega}(\cos\Omega_1 t - \cos\Omega_2 t)\,, \tag{3.12}$$

$$\Omega_1 = (\omega + \sqrt{\omega^2 - \gamma^2})/2 = (\omega + \Omega)/2\,, \tag{3.13}$$

$$\Omega_2 = (\omega - \sqrt{\omega^2 - \gamma^2})/2 = (\omega - \Omega)/2\,, \tag{3.14}$$

and

$$\Omega = \sqrt{\omega^2 - \gamma^2}\,. \tag{3.15}$$

The operators (3.5)-(3.8) satisfy the following commutation relations:

$$[a_1(t), a_1^\dagger(t)] = 1, \qquad [a_2(t), a_2^\dagger(t)] = 1\,, \tag{3.16}$$

$$[a_1(t), a_2(t)] = [a_1^\dagger(t), a_2^\dagger(t)] = [a_1(t), a_2^\dagger(t)] = [a_1^\dagger(t), a_2(t)] = 0, \tag{3.17}$$

which are similar to the commutation relations (2.7).

As has been shown by Jannussis *et al.* [12] for the Yuen [2] operators in the

---

([12]) A. JANNUSSIS, N. PATARGIAS and L. PAPALOUCAS: *Lett. Nuovo Cimento*, **29**, 87 (1980); A. PAJAGORAL and J. MARSAL: *Phys. Rev. A*, 26, 2977 (1982); M. SHULDERT and W. VOGEL: *Phys. Rev. A*, 28, 3668 (1983).

phase space, the operators (3.5)-(3.8) are of the Yuen form and describe squeezed states. The study of the operator (1.3) in the Schrödinger picture is out of our concern here because it has been studied by Dodonov and Man'ko [13] for quadratic operators in N-dimensions.

The case $\gamma = 0$ is very interesting because it leads to the Yuen operators too. In fact, from operators (3.5)-(3.8), for $\gamma = 0$, we obtain $\Omega_1 = \omega$, $\Omega_2 = 0$:

$$
\begin{cases}
f_1(t) = (1+\cos\omega t)/2 - \dfrac{i}{2}\sin\omega t = \exp\left[-i\dfrac{\omega t}{2}\right]\cos\dfrac{\omega t}{2}, \\
f_2(t) = 0, \quad f_4(t) = 0, \\
f_3(t) = \dfrac{\sin\omega t}{2} - \dfrac{i}{2}(1-\cos\omega t) = \exp\left[-i\dfrac{\omega t}{2}\right]\sin\dfrac{\omega t}{2}.
\end{cases} \tag{3.18}
$$

So, operators (3.5)-(3.8) can be written as

$$a_1(t) = \exp[-i\omega t/2]\cos\frac{\omega t}{2}a_1(0) + \exp[-i\omega t/2]\sin\frac{\omega t}{2}a_2(0), \tag{3.19}$$

$$a_1^\dagger(t) = \exp[i\omega t/2]\cos\frac{\omega t}{2}a_1^\dagger(0) + \exp[i\omega t/2]\sin\frac{\omega t}{2}a_2^\dagger(0), \tag{3.20}$$

$$a_2(t) = \exp[-i\omega t/2]\cos\frac{\omega t}{2}a_2(0) - \exp[-i\omega t/2]\sin\frac{\omega t}{2}a_1(0), \tag{3.21}$$

$$a_2^\dagger(t) = \exp[i\omega t/2]\cos\frac{\omega t}{2}a_2^\dagger(0) - \exp[i\omega t/2]\sin\frac{\omega t}{2}a_1^\dagger(0) \tag{3.22}$$

and satisfy the commutation relations (3.16) and (3.17).

In the following we note that the operator of the free electron in a uniform magnetic field (1.3) for $\gamma = 0$, with symmetrical vector potential of the form $\boldsymbol{A} = (-H/2q_2, H/2q_1, 0)$, $\omega = eH/2mc = \omega_L/2$ and with the use of the operators

$$a_1 = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,q_1 + i\frac{p_1}{\sqrt{m\omega}}\right), \quad a_1^\dagger = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,q_1 - i\frac{p_1}{\sqrt{m\omega}}\right), \tag{3.23}$$

$$a_2 = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,q_2 + i\frac{p_2}{\sqrt{m\omega}}\right), \quad a_2^\dagger = \frac{1}{\sqrt{2\hbar}}\left(\sqrt{m\omega}\,q_2 - i\frac{p_2}{\sqrt{m\omega}}\right), \tag{3.24}$$

([13]) V. Dodonov, I. Malkin and V. Man'ko: *Physica*, **59**, 241 (1972); V. Dodonov, V. Man'ko and V. Skarzhinsky: *Nuovo Cimento B*, **69**, 185 (1982); V. Dodonov and V. Man'ko: *Invariants and Evolution of Nonstationary Quantum Systems*, Vol. **183** (Moscow, 1987), Lebedev Institute of Physics and references therein.

takes the form

$$H = \hbar\omega(a_1^\dagger a_1 + a_2^\dagger a_2 + 1) + i\hbar\omega(a_2 a_1^\dagger - a_1 a_2^\dagger), \tag{3.25}$$

i.e. it has the form of a two-mode squeeze operator.

From the above results we conclude that the time evolution of the corresponding annihilation and creation operators of the free electrons in a uniform magnetic field are of the Yuen form and describe squeezed states. Since the corresponding Hamilton operator is a two-mode squeeze operator [14].

In the following we study the case of the free electrons in a uniform magnetic field for the Caldirola Hamiltonian.

4. – Caldirola Hamiltonian for electrons in a uniform magnetic field.

According to Jannussis *et al.* [15] the Caldirola Hamiltonian for the vector potential $\boldsymbol{A}(\boldsymbol{q}, t) \neq 0$ has the following expression:

$$H = \frac{p^2}{2m}\exp[-\gamma t] - \frac{e}{2mc}(\boldsymbol{A}\boldsymbol{p} + \boldsymbol{p}\boldsymbol{A}) + \left[V(\boldsymbol{q}, t) + \frac{e^2}{2mc^2}\boldsymbol{A}^2(\boldsymbol{q}, t)\right]\exp[\gamma t], \tag{4.1}$$

where $V(\boldsymbol{q}, t)$ is the potential energy. For $\boldsymbol{A} = 0$ we obtain the usual Caldirola Hamiltonian [16]

$$H = (p^2/2m)\exp[-\gamma t] + V(q, t)\exp[\gamma t]. \tag{4.2}$$

For the case of the free electrons in a uniform magnetic field, in two-dimensions, with the symmetrical vector potential $\boldsymbol{A} = (-H/2q_2, H/2q_1, 0)$ Hamiltonian (4.1) takes the form

$$H = \frac{p_1^2 + p_2^2}{2m}\exp[-\gamma t] + \omega(p_1 q_2 - q_1 p_2) + \frac{m}{2}\omega^2(q_1^2 + q_2^2)\exp[\gamma t] \tag{4.3}$$

with $\omega = eH/2mc = \omega_L/2$.

For Hamiltonian (4.3) the Heisenberg equations of motion are the

---

[14] C. CAVES and B. SCHUMAKER: *Phys. Rev. A*, **31**, 3068 (1985); B. SCHUMAKER: *Phys. Rep.*, **135**, 317 (1986).

[15] A. JANNUSSIS, G. BRODIMAS, V. PAPATHEOU, G. KARAYANNIS, P. PANAGOPOULOS and H. IOANNIDOU: *Hadronic J.*, **6**, 1434 (1989).

[16] P. CALDIROLA: *Hadronic J.*, **6**, 1400 (1983).

following:

$$\text{(4.4)} \qquad \mathrm{d}q_1/\mathrm{d}t = \frac{p_1}{m}\exp[-\gamma t] + \omega q_2, \qquad \mathrm{d}q_2/\mathrm{d}t = \frac{p_2}{m}\exp[-\gamma t] - \omega q_1,$$

$$\text{(4.5)} \qquad \mathrm{d}p_1/\mathrm{d}t = \omega p_2 - m\omega^2 q_1 \exp[\gamma t], \qquad \mathrm{d}p_2/\mathrm{d}t = -\omega p_1 - m\omega^2 q_2 \exp[\gamma t].$$

The above equations with the help of the contact transformation

$$\text{(4.6)} \qquad \begin{cases} q_1 = \exp\left[-\frac{\gamma}{2}t\right]Q_1, & q_2 = \exp\left[-\frac{\gamma}{2}t\right]Q_2, \\ p_1 = \exp\left[\frac{\gamma}{2}t\right]P_1, & p_2 = \exp\left[\frac{\gamma}{2}t\right]P_2 \end{cases}$$

take the form

$$\text{(4.7)} \qquad \mathrm{d}Q_1/\mathrm{d}t = P_1/m + \omega Q_2 + \frac{\gamma}{2}Q_1, \qquad \mathrm{d}Q_2/\mathrm{d}t = P_2/m - \omega Q_1 + \frac{\gamma}{2}Q_2,$$

$$\text{(4.8)} \qquad \mathrm{d}P_1/\mathrm{d}t = \omega P_2 - m\omega^2 Q_1 - \frac{\gamma}{2}P_1, \qquad \mathrm{d}P_2/\mathrm{d}t = -\omega_1 P_1 - m\omega^2 Q_2 - \frac{\gamma}{2}P_2,$$

which are exactly eqs. (3.1)-(3.4) with $\omega_L = 2\omega$ and they follow from the new Hamiltonian

$$\text{(4.9)} \qquad H = \frac{1}{2m}\left[\boldsymbol{P} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{R})\right]^2 + \frac{\gamma}{4}(\boldsymbol{PR} + \boldsymbol{RP}),$$

where

$$\boldsymbol{R} = \boldsymbol{R}(Q_1, Q_2).$$

Hamiltonian (4.3) in the coordinate representation takes the form

$$\text{(4.10)} \qquad H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial q_1^2} + \frac{\partial^2}{\partial q_2^2}\right)\exp[-\gamma t] - \\ - i\hbar\omega\left(q_2\frac{\partial}{\partial q_1} - q_1\frac{\partial}{\partial q_2}\right) + \frac{m}{2}\omega^2(q_1^2 + q_2^2)\exp[\gamma t].$$

By the use of the contact transformation

$$\text{(4.11)} \qquad Q_1 = \exp\left[\frac{\gamma}{2}t\right]q_1, \qquad Q_2 = \exp\left[\frac{\gamma}{2}t\right]q_2$$

the time-dependent Schrödinger equation

$$\text{(4.12)} \qquad i\hbar\frac{\partial \Psi}{\partial t} = H\,\Psi(q_1, q_2, t),$$

because of the correspondence

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \frac{1}{2}\left(\frac{\partial Q_1}{\partial t}\frac{\partial}{\partial Q_1} + \frac{\partial}{\partial Q_1}\frac{\partial Q_1}{\partial t} + \frac{\partial Q_2}{\partial t}\frac{\partial}{\partial Q_2} + \frac{\partial}{\partial Q_2}\frac{\partial Q_2}{\partial t}\right), \tag{4.13}$$

takes the form:

$$i\hbar\frac{\partial \Psi}{\partial t} = H\,\Psi(Q_1, Q_2, t) - \tag{4.14}$$

$$- \frac{i\hbar\gamma}{4}\left(Q_1\frac{\partial}{\partial Q_1} + \frac{\partial}{\partial Q_1}Q_1 + Q_2\frac{\partial}{\partial Q_2} + \frac{\partial}{\partial Q_2}Q_2\right)\Psi(Q_1, Q_2, t),$$

or

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m}\left(\boldsymbol{P} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{R})\right)^2 + \frac{\gamma}{4}(\boldsymbol{PR} + \boldsymbol{RP})\right]\Psi(\boldsymbol{R}, t), \tag{4.15}$$

i.e. we obtain the new Hamiltonian (4.9) which is equivalent to (4.3). Therefore, Hamiltonian (4.9) describes also squeezed states.

A more general case for time-dependent magnetic field is referred in a recent paper by Abdalla ([17]) which studies the Hamiltonian

$$H(t) = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}\Omega^2(t)(q_1^2 + q_2^2) - \frac{\lambda(t)}{2}(q_1 p_2 - p_1 q_2) \tag{4.16}$$

with

$$\Omega(t) = \sqrt{\omega^2(t) + \lambda^2(t)/4}\,. \tag{4.17}$$

In the following we will determine the time evolution of the corresponding creation and annihilation operators.

5. – Creation and annihilation operators for Hamiltonian (4.16).

According to Abdalla ([17]) the solution of the Heisenberg equations of motion is the following:

$$q_1(t) = r(t)[q_1(0)\cos f(t) + q_2(0)\sin f(t)] + s(t)[p_1(0)\cos f(t) + p_2(0)\sin f(t)], \tag{5.1}$$

$$q_2(t) = r(t)[q_2(0)\cos f(t) - q_1(0)\sin f(t)] + s(t)[p_2(0)\cos f(t) - p_1(0)\sin f(t)], \tag{5.2}$$

([17]) M. SEBAWE ABDALLA: *Nuovo Cimento B*, **101**, 267 (1988) and references therein.

$$(5.3)\quad p_1(t) = v(t)[p_1(0)\cos f(t) + p_2(0)\sin f(t)] + w(t)[q_1(0)\cos f(t) + q_2(0)\sin f(t)],$$

$$(5.4)\quad p_2(t) = v(t)[p_2(0)\cos f(t) - p_1(0)\sin f(t)] + w(t)[q_2(0)\cos f(t) - q_1(0)\sin f(t)],$$

where

$$(5.5)\qquad f(t) = 1/2\int_0^t \lambda(t')\,\mathrm{d}t',$$

$$(5.6)\qquad r(t) = \sqrt{\frac{\mu(0)}{\mu(t)}}\cos\eta(t) + \frac{\dot{\mu}(0)/2\mu(0)}{\sqrt{\mu(t)\mu(0)}}\sin\eta(t),$$

$$(5.7)\qquad s(t) = \frac{\sin\eta(t)}{\sqrt{\mu(t)\mu(0)}},$$

$$(5.8)\qquad v(t) = \sqrt{\frac{\mu(t)}{\mu(0)}}\cos\eta(t) - \frac{\dot{\mu}(t)/2\mu(t)}{\sqrt{\mu(t)\mu(0)}}\sin\eta(t)$$

and

$$(5.9)\quad w(t) = \left[\sqrt{\frac{\mu(t)}{\mu(0)}}\left(\frac{\dot{\mu}(0)}{2\mu(0)}\right) - \sqrt{\frac{\mu(0)}{\mu(t)}}\left(\frac{\dot{\mu}(t)}{2\mu(t)}\right)\right]\cos\eta(t) - \\ - \sqrt{\mu(t)\mu(0)}\left[1 + \frac{(\dot{\mu}(t)/2\mu(t))(\dot{\mu}(0)/2\mu(0))}{\mu(t)\mu(0)}\right]\sin\eta(t),$$

while

$$(5.10)\qquad \eta(t) = \int_0^t \mu(t')\,\mathrm{d}t'$$

and $\mu(t)$ is given by

$$(5.11)\qquad \mu(t) = 1/\rho^2(t), \qquad \ddot{\rho}(t) + \Omega^2(t)\rho(t) = 1/\rho^3(t).$$

By introduction of the corresponding creation and annihilation operators, *i.e.*

$$(5.12)\quad q_1(t) = \sqrt{\frac{\hbar}{2\Omega(t)}}[a_1(t) + a_1^\dagger(t)], \qquad p_1(t) = -i\sqrt{\frac{\hbar\Omega(t)}{2}}[a_1(t) - a_1^\dagger(t)],$$

$$(5.13)\quad q_2(t) = \sqrt{\frac{\hbar}{2\Omega(t)}}[a_2(t) + a_2^\dagger(t)], \qquad p_2(t) = -i\sqrt{\frac{\hbar\Omega(t)}{2}}[a_2(t) - a_2^\dagger(t)],$$

and after some minor algebra we obtain the following expressions for the above operators:

$$a_1(t) = F_1(t)[a_1(0)\cos f(t) + a_2(0)\sin f(t)] + \\ + F_2(t)[a_1^\dagger(0)\cos f(t) + a_2^\dagger(0)\sin f(t)], \tag{5.14}$$

$$a_1^\dagger(t) = F_1^*(t)[a_1^\dagger(0)\cos f(t) + a_2^\dagger(0)\sin f(t)] + \\ + F_2^*(t)[a_1(0)\cos f(t) + a_2(0)\sin f(t)], \tag{5.15}$$

$$a_2(t) = F_1(t)[a_2(0)\cos f(t) - a_1(0)\sin f(t)] + \\ + F_2(t)[a_2^\dagger(0)\cos f(t) - a_1^\dagger(0)\sin f(t)], \tag{5.16}$$

$$a_2^\dagger(t) = F_1^*(t)[a_2^\dagger(0)\cos f(t) - a_1^\dagger(0)\sin f(t)] + \\ + F_2^*(t)[a_2(0)\cos f(t) - a_1(0)\sin f(t)], \tag{5.17}$$

where

$$F_1(t) = (1/2)[r(t) - i\Omega(t)\,s(t) + v(t) + iw(t)/\Omega(t)], \tag{5.18}$$

$$F_2(t) = (1/2)[r(t) + i\Omega(t)\,s(t) - v(t) + iw(t)/\Omega(t)]. \tag{5.19}$$

The above operators satisfy the commutation relations:

$$\begin{cases} [a_j(t), a_j^\dagger(t)] = 1, & j = 1, 2, \\ [a_j(t), a_k^\dagger(t)] = 0, & \text{for } j \neq k. \end{cases} \tag{5.20}$$

If we introduce the linear transformation

$$\begin{cases} A_1(t) = a_1(0)\cos f(t) + a_2(0)\sin f(t), \\ A_1^\dagger(t) = a_1^\dagger(0)\cos f(t) + a_2^\dagger(0)\sin f(t), \end{cases} \tag{5.21}$$

with

$$[A_1, A_1^\dagger] = 1,$$

operators (5.14) and (5.15) take the form

$$a_1(t) = F_1(t)A_1 + F_2(t)A_1^\dagger, \qquad a_1^\dagger(t) = F_1^*(t)A_1^\dagger + F_2^*(t)A_1 \tag{5.22}$$

and it is easy to show that the functions $F_1(t)$ and $F_2(t)$ satisfy the relation

$$|F_1(t)|^2 - |F_2(t)|^2 = 1\,. \tag{5.23}$$

The same is valid for the other pair of operators $a_2(t)$, $a_2^\dagger(t)$, *i.e.*

$$a_2(t) = F_1(t)A_2 + F_2(t)A_2^\dagger\,, \qquad a_2^\dagger(t) = F_1^*(t)A_2^\dagger + F_2^*(t)A_2 \tag{5.24}$$

with

$$[A_2, A_2^\dagger] = 1\,.$$

From the above results we conclude that the operators $a_j(t)$, $a_j^\dagger(t)$, $j = 1, 2$ are of the Yuen form and describe squeezed states.

6. – Conclusion.

In the present paper the prototype squeeze operator (1.2) is studied in phase space.

We find the corresponding Wigner operator which is exactly a prototype two-mode operator and describes squeezed states in phase space.

The extended prototype operator (1.3), for the case of an electron in a uniform magnetic field, describes also squeezed states. The case $\gamma = 0$, *i.e.* the case of the free electron in a uniform magnetic field is very interesting because it is proved that we have squeezed states in this case.

Also, the case of the free electrons in uniform magnetic field for the Caldirola Hamiltonian is studied. This Hamiltonian describes squeezed states too.

Finally, for the general case of time-varying magnetic field, the time evolution of the corresponding creation and annihilation operators leads to Yuen operators and therefore to squeezed states.

● RIASSUNTO (*)

In questo lavoro si studiano varie proprietà dell'operatore squeeze prototipo in un campo magnetico uniforme in due dimensioni. Gli operatori di creazione e di annichilazione corrispondenti sono della forma di Yen e descrivono stati squeezed. Il caso dell'elettrone libero in un campo magnetico che varia nel tempo porta anche a stati squeezed.

(*) *Traduzione a cura della Redazione.*

Сжатые состояния в присутствии зависящего от времени магнитного поля.

Резюме (*). — В этой статье мы исследуем различные свойства прототипа оператора сжатия в однородном магнитном поле в двух измерениях. Соответствующие операторы рождения и уничтожения представляют операторы Юена и описывают сжатые состояний. Случай свободного электрона в зависящем от времени магнитном поле также приводит к сжатым состояниям.

(*) *Переведено редакцией.*