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Relativistic Thermodynamics: Temperature Transformations, Invariance and Measurement.

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Summary. — Analysis of the physical foundations of the Planck and Ott relativistic temperature transformations shows that nearly all derivations have been based on dynamic physical processes. Different processes lead to different results, and we conclude that there is no universally general temperature transformation. The rule of temperature invariance proposed by Landsberg and developed by Cavalleri and Salgarelli is discussed. For the purely kinematic case in which the temperature of a single body is compared by measurement in two inertial frames, we show that the Ott rule holds. A new derivation of it is given which is independent of any material assumptions and which is based on two Lorentz invariants—entropy and radiated power. The significance of this new derivation is compared with the role of the Landsberg rule in experimental measurement.

1. – Introduction.

Within two years of the publication of Einstein's first paper on the special theory of relativity (1), MOSENGEIL (2), a student of Planck, had applied the results to radiation from a moving cavity. A year later a paper by PLANCK (3) appeared, which for fifty-five years was held to be the definitive statement on

(1) A. EINSTEIN: *Ann. der Phys.*, **17**, 891 (1905).
(2) K. VON MOSENGEIL: *Ann. der Phys.*, **22**, 867 (1907).
(3) M. PLANCK: *Ann. der Phys.*, **26**, 1 (1908).

relativistic transformations of thermodynamic quantities. In his paper Planck applied the principle of least action which, as he said, encompasses mechanics, electrodynamics and the first two laws of thermodynamics. He demonstrated the Lorentz invariance of entropy as measured in various inertial systems and arrived at a relativistic transformation for temperature.

Planck considered two inertial systems, K and K', such that K' moves with velocity v in the positive x-direction with respect to K. Using a black-body cavity at rest in K (hence moving in K' with velocity $-v$), Planck showed that its temperature T' in K' is related to its temperature T in K as

$$T' = T\,(1 - v^2/c^2)^{\frac{1}{2}}\,, \tag{1}$$

where c is the velocity of light.

Planck's temperature relation and his other thermodynamic transformations remained unchallenged until 1963 when Ott [4] in a posthumous paper asserted that Planck had introduced nonphysical forces in calculating the mechanical work done on a thermodynamic system. Ott's temperature relation is just the inverse of Planck's, namely

$$T' = T/(1 - v^2/c)^{\frac{1}{2}}\,. \tag{2}$$

While Planck had concluded that the K' observer saw the body as colder than did the K observer, Ott's calculations indicated that the K' observer saw it as warmer. Møller [5] has written a detailed account of the Ott point of view with many references to recent work.

There is a curious, almost solipsistic quality to papers supporting one school or the other. While each acknowledges the existence of the other, there seems to be little effort to show the flaw in the other's argument.

Given two such contradictory solutions to what is seemingly the same question, we can approach the problem in a different way. This is to determine if the Planck and Ott transformations are truly solutions to the same physical problem. In a recent paper, Treder [6] makes the important point that Einstein, in discussing the comparison of physical data between two relatively moving systems, always started from the viewpoint that information exchange occurs by means of radiation, specifically electromagnetic waves. Examining the different formulations with this as our touchstone, we conclude that the transformations obtained depend markedly on the physical processes postulated. The result is that we cannot claim correctness for one or the other; each has restricted validity.

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[4] H. Ott: *Zeits. Phys.*, **175**, 70 (1963).
[5] C. Møller: *Old and New Problems in Elementary Particles*, edited by G. Puppi (New York, N. Y., and London, 1968), p. 202.
[6] H. J. Treder: *Ann. der Phys.*, **34**, 23 (1977).

This conclusion raises a further question, however. Is there a way of defining temperature which leads to a completely general transformation law? Our conclusion is that such a law does not exist. There is, though, a new importance for an ideal black-body which allows us to give a well-defined prescription for a large class of temperature measurements in different inertial frames. This point will be discussed in sect. **5**.

2. – Physical bases of the Planck and Ott transformations.

There are many different derivations of the Planck and Ott transformations. Almost without exception they make no distinction between kinematic and dynamic processes, treating them interchangeably. This confusion between kinematic and dynamic processes is quite similar to that found in many discussions of the Ehrenfest paradox, a point to which we will return.

Consider a physical entity $\mathscr{A}$—it can be temperature, length, mass, volume—whatever one wants—as long as it is allowable of quantitative measurement. To carry out our measurements, we introduce the two inertial frames K and K' described in the introduction. Let the body with which $\mathscr{A}$ is associated be at rest in K and, therefore, moving with velocity $-v$ in K'. Further let an observer at rest in K measure $\mathscr{A}$. His instruments—rods, clocks, meters—are all at rest in K. He finds the value A for the entity $\mathscr{A}$.

There are now two questions one can ask. Each question corresponds to a different physical situation, and the answer to each requires a different experiment. The first of these is what value of $\mathscr{A}$ will an observer at rest in K' (with instruments also at rest in K') measure? This is a purely kinematic question. The two observers in K and K' make their measurements on the same body and—if we neglect any quantum effects—leave the state of the body unchanged by their actions. Our goal is a general transformation which will relate the value A' found by the observer at rest in K' to the value A found in K.

The second question is a dynamic one and involves one frame only, the body's proper frame K. The body is at rest in K and, as before, the observer measures the value A for the entity $\mathscr{A}$. Now set the body into motion so that it moves with velocity v in K. What value for $\mathscr{A}$ will the observer at rest in K now measure? Clearly this question is not identical with the first. He will obtain a value A'' which may or may not equal A'. Setting the body into motion has changed its thermodynamic state. Just how it was done—the forces applied, the constraints on heat flow, the material properties of the body—will influence the value A''. For example, it may be a Born rigid motion as described by NEWBURGH (7) or a motion which introduces stresses and defor-

(7) R. G. NEWBURGH: *Nuovo Cimento*, **23** B, 365 (1974).

mations. Obviously an infinite number of stressed motions is possible. Therefore, unless a complete prescription of the forces and the time schedule are available, one cannot say what the end state of $\mathscr{A}$ and hence the value A'' will be.

All the derivations of temperature transformations I have seen—with one notable exception, that of Treder (6)—involve some change in state of a thermodynamic fluid, a calculation of work done by a piston or strains set up in a container. Several authors, for example TOLMAN (8), discuss the acceleration of an ideal thermodynamic fluid which implies that forces are present. Yet, when one examines their arguments carefully, one is hard put to say how they use the acceleration in calculating the heat transformation. TREDER has succeeded in excising the accelerated fluid from the argument and used radiated energy alone. He based his derivation on the transformation laws of the specific radiation intensities demonstrated by VON LAUE (9) in his relativistic deduction of Wien's displacement law. TREDER coupled these transformation laws with Einstein's theory of the transverse Doppler effect and acceleration. He did not introduce a thermodynamic process to carry a fluid from one equilibrium state to another. His sole physical model was a Planck black-body. Specifically, he made no use of force, work, volume, or pressure transformations as used in most derivations of the Planck and Ott relations.

At first one would conclude that TREDER has argued purely kinematically. However, his radiating body, the temperature of which he wishes to determine, describes a rigid rotation about the observer. At all points of his radiating body there exist a velocity and an acceleration field. There is no rest frame for the radiating body. We conclude that TREDER has not arrived at a transformation valid for two inertial frames.

At the same time the derivations of the Ott relation are equally dynamic, as described, for example, in Møller's paper (5). MØLLER began with a fluid enclosed in a container of changeable volume. He then considered a thermodynamic process which brought the fluid from state I to state II. Work was done on and heat supplied to the fluid during the process. As a result, the internal state of the fluid changed. He further specified that the fluid was at rest in frame K at the start and end of the process though not during the process. As a consequence, momentum, including that corresponding to the inertial mass of energy, figured in the derivation.

MØLLER went further and carried out the gedanken experiment corresponding to our second question. He took his thermodynamic system originally at rest in K and accelerated it adiabatically to a final velocity v and again found the Ott result. His approach is clearly dynamic, involving work, force, pressure and volume transformations. In addition, he introduced extended struc-

(8) R. C. TOLMAN: *Relativity, Thermodynamics, and Cosmology*, sect. **68**, **69** (Oxford, 1934), p. 152-159.
(9) M. VON LAUE: *Ann. der Phys.*, **43**, 220 (1943).

tures with all their concomitant complexities. Extended structures are far from simple, as GRØN [10] has shown in his discussion of the synchronous and asynchronous formulations for equilibrium. Moreover, MØLLER did not consider a further complication, that of Born rigid motions. It is not enough to specify whether the process be adiabatic or not. It is necessary to describe the type of motion and consequent internal strains produced on the extended system—fluid and walls. There is an infinite number of ways to set a body into motion. To calculate the result requires a complete prescription of the forces.

The paper on Born rigid motions [7], referred to earlier, examined a similar confusion in the formulation of the rotating disk or Ehrenfest paradox. There the concern was the measure of the ratio of disk circumference to radius. Again there are two questions one may ask. The first is what are the values of this ratio for an observer at rest in the center and for one co-moving with the rim? The second is how does this ratio change as the stationary disk is set rotating? It is just this confusion between kinematics and dynamics which has led to so much strident argumentation in the special theory.

3. – Lorentz invariance of temperature.

In addition to the Planck and Ott views, there is still a third approach to the question which takes the temperature to be a Lorentz invariant. First proposed by LANDSBERG [11], the same view has been developed in papers by LANDSBERG and JOHNS [12], VAN KAMPEN [13], and CAVALLERI and SALGARELLI [14]. These papers are not in complete agreement, though all do discuss Lorentz invariance of temperature. That of Cavalleri and Salgarelli established the broadest general base for the argument of invariance and is most significant for its implications in the domain of temperature measurement.

LANDSBERG in his first paper [11] offered an interesting plausibility argument for the Lorentz invariance of temperature. It is that relativistic changes are caused by the nonabsolute concept of time. But, since time is not a variable in reversible thermodynamics, the classical concept of temperature remains unaffected. The paper written with JOHNS [12] generalized the earlier ideas by classifying thermodynamic systems as free, confined and inclusive. Free systems are not restricted to a definite volume as are confined systems. Inclusive systems are confined systems to which are added the energy and momentum arising from stresses in the moving, confining container.

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(10) Ø. GRØN: *Nuovo Cimento*, **17** B, 141 (1973).
(11) P. T. LANDSBERG: *Nature*, **212**, 571 (1966).
(12) P. T. LANDSBERG and K. A. JOHNS: *Nuovo Cimento*, **52** B, 28 (1967).
(13) N. G. VAN KAMPEN: *Phys. Rev.*, **173**, 295 (1968).
(14) G. CAVALLERI and G. SALGARELLI: *Nuovo Cimento*, **62** A, 792 (1969).

Van Kampen [13] also defines temperature as being Lorentz invariant and distinguishes between the four-vector, called the thermal energy-momentum transfer, and the scalar heat supply $\mathrm{d}Q$, invariant in all frames. In this formulation, van Kampen's covariant generalization of the first law is a function of the Minkowski force acting on the system.

However, the clearest exposition of the Lorentz invariance of temperature is that of Cavalleri and Salgarelli [14], who advance an extremely important experimental argument. In the final analysis, temperature is a meaningless concept unless it is measurable. To discuss the transformation of temperature from one frame to another is a sterile exercise if there is no prescription given for its measurement. This prescription Cavalleri and Salgarelli do provide. They write:

« The temperature of a fluid element is defined classically as a quantity proportional to the mean kinetic energy relevant to the mass center of the element considered and not to the kinetic energy relevant to the observer. Consequently, temperature is invariant. That temperature must be calculated in the rest system is clear also operationally since temperature must be measured by a thermometer at rest with the element considered ».

By bringing the act of measurement into the argument, they have removed the scholastic element from relativistic thermodynamics. We shall return to this point when we consider the alternative approach to temperature measurement discussed in the next sections.

They go beyond this and carry out a careful and consequent analysis of the problem in terms of the synchronous and asynchronous formulations of dynamics for extended structures. They point out that supplying heat to a body increases the energy and, therefore, the rest mass. This leads them to rewrite the relation

$$\mathrm{d}Q = T\,\mathrm{d}S \tag{3}$$

as

$$\mathrm{d}Q' = \gamma\,\mathrm{d}Q = T'\,\mathrm{d}S' + (\gamma - 1)\,\mathrm{d}Q = T'\,\mathrm{d}S + (\gamma - 1)c^2\,\mathrm{d}m \tag{4}$$

in the *asynchronous* formulation with T' always equal to T. Here Q is heat, S entropy and γ is $(1 - v^2/c^2)^{-\frac{1}{2}}$. In the *synchronous* formulation, they point out that (4) holds for point bodies only. For extended bodies it could be $\mathrm{d}Q' = f(\beta^2)\,\mathrm{d}Q$, where $f(\beta^2)$ is a generic function of β^2.

Again it should be noted that they are discussing a dynamic rather than a kinematic situation with an exact description of the processes in the two frames.

4. – Lorentz invariance of radiated power and the Ott transformation of temperature.

The discussion of the previous sections indicates most clearly that there is no simple answer to the question « how does temperature transform? ».

However, if, following TREDER and EINSTEIN, we argue that information exchange between relatively moving systems occurs by means of electromagnetic radiation and consider inertial systems only, we can obtain the Ott transformation in a remarkably simple manner. Though closely related to Treder's approach, it makes a sharp break with it by eliminating any rotations. The derivation makes no assumptions whatsoever about material bodies, uses two Lorentz invariances only and thereby shows the manifestly kinematic nature of the Ott relation. The two invariances are those of entropy and of radiated power. That of entropy was first shown by PLANCK ([3]). The invariance of radiated power is a powerful tool which is not so well known as should be. PAULI ([15]) applied it to the discussion of radiation from a moving dipole. SCHWINGER ([16]) used it in obtaining the intensity relations for synchrotron radiation. NEWBURGH ([17]) found it useful in the problem of radiation reaction. The invariance is stated simply. A body emitting energy isotropically at the rate P, in its inertial rest system, will radiate at the same rate in any other inertial frame. If we consider systems with relative motion along a given direction, it is sufficient that there be axial symmetry around such a direction in the rest frame.

Consider a body at rest in K so that energy is emitted *isotropically* at the rate

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = P\,. \tag{5}$$

An observer in K' (as defined in the introduction) measures the same rate

$$P = \frac{\mathrm{d}Q'}{\mathrm{d}t'}\,, \tag{6}$$

so that

$$\mathrm{d}Q' = \frac{\mathrm{d}t'}{\mathrm{d}t}\,\mathrm{d}Q\,. \tag{7}$$

(This assumes that the K' observer can measure the *total* emitted power or, at the least, infer it from an axially symmetric measurement.) The time transformation

$$t' = \gamma(t - xv/c^2) \tag{8}$$

gives

$$\mathrm{d}t' = \gamma(\mathrm{d}t - \mathrm{d}xv/c^2)\,.$$

However, since $\mathrm{d}x$ is zero because K is the proper frame of the body, time

([15]) W. PAULI: *Theory of Relativity*, subsect. **32**(e) (London, 1958), p. 98.
([16]) J. SCHWINGER: *Phys. Rev.*, **75**, 1912 (1946).
([17]) R. G. NEWBURGH: *Amer. Journ. Phys.*, **36**, 399 (1968).

dilatation holds and

$$dt' = \gamma\, dt\,. \tag{9}$$

Therefore,

$$dQ' = \gamma\, dQ = dQ(1 - v^2/c^2)^{-\frac{1}{2}}\,, \tag{10}$$

which is the Ott heat transformation. Entropy invariance states that

$$\frac{dQ}{T} = \frac{dQ'}{T'}\,, \tag{11}$$

which combined with eq. (7) gives the Ott temperature transformation

$$T' = T/(1 - v^2/c^2)^{\frac{1}{2}}\,. \tag{12}$$

This derivation is the most general and simple I have found. It makes no assumption about the material nature of bodies. It is, as stated before, manifestly kinematic and makes no predictions as to the consequences of setting bodies in motion.

5. – Discussion.

The previous sections have demonstrated clearly how the temperature transformation obtained is a function of the physical processes applied. Nearly all derivations of the Planck and Ott relations as well as those using Lorentz invariance involve material bodies (and, therefore, require knowledge of their physical properties), applied forces and changes of physical state. The fullest treatment of Lorentz invariance, that of Cavalleri and Salgarelli, distinguishes carefully between mass points and extended bodies and discusses changes of state in terms of the asynchronous formulation of dynamics. In contrast, the derivation of the Planck relation by TREDER and that by OTT given in the previous section require a photon gas only. Unlike the other derivations, they postulate no thermodynamic process, no change of state. Moreover, the use of a photon gas, a gas with zero rest mass, allows us to avoid deciding between the synchronous and asynchronous formulations. The Treder derivation is still dynamic, however, requiring as it does a rigid rotation of the radiating body about the observer. The result is that the transformation does not compare temperature measurements of a single unaccelerated body in two inertial frames. In contrast, the Ott derivation of sect. **4** is manifestly kinematic.

Having established this difference, we ask now under what conditions is each transformation valid. It is not enough to say that a body, originally at rest in an inertial frame, is set into motion. We must have a full prescription of the forces acting and knowledge of the properties of the body. As shown by CAVALLERI and SALGARELLI, the dynamics of forces acting on extended bodies must be discussed within a consistent framework—be it synchronous or asychronous. Although our derivation of the Ott rule is far simpler, a *sine* *qua non* for its application is the existence of a radiating system. This brings us to the second main point of the paper, the measurement of temperature.

In spite of the extensive literature on the subject of relativistic temperature transformations, LANDSBERG (11,12) and CAVALLERI and SALGARELLI (14) are alone in discussing actual temperature measurement. From their viewpoint, temperature must be calculated in the proper frame of the body since its measurement demands a thermometer at rest with respect to the body. As they themselves point out, the asynchronous formulation does make an observer at rest in the proper frame somewhat privileged, a fact which is true for the proper thermometer. However, such a prescription imposes a serious limitation on our ability to measure. How can we measure the temperature of an object either placed at such a large distance from us or moving so fast with respect to us that we cannot juxtapose a thermometer at rest with respect to the body? The constraint this places on extraterrestrial investigations is evident. I believe that the alternative suggested by the power invariance used in the derivation of the Ott relation offers a solution to the problem.

Temperature is an equilibrium concept, a concept that is preserved in a kinematic description, since equilibrium is, of necessity, a Lorentz invariant. Consider a body B at rest in a frame K in thermal equilibrium with a black-body also at rest in K. If the black-body is at temperature T—which an observer at rest in K can determine by measuring the properties of its radiation—the body B is also at temperature T. This is, of course, completely classical. However, now consider a second frame K' as defined in sect. **2**. An observer at rest in K' can measure the power radiated from the black-body by using a radiometer at rest in K'. (There is now no thermometer at rest with respect to the black-body.) He will conclude from his power measurement that the black-body and, therefore, body B has a temperature T' in K'. By applying the Ott rule, he can then infer the temperature T in K.

This approach can be generalized with some limitations. All bodies with temperatures greater than absolute zero radiate. Therefore, the measurement of the power radiated from a body gives us an equivalent black-body temperature. We relate this equivalent temperature in K' to that in K through the Ott rule. This is a well-defined prescription, operationally clear, which replaces a « thermometer » at rest in the proper frame K with a « power meter » or radiometer at rest in an arbitrary inertial frame K'. This is obviously more in harmony with the principle of relativity. In a sense, we are using a black-body as an ideal thermometer. Indeed the Ott transformation and the Planck black-body law are bound up inextricably.

It is clear that we do not have a universally « correct » law for relativistic temperature transformations. The distinction between kinematic and dynamic measurements is often overlooked, as is the confusion between synchronous and asynchronous formulations. The road to the Ott transformation given in this paper does introduce time into thermodynamics through the use of power, which contradicts Landsberg's plausibility argument. However, all three of the rules discussed here involve the proper frame of the body, for even the kinematic Ott rule goes back to the Planck black-body distribution in a proper frame. This fact precludes a truly relativistic temperature transformation, though, through the Ott rule, our actual measurements are not restricted to the body's proper frame. Ott's legacy is that we have been forced to re-examine the physical foundations of relativistic thermodynamics and to recognize once again the necessity for a complete description of the experiments by which we measure.

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I must thank Prof. G. Cavalleri for his careful reading of the paper. His comments prevented me from making an egregious error and led me to consider the problem of actual measurement.

● RIASSUNTO (*)

Un'analisi dei fondamenti fisici delle trasformazioni relativistiche di temperatura di Plank e Ott mostra che quasi tutte le derivazioni sono basate su processi fisici dinamici. Processi diversi portano a risultati diversi, e si conclude che non c'è nessuna trasformazione di temperatura universalmente valida. Si discute il ruolo dell'invarianza della temperatura proposto da Landsberg e sviluppato da Cavalleri e Salgarelli. Per il caso puramente cinematico in cui la temperatura di un singolo corpo è confrontata mediante la misurazione in due sistemi inerziali, si mostra che vale la regola di Ott. Si fornisce una nuova derivazione di questa, che è indipendente da qualsiasi dato materiale e che è basata su due invarianti di Lorentz — l'entropia e la potenza irradiata. Si confronta il significato di questa nuova derivazione con il ruolo della regola di Landsberg nelle misurazioni sperimentali.

(*) *Traduzione a cura della Redazione.*

Релятивистская термодинамика: Преобразования температуры, инвариантность и измерения.

Резюме (*). — Анализ физических основ релятивистских преобразований температуры Планка и Отта показывает, что почти все выводы этих преобразований основаны на динамических физических процессах. Различные процессы приводят к различным результатам. Мы утверждаем, что не существует универсального общего преобразования температуры. Обсуждается правило инвариантности температуры, предложенное Ландсбергом и развитое Каваллери и Салгарелли. Для чисто кинематического случая, в котором температура отдельного тела сравнивается с измерениями в двух инерциальных системах отсчета, мы показываем, что справедливо правило Отта. Предлагается новый вывод этого правила, который не зависит от предположений о свойствах вещества и который основан на двух Лоренц-инвариантах — энтропии и мощности излучения. Значимость нового вывода сравнивается с ролью правила Ландсберга в экспериментальных измерениях.

(*) *Переведено редакцией.*