IL NUOVO CIMENTO VOL. 12 A, N. 3 1 Dicembre 1972

Evaluation of the Massive Superpropagator in the f-Meson–Graviton Mixing Model.

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(ricevuto il 16 Maggio 1972)

Summary. — Working within the framework of the strong gravity theory using nonpolynomial Lagrangians, we have investigated the *massive* superpropagator for a mixed *tensor* field consisting of Einstein's massless gravity field and the strong gravity field of the massive f-meson. The final compact expression for the massive superpropagator in Euclidean co-ordinate space has the form of a one-dimensional integral characterized by poles and branch cuts. A similar integral representation has been derived for the « pure » f-meson superpropagator. The reality of both integrals is guaranteed by an averaging prescription. Numerical calculations of the massive superpropagator have been carried out both in Euclidean x-space and, for the corresponding Fourier transform, in the Symanzik region of the external momenta.

1. – Introduction.

One of the most intractable problems in theoretical physics during the past four decades has been the subject of the ultraviolet infinities in the theory of interacting particle fields. Several ingenious methods have been developed to cope with these infinities, such as the successful renormalization procedure in

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(**) Supported in part by the National Research Council of Canada under Grant No. A8063 and C1006.

quantum electrodynamics (¹), or the peratization approach of FEINBERG and PAIS (²) in the theory of weak interactions. Yet it is fair to say that no individual technique has been sufficiently powerful to deal consistently and in a physically meaningful manner with the divergences arising in all three basic interactions: strong, electromagnetic and weak. What is crucial in the context of this article is the fact that the only Lagrangians seriously considered until recently have been *polynomial* Lagrangians. The rather limited success of polynomial Lagrangians in renormalization theory has prompted EFIMOV (³), FRADKIN (⁴) and others to employ *nonpolynomial* Lagrangians as a means of damping the ultraviolet divergences. In the case of a nonpolynomial theory, this damping is achieved specifically by a sum of distributions of the form $(\Delta_F(x^2, m))^n$, called superpropagators, which describe the propagation of N particles, $N = 0, 1, 2, \ldots, \infty$, of mass m between two points in space-time ($\Delta_F(x^2, m)$ being the causal free massive propagator). The use of superpropagators is equivalent to taking *all* terms in the perturbative expansion. Superpropagators will be discussed more fully near the end of this Section.

The nonpolynomiality mentioned above may be introduced in a variety of ways. One prescription, due to SALAM and his collaborators (⁵,⁶), is to make the Lagrangian generally covariant. This introduces in the Lagrangian factors of the form $(-\det g^{\mu\nu})^{-\frac{1}{2}}$, where

$$g^{\mu\nu} \equiv \eta^{\mu\nu} + \varkappa_g h^{\mu\nu}$$

is the Einstein gravitational field, $\eta^{\mu\nu}$ is the Minkowski metric defined by diag $(1, -1, -1, -1)$ and $h^{\mu\nu}$ describes the deviation from $\eta^{\mu\nu}$ and contains all the physics (*). The nonpolynomial character originates from the nonlinearity of $\det g^{\mu\nu}$ in the field variables $h^{\mu\nu}$ and requires a nonperturbative treatment of the modified Lagrangian. It is an appealing feature of Salam's prescription that the inbuilt cut-off $\varkappa_g^{-1} = 10^{19}$ GeV enters the theory in a natural way through the gravitational field $g^{\mu\nu}$. In quantum electrodynamics this cut-off regular-

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(¹) F. J. DYSON: *Phys. Rev.*, **75**, 486, 1736 (1949); A. SALAM: *Phys. Rev.*, **82**, 217 (1951); **84**, 426 (1951).

(²) G. FEINBERG and A. PAIS: *Phys. Rev.*, **131**, 2724 (1963).

(³) G. V. EFIMOV: *Sov. Phys. JETP*, **17**, 1417 (1963); *Phys. Lett.*, **4**, 314 (1963); *Nuovo Cimento*, **32**, 1046 (1964); *Nucl. Phys.*, **74**, 657 (1965).

(⁴) E. S. FRADKIN: *Nucl. Phys.*, **49**, 624 (1963); **76**, 588 (1966).

(⁵) R. DELBOURGO, A. SALAM and J. STRATHDEE: *Lett. Nuovo Cimento*, **2**, 354 (1969).

(⁶) A. SALAM and J. STRATHDEE: *Lett. Nuovo Cimento*, **4**, 101 (1970).

(*) We shall employ natural units, $\hbar = c = 1$, throughout this paper, in which case the gravitational constant $\varkappa_g = (8\pi G)^{\frac{1}{2}} \simeq 10^{-22}\,(m_e)^{-1} \simeq 10^{-19}\,(\text{GeV})^{-1}$, where G is the Newtonian constant and m_e the mass of the electron.

izes the logarithmic infinities which appear in the self-mass and self-charge of the electron, as shown by ISHAM, SALAM and STRATHDEE (I.S.S.) ([7]). Non-polynomial Lagrangians have of late been discussed in several papers, and we refer the reader to the literature on this subject ([8,9]).

Of immediate interest to us is another article by I.S.S. on the f-dominance of gravity ([10]) in which they describe the mixing of the Einstein graviton field $g^{\mu\nu}$ with the massive spin-2^+ f-meson field $f^{\mu\nu}$. The presence of the strong gravity field $f^{\mu\nu}$ leads to a strong-interaction cut-off $\varkappa_f^{-1} \approx$ a few GeV ([*]), which is seen to be appreciably smaller than the weak gravitational cut-off $\varkappa_g^{-1} = 10^{19}$ GeV. The f-meson in this f-g mixing model is taken to be the «representative» of the (f^0, $f^{0'}$, A_2^0) complex and is shown to couple universally to the hadronic stress tensor ([10,11]).

The purpose of this article is to evaluate, within the framework of the f-g mixing hypothesis, the *massive* superpropagator for a mixed tensor field consisting of Einstein's massless gravity field $g^{\mu\nu}$ and the massive strong gravity field $f^{\mu\nu}$. We shall now briefly describe some of the work that has been carried out on superpropagators.

The *massless* graviton superpropagator was first evaluated by DELBOURGO and HUNT ([12]) in both configuration and momentum space, and subsequently by I.S.S. ([7]) using vierbein (tetrad) fields and the complex z-space approach. Compared with the massless case, progress for the *massive* superpropagator has been rather slow, owing largely to the difficulties connected with solving multiple four-dimensional integrals. Nevertheless a number of definite results have been published, for example, by VOLKOV ([13]), who has studied the analytic structure of the superpropagator in the coupling constant for massive *scalar* particles, and by KAROWSKI ([14]), who investigated the mathematical properties of a few specific superpropagators. Although SALAM and STRATHDEE ([9]) have derived some useful rules for the momentum-space behaviour of these complicated objects, an exact solution for the massive superpropagator in momentum

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([7]) C. J. ISHAM, A. SALAM and J. STRATHDEE: *Phys. Rev. D*, **3**, 1805 (1971).

([8]) R. DELBOURGO, A. SALAM and J. STRATHDEE: *Phys. Rev.*, **187**, 1999 (1969); A. SALAM: in *Fundamental Interactions at High Energy, Proceedings of the 1970 Coral Gables Conference* (New York, 1970), p. 221.

([9]) A. SALAM and J. STRATHDEE: *Phys. Rev. D*, **1**, 3296 (1970).

([10]) C. J. ISHAM, A. SALAM and J. STRATHDEE: *Phys. Rev. D*, **3**, 867 (1971).

([*]) We shall take the mass of the f-meson $M = 1500$ MeV so that its coupling constant is approximately $\varkappa_f = M^{-1} = (1.5\ \text{GeV})^{-1}$. We thank Dr. P. ROTELLI for clarifying remarks about the mass of the f-meson.

([11]) S. N. GUPTA: *Phys. Rev.*, **96**, 1683 (1954).

([12]) R. DELBOURGO and A. P. HUNT: *Lett. Nuovo Cimento*, **4**, 1010 (1970).

([13]) M. K. VOLKOV: *Comm. Math. Phys.*, **7**, 289 (1969); **15**, 69 (1969); *Ann. of Phys.*, **49**, 202 (1968).

([14]) M. KAROWSKI: *Comm. Math. Phys.*, **19**, 289 (1970).

space has not been found as yet (*). The prime obstacle has been the lack of a suitable Fourier transform [15] for

$$(\Delta_F(x^2, m))^n .$$

The outline of our paper is as follows. In Sect. **2** we review briefly the central features of the I.S.S. f-g mixing model. We then eliminate the interaction term $\mathscr{L}_{fg}$ by diagonalizing the total Lagrangian $\mathscr{L}_{tot}$. The purpose of this diagonalization procedure is to simplify the vacuum expectation values occurring in the integral representation of the superpropagator $S(x)$.

In Sect. **3** we shall derive a compact expression for $S(x)$ in Euclidean coordinate space. In Sect. **4** we evaluate the Fourier transform of $S(x)$ in the Symanzik region of the external momenta ($p^2 < 0$). Sect. **5** consists of a brief summary and discussion.

2. – On the f-meson–graviton mixing hypothesis.

In the first part of this Section, we shall summarize the salient features of the I.S.S. [10] theory on the f-dominance of gravity. We shall begin by listing the Lagrangians for the graviton and the f-meson, together with a mixing term $\mathscr{L}_{fg}$ describing the interaction between leptonic matter, to which the graviton couples, and hadronic matter. In the second part of this Section, beginning with (2.10), we shall define appropriate fields which diagonalize the total Lagrangian. This will simplify subsequent calculations of the massive superpropagator.

For the pure gravity Lagrangian, we take the usual Einstein Lagrangian with the second derivatives of the metric tensor $g^{\mu\nu}$ removed:

$$\mathscr{L}_g = -\frac{1}{\varkappa_g^2}(-g)^{-\frac{1}{2}} g^{\alpha\beta}\{\Gamma^\gamma_{\alpha\delta}\Gamma^\delta_{\gamma\beta} - \Gamma^\gamma_{\alpha\beta}\Gamma^\delta_{\gamma\delta}\} , \tag{2.1}$$

where $\varkappa_g$ is the weak gravitational constant and $g \equiv \det g^{\mu\nu}$. The Christoffel symbol of the second kind, $\Gamma^\alpha_{\nu\mu}$, may be expressed in terms of the metric tensor by

$$\Gamma^\alpha_{\mu\nu} = \tfrac{1}{2} g^{\alpha\beta}\{(g^{-1})_{\beta\mu,\nu} + (g^{-1})_{\beta\nu,\mu} - (g^{-1})_{\mu\nu,\beta}\} \tag{2.2}$$

(*) The method of generalized functions employed in finding the Fourier transform of the massless superpropagator $(D_F(x))^n$ fails in the *massive* case, where one needs to know the Fourier transform of $(K_1(mr)/r)^n$; here K_1 is the modified Bessel function, m the mass and $r = \sqrt{x^2 + x_4^2}$.

([15]) I. M. GEL'FAND and G. E. SHILOV: *Generalized Functions*, Vol. **1** (New York, 1964).

with a comma denoting the ordinary derivative. We have taken $g^{\mu\nu}$ as the graviton field; the covariant tensor $(g^{-1})_{\mu\nu}$ is dependent on it and satisfies

$$g^{\mu\alpha}(g^{-1})_{\alpha\nu} = \delta^\mu_\nu \,, \tag{2.3}$$

where δ^μ_ν is the Kronecker symbol:

$$\delta^\mu_\nu = \begin{cases} 1 \,, & \text{if } \mu = \nu \,, \\ 0 \,, & \text{if } \mu \neq \nu \,, \end{cases} \qquad \mu, \nu = 0, 1, 2, 3 \,.$$

ISHAM, SALAM and STRATHDEE now assume that the pure f-meson part of the total Lagrangian has the same form as (2.1):

$$\mathscr{L}_f = -\frac{1}{\varkappa_f^2}(-f)^{-\frac{1}{2}} f^{\alpha\beta}\{\Gamma^\gamma_{\alpha\delta}\Gamma^\delta_{\gamma\beta} - \Gamma^\gamma_{\alpha\beta}\Gamma^\delta_{\gamma\delta}\} \,, \tag{2.4}$$

where $f \equiv \det f^{\mu\nu}$ and $\Gamma^\alpha_{\mu\nu}$ has the usual definition in terms of $f^{\mu\nu}$ as the metric tensor.

The form of the mixing term is based on the mass term for a spin-two field of mass M:

$$\mathscr{L}_{\text{mass}} = \frac{M^2}{4}(F^{\alpha\beta}F^{\alpha\beta} - F^{\alpha\alpha}F^{\beta\beta}) \,, \tag{2.5}$$

$F^{\alpha\beta}$ being a symmetric, pure spin-two field ([16,17]). A covariant generalization of (2.5), in terms of $f^{\mu\nu}$ and $g^{\mu\nu}$, gives

$$\mathscr{L}_{fg} = \frac{M^2}{4\varkappa_f^2}(-f)^{-\frac{1}{2}}\{f^{\alpha\beta}(g^{-1})_{\alpha\varkappa}f^{\varkappa\lambda}(g^{-1})_{\beta\lambda} - (f^{\alpha\beta}(g^{-1})_{\alpha\beta})^2 + 6f^{\alpha\beta}(g^{-1})_{\alpha\beta} - 12\} \,. \tag{2.6}$$

By writing

$$f^{\mu\nu} = \eta^{\mu\nu} + \varkappa_f F^{\mu\nu} \,, \tag{2.7}$$

$$g^{\mu\nu} = \eta^{\mu\nu} + \varkappa_g h^{\mu\nu} \,, \tag{2.8}$$

and by working to zeroth order in $\varkappa_f$ and $\varkappa_g$, one can show that the expression (2.6) reduces to (2.5).

For purely calculational reasons, we now express the total Lagrangian

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_f + \mathscr{L}_g + \mathscr{L}_{fg} \tag{2.9}$$

([16]) W. PAULI and M. FIERZ: *Proc. Roy. Soc.*, A **173**, 211 (1939).
([17]) M. FIERZ: *Helv. Phys. Acta*, **12**, 3 (1939).

in the form

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_{\tilde{f}} + \mathscr{L}_{\tilde{g}} \; ; \tag{2.10}$$

the new fields $\tilde{f}$ and $\tilde{g}$, defined in terms of the original fields, f and g, must be chosen so as to *diagonalize* the Lagrangian (2.9). We proceed by expanding $\mathscr{L}_{\text{tot}}$ in terms of the quantum (in the I.S.S. sense ([10])) fields $F^{\mu\nu}$ and $h^{\mu\nu}$ introduced respectively in (2.7) and (2.8). Working to first order in $\varkappa_g$ we obtain

$$(g^{-1})_{\mu\nu} = \eta_{\mu\nu} - \varkappa_g h_{\mu\nu} \, , \tag{2.11}$$

in which case the Christoffel symbol (2.2) reduces to (*)

$$\Gamma^{\gamma}_{\alpha\delta} = -\frac{\varkappa_g}{2}\left(h_{\gamma\alpha,\delta} + h_{\gamma\delta,\alpha} - h_{\alpha\delta,\gamma}\right) + O(\varkappa_g^2) \, . \tag{2.12}$$

The factor $(-g)^{-\frac{1}{2}}$ appearing in (2.1) can be expanded according to the formula

$$g^{-a} = 1 - a\varkappa_g \operatorname{Tr}(h^{\mu\nu}) = O(\varkappa_g^2) \, . \tag{2.13}$$

The final expression for (2.1) then reads

$$\mathscr{L}_g = \tfrac{1}{4}\left(h_{\mu\nu,\alpha}h_{\mu\nu,\alpha} - h_{\mu\mu,\alpha}h_{\nu\nu,\alpha} + 2h_{\mu\mu,\alpha}h_{\alpha\nu,\nu} - 2h_{\mu\nu,\alpha}h_{\nu\alpha,\mu}\right) + O(\varkappa_g) \, . \tag{2.14}$$

For the f-meson field, we have similarly

$$(f^{-1})_{\mu\nu} = \eta_{\mu\nu} - \varkappa_f F_{\mu\nu} + \text{higher-order terms in } \varkappa_f F \, , \tag{2.15}$$

so that

$$\mathscr{L}_f = \tfrac{1}{4}\left(F_{\mu\nu,\alpha}F_{\mu\nu,\alpha} - F_{\mu\mu,\alpha}F_{\nu\nu,\alpha} + 2F_{\mu\mu,\alpha}F_{\alpha\nu,\nu} - 2F_{\mu\nu,\alpha}F_{\nu\alpha,\mu}\right) + \text{higher-order terms in } F \, . \tag{2.16}$$

The expansion of $\mathscr{L}_{fg}$ in (2.6) yields

$$\mathscr{L}_{fg} = \frac{M^2}{4\varkappa_f^2}\Big(\varkappa_f^2(F_{\alpha\beta}F_{\alpha\beta} - F_{\alpha\alpha}F_{\beta\beta}) + \varkappa_g^2(h_{\alpha\beta}h_{\alpha\beta} - h_{\alpha\alpha}h_{\beta\beta}) - 2\varkappa_f\varkappa_g(F_{\alpha\beta}h_{\alpha\beta} - F_{\alpha\alpha}h_{\beta\beta}) + \text{higher-order terms}\Big) \, . \tag{2.17}$$

(*) For simplicity, we use a Euclidean metric, replacing $\eta_{\mu\nu}$ by $\delta_{\mu\nu}$ and removing minus signs from determinants. The change back to the Minkowski metric may be made at the end of the calculation.

In order to diagonalize $\mathscr{L}_{\text{tot}}$, we define new fields $\tilde{F}^{\mu\nu}$ and $\tilde{h}^{\mu\nu}$ by

$$\tilde{F}^{\mu\nu} = \frac{1}{(\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}}} (\varkappa_f F^{\mu\nu} - \varkappa_g h^{\mu\nu}) , \tag{2.18}$$

$$\tilde{h}^{\mu\nu} = \frac{1}{(\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}}} (\varkappa_g F^{\mu\nu} + \varkappa_f h^{\mu\nu}) , \tag{2.19}$$

in terms of which

$$\mathscr{L}_f + \mathscr{L}_g = \tfrac{1}{4}\{\tilde{F}_{\mu\nu,\alpha}\tilde{F}_{\mu\nu,\alpha} + \tilde{h}_{\mu\nu,\alpha}\tilde{h}_{\mu\nu,\alpha} + \text{terms with other indices, as in (2.14) and (2.16)}\} \tag{2.20}$$

and

$$\mathscr{L}_{fg} = \frac{M^2}{4} \frac{\varkappa_f^2 + \varkappa_g^2}{\varkappa_f^2} (\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta} - \tilde{F}_{\alpha\alpha}\tilde{F}_{\beta\beta}) . \tag{2.21}$$

We see that all factors bilinear in $\tilde{h}$ have vanished from $\mathscr{L}_{fg}$ and that the « new » mass in our total Lagrangian is given by

$$m = \frac{(\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}}}{\varkappa_f} M , \tag{2.22}$$

where M is the mass of the f-meson. We also note that the diagonalization of $\mathscr{L}_{\text{tot}}$ has been carried out up to terms quadratic in the fields $F^{\mu\nu}$ and $h^{\mu\nu}$.

Since $\tilde{F}^{\mu\nu}$ and $\tilde{h}^{\mu\nu}$ are, in general, *not* tensors, it is desirable to rewrite these fields in terms of the (diagonalized) *tensor* fields $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$. It follows from (2.7) and (2.8), together with (2.18) and (2.19), that

$$f^{\mu\nu} - g^{\mu\nu} = (\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}} \tilde{F}^{\mu\nu} , \tag{2.23}$$

$$\left(1 + \frac{\varkappa_g^2}{\varkappa_f^2}\right)^{-1} \left(g^{\mu\nu} + \frac{\varkappa_g^2}{\varkappa_f^2} f^{\mu\nu}\right) = \eta^{\mu\nu} + \frac{\varkappa_f\varkappa_g}{(\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}}} \tilde{h}^{\mu\nu} . \tag{2.24}$$

The left-hand sides of these equations, being tensor expressions, can now be defined as the new fields $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ respectively:

$$\tilde{f}^{\mu\nu} = \lambda_f \tilde{F}^{\mu\nu} \tag{2.25}$$

and

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} + \lambda_g \tilde{h}^{\mu\nu} , \tag{2.26}$$

where

$$\lambda_f = (\varkappa_f^2 + \varkappa_g^2)^{\frac{1}{2}} , \tag{2.27a}$$

$$\lambda_g = \frac{\varkappa_f \varkappa_g}{\lambda_f} . \tag{2.27b}$$

Relations between $\tilde{f}$, $\tilde{g}$ and the original fields f, g, can be derived with the aid of (2.23)-(2.26). The result is

$$f^{\mu\nu} = a_1 \tilde{f}^{\mu\nu} + \tilde{g}^{\mu\nu} , \tag{2.28a}$$

$$g^{\mu\nu} = a_2 \tilde{f}^{\mu\nu} + \tilde{g}^{\mu\nu} , \tag{2.28b}$$

with

$$a_1 = \frac{\varkappa_f^2}{\varkappa_f^2 + \varkappa_g^2} , \tag{2.29a}$$

$$a_2 = \frac{-\varkappa_g^2}{\varkappa_f^2 + \varkappa_g^2} . \tag{2.29b}$$

In summary, we have shown how to write

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_{\tilde{f}} + \mathscr{L}_{\tilde{g}} \tag{2.30}$$

with

$$\mathscr{L}_{\tilde{f}} = \frac{1}{4} (\tilde{F}_{\mu\nu,\alpha}\tilde{F}_{\mu\nu,\alpha} - \tilde{F}_{\mu\mu,\alpha}\tilde{F}_{\nu\nu,\alpha} + 2\tilde{F}_{\mu\mu,\alpha}\tilde{F}_{\alpha\nu,\nu} - 2\tilde{F}_{\mu\nu,\alpha}\tilde{F}_{\nu\alpha,\mu}) + \frac{M^2}{4}\frac{\varkappa_f^2 + \varkappa_g^2}{\varkappa_f^2} (\tilde{F}_{\alpha\beta}\tilde{F}_{\alpha\beta} - \tilde{F}_{\alpha\alpha}\tilde{F}_{\beta\beta}) \tag{2.31}$$

and

$$\mathscr{L}_{\tilde{g}} = \tfrac{1}{4} (\tilde{h}_{\mu\nu,\alpha}\tilde{h}_{\mu\nu,\alpha} - \tilde{h}_{\mu\mu,\alpha}\tilde{h}_{\nu\nu,\alpha} + 2\tilde{h}_{\mu\mu,\alpha}\tilde{h}_{\alpha\nu,\nu} - 2\tilde{h}_{\mu\nu,\alpha}\tilde{h}_{\nu\alpha,\mu}) . \tag{2.32}$$

$\tilde{F}$ and $\tilde{h}$ are related to $\tilde{f}$ and $\tilde{g}$ respectively by (2.25) and (2.26). The diagonalized fields $\tilde{f}$ and $\tilde{g}$ have no direct physical interpretation in terms of the f-meson and graviton fields, but have been introduced for purely calculational purposes. They remove the need for evaluating expressions like $\langle 0|T(f^{\mu\nu}(x_1)\cdot \cdot g^{\alpha\beta}(x_2))|0\rangle$. Such terms would occur in the superpropagator if we were to work with a Lagrangian involving cross-terms in f and g. The diagonalized Lagrangian « conceals » such interaction terms.

3. – Evaluation of the massive superpropagator.

3·1. *Integral representation.* – The massive superpropagator for the f-meson–graviton fields, corresponding to the Lagrangian (2.9), is defined by

$$S^{\mu\nu,\alpha\beta}(x_1 - x_2) = \langle 0|T\left(\frac{f^{\mu\nu}(x_1)}{\sqrt{-\det f^{\mu\nu}(x_1)}} \cdot \frac{g^{\alpha\beta}(x_2)}{\sqrt{-\det g^{\mu\nu}(x_2)}}\right)|0\rangle , \tag{3.1}$$

where T is the covariant time-ordered product. Since the general expression (3.1) cannot be handled computationally at this juncture with any degree of exactness, we shall consider the simpler form

$$S_0^{\mu\nu,\alpha\beta}(x_1-x_2)=\eta^{\mu\nu}\eta^{\alpha\beta}S(x_1-x_2)\,, \tag{3.2}$$

$$S(x_1-x_2)=\langle 0|T\left(\frac{1}{\sqrt{-\det f^{\mu\nu}(x_1)}}\cdot\frac{1}{\sqrt{-\det g^{\alpha\beta}(x_2)}}\right)|0\rangle\,, \tag{3.3}$$

which contains the basic features of the general superpropagator (3.1).

Our plan is first to write down an integral representation for $S(x_1-x_2)$ and then to compute the resulting parameter integrals. Using the formula [12]

$$\frac{1}{\sqrt{+\det A^{\alpha\beta}}}=\frac{1}{\pi^2}\int \mathrm{d}^4u\,\exp\left[-u_\alpha u_\beta A^{\alpha\beta}\right], \tag{3.4}$$

where the integration contours for the u_α, $\alpha=1,2,3,4$, depend on the form of $A^{\alpha\beta}$ and are chosen to make the integral converge, we arrive at the following integral representation for $S(x_1-x_2)$:

$$S(x_1-x_2)=\frac{1}{\pi^4}\iint \mathrm{d}^4u\,\mathrm{d}^4v\,\langle 0|T\{\exp[-u_\mu u_\nu f^{\mu\nu}(x_1)]\exp[-v_\alpha v_\beta g^{\alpha\beta}(x_2)]\}|0\rangle \tag{3.5}$$

with $\alpha,\beta,\mu,\nu=0,1,2,3$. It is convenient to express the fields $f^{\nu\mu}$ and $g^{\mu\nu}$ in terms of $\tilde{F}^{\mu\nu}$ and $\tilde{h}^{\mu\nu}$ with the aid of (2.25), (2.26) and (2.28). The result is

$$f^{\mu\nu}=\eta^{\mu\nu}+a_1\lambda_f\tilde{F}^{\mu\nu}+\lambda_g\tilde{h}^{\mu\nu}\,, \tag{3.6a}$$

$$g^{\mu\nu}=\eta^{\mu\nu}+a_2\lambda_f\tilde{F}^{\mu\nu}+\lambda_g\tilde{h}^{\mu\nu}\,, \tag{3.6b}$$

where the coefficients λ_f, λ_g, a_1 and a_2 have already been defined in (2.27) and (2.29), which show that

$$a_1a_2\,\lambda_f^2=-\lambda_g^2\,. \tag{3.6c}$$

Substituting (3.6) into (3.5) and noting that

$$\langle 0|T\{\exp[a_{\mu\nu}\varphi^{\mu\nu}(x_1)]\exp[b_{\alpha\beta}\,\chi^{\alpha\beta}(x_2)]\}|0\rangle=\langle 0|\exp\left[a_{\mu\nu}b_{\alpha\beta}T\{\varphi^{\mu\nu}(x_1)\cdot\chi^{\alpha\beta}(x_2)\}\right]|0\rangle\,,$$

we obtain

$$S(x_1-x_2)=\frac{1}{\pi^4}\iint \mathrm{d}^4u\,\mathrm{d}^4v\,\exp[-u^2-v^2]\cdot \\ \cdot\exp\left[u_\mu u_\nu v_\alpha v_\beta\langle 0|T\big((a_1\lambda_f\tilde{F}^{\mu\nu}+\lambda_g\tilde{h}^{\mu\nu})(a_2\lambda_f\tilde{F}^{\alpha\beta}+\lambda_g\tilde{h}^{\alpha\beta})\big)|0\rangle\right]. \tag{3.7}$$

Before we can perform the integrations in (3.7) it is necessary to evaluate

$$\langle 0|T(\tilde{F}^{\mu\nu}\tilde{h}^{\alpha\beta})|0\rangle \tag{3.8a}$$

and

$$\langle 0|T(\tilde{F}^{\mu\nu}\tilde{F}^{\alpha\beta})|0\rangle\,, \qquad \langle 0|T(\tilde{h}^{\mu\nu}\tilde{h}^{\alpha\beta})|0\rangle\,. \tag{3.8b}$$

We shall discuss first the time-ordered product (3.8*a*).

i) *The quantity* $\langle 0|T(\tilde{F}^{\mu\nu}(x_1)\tilde{h}^{\alpha\beta}(x_2))|0\rangle$. – The diagonalized free fields $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ possess the following vacuum-expectation values:

$$\langle 0|\tilde{f}^{\mu\nu}|0\rangle = 0\,, \qquad \langle 0|\tilde{g}^{\mu\nu}|0\rangle = \eta^{\mu\nu}\,, \tag{3.9}$$

where we have taken

$$\langle 0|\tilde{h}^{\mu\nu}|0\rangle = 0 = \langle 0|\tilde{F}^{\mu\nu}|0\rangle \tag{3.10}$$

in order that $\langle 0|f^{\mu\nu}|0\rangle$ coincide with the flat-space value (similarly $\langle 0|g^{\mu\nu}|0\rangle = \eta^{\mu\nu}$). Furthermore

$$\langle 0|T(\tilde{f}^{\mu\nu}(x_1)\tilde{g}^{\alpha\beta}(x_2))|0\rangle = 0\,, \tag{3.11}$$

in view of the absence of an interaction term in the diagonalized Lagrangian (2.30). Substituting (2.25), (2.26) into the l.h.s. of (3.11) and using (3.10), we obtain immediately

$$\langle 0|T(\tilde{F}^{\mu\nu}(x_1)\tilde{h}^{\alpha\beta}(x_2))|0\rangle = 0\,. \tag{3.12}$$

ii) *The propagator* $\langle 0|T(\tilde{F}^{\mu\nu}(x_1)\tilde{F}^{\alpha\beta}(x_2))|0\rangle$. – For the massive tensor field $\tilde{F}^{\mu\nu} = (1/\lambda_t)\,\tilde{f}^{\mu\nu}$ we find that [18]

$$\begin{aligned}\langle 0|T(\tilde{F}^{\mu\nu}(x_1)\tilde{F}^{\alpha\beta}(x_2))|0\rangle &= \\ &= \tfrac{1}{2}\,(d^{\mu\alpha}d^{\nu\beta} + d^{\mu\beta}d^{\nu\alpha} - \tfrac{2}{3}\,d^{\mu\nu}d^{\alpha\beta})\,\Delta_F(x^2 - i\varepsilon, m)\,, \qquad \varepsilon > 0\,,\end{aligned} \tag{3.13}$$

where m is given by (2.22), $x_\mu \equiv (x_1 - x_2)_\mu$, $\mu = 0, 1, 2, 3$, and

$$d^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{m^2}\cdot\frac{\partial^2}{\partial x_\mu\,\partial x_\nu}\,. \tag{3.14}$$

([18]) B. S. De Witt: in *Relativity, Groups and Topology* (New York, 1964), p. 622.

The general form of the causal propagator $\Delta_F(x^2 - i\varepsilon, m)$ reads ([19])

$$(3.15) \qquad \Delta_F(x^2 - i\varepsilon, m) = \frac{1}{4\pi i}\,\delta(x^2) + \frac{im\theta(x^2)}{8\pi\sqrt{x^2 - i\varepsilon}}\,H_1^{(2)}(m\sqrt{x^2 - i\varepsilon}) + \\ + \frac{m\theta(-x^2)}{4\pi^2\sqrt{-x^2 + i\varepsilon}}\,K_1(m\sqrt{-x^2 + i\varepsilon})\;;$$

here $\theta(x^2)$ is the usual step function, $x^2 = x_0^2 - \boldsymbol{x}^2$, while $H_1^{(2)}$ and K_1 denote the Hankel and modified Bessel functions respectively.

We shall work in the *Symanzik region* of the external momenta ($p^2 < 0$) in which case it is sufficient to consider $S(x)$ or, more specifically, $\Delta_F(x^2 - i\varepsilon, m)$ for Euclidean x-space only ([9]). In this region the contour of integration over x_0 for the Fourier transform

$$(3.16) \qquad \tilde{S}^{\mu\nu,\alpha\beta}(p) = \eta^{\mu\nu}\eta^{\alpha\beta}\tilde{S}(p)\,,$$

$$(3.17) \qquad \tilde{S}(p) = \int \exp\,[ip\cdot x]\,S(x)\,\mathrm{d}^4x\,,$$

may be rotated counterclockwise through 90° without crossing a pole. Thus $x_0 \to ix_4$ so that $-x^2 = \boldsymbol{x}^2 + x_4^2 = r^2$, while $\Delta_F(x^2 - i\varepsilon, m)$ in (3.15) reduces to

$$\Delta_F(x^2 - i\varepsilon, m) = \frac{mK_1(m\sqrt{-x^2 + i\varepsilon})}{4\pi^2\sqrt{-x^2 + i\varepsilon}}\,, \qquad \varepsilon > 0\,.$$

We shall use the following single-valued branch of Δ_F which is analytic for $r > 0$:

$$(3.18) \qquad \Delta_F(x^2, m) = \frac{mK_1(mr)}{4\pi^2 r}\,.$$

For momentum values which lie *outside* the Symanzik region, $\tilde{S}(p)$ in (3.17) is obtained by analytic continuation.

It is clear from (3.13) that the major technical complication in the evaluation of $S(x)$ can be expected to arise from the mass-dependent $d^{\mu\nu}$-operators in the $\tilde{F}^{\mu\nu}$-propagator. Despite their troublesome character, the operators $d^{\mu\nu}$ are essential in determining the analytic structure of the massive superpropagator. It follows from (3.13) and (3.14) that this structure will be affected not only by the mass of the f-meson, but more decisively by the differential operators $\partial^\mu\partial^\nu$, which will alter the singularity structure of (3.13) and hence of $S(x)$.

([19]) N. N. Bogoliubov and D. V. Shirkov: *Introduction to the Theory of Quantized Fields* (New York, 1959), p. 650.

The operators $\partial^\mu\partial^\nu$ will have a particularly strong effect in the vicinity of the light-cone, where

$$(3.19)\qquad \partial^\mu\partial^\nu \Delta_F \simeq \frac{-x^\mu x^\nu}{4\pi^2 x^6}\,, \qquad x^2\to 0\,,$$

as compared with $\Delta_F \approx -(4\pi^2x^2)^{-1}$. We shall return to a discussion of the analytic structure of $S(x)$ at a later point.

iii) *The propagator* $\langle 0|T(\tilde{h}^{\mu\nu}(x_1)\tilde{h}^{\alpha\beta}(x_2))|0\rangle$. – The propagator for the massless field $\tilde{h}^{\mu\nu}$ is much simpler and is given, in a specific gauge ([20]), by

$$(3.20)\qquad \langle 0|T(\tilde{h}^{\mu\nu}(x_1)\tilde{h}^{\alpha\beta}(x_2))|0\rangle = \tfrac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}-\eta^{\mu\nu}\eta^{\alpha\beta})\,D_F(x)\,,$$

where

$$(3.21)\qquad D_F(x) = \frac{-1}{4\pi^2x^2-i\varepsilon}\,, \qquad \varepsilon>0\,,$$

is the well-known causal propagator for massless particles. In the Euclidean region of x-space, the function

$$(3.22)\qquad D_F(x) = \frac{1}{4\pi^2r^2}\,, \qquad r>0\,,$$

is everywhere positive and differentiable.

Before substituting (3.12), (3.13) and (3.20) into the double integral (3.7), we remark that the computation of our four-dimensional integrals over u- and v-space is simplified by replacing the Minkowski metric $\eta^{\alpha\beta}$ with a Euclidean metric $\delta^{\mu\nu}$ ($\mu,\nu=1,2,3,4$). After the integrations have been performed one may « convert » back to Minkowski space (cf. footnote (*) on p. 616). With this alteration in the metric the r.h.s. of (3.7) assumes the form

$$(3.23)\qquad S(x) = \frac{1}{\pi^4}\iint \mathrm{d}^4u\,\mathrm{d}^4v\,\exp\Bigg[-u_\mu u_\nu\delta^{\mu\nu}-v_\alpha v_\beta\delta^{\alpha\beta}+ + u_\mu u_\nu v_\alpha v_\beta\Bigg(\frac{a_1a_2}{2}\lambda_f^2\Big(d^{\mu\alpha}d^{\nu\beta}+d^{\mu\beta}d^{\nu\alpha}-\frac{2}{3}d^{\mu\nu}d^{\alpha\beta}\Big)\Delta_F + + \frac{1}{2}\lambda_g^2(\delta^{\mu\alpha}\delta^{\nu\beta}+\delta^{\mu\beta}\delta^{\nu\alpha}-\delta^{\mu\nu}\delta^{\alpha\beta})\,D_F\Bigg)\Bigg]\,.$$

([20]) T. W. B. Kibble: in *High-Energy Physics and Elementary Particles* (Vienna, 1965), p. 885.

Remembering that $d^{\mu\nu} = \delta^{\mu\nu} - m^{-2}\partial^\mu\partial^\nu$, we finally obtain

$$S(x) = \frac{1}{\pi^4}\iint \mathrm{d}^4u\,\mathrm{d}^4v \exp\big[-u^2 - v^2 - (\alpha_1 u^2 v^2 + \alpha_2 (u\cdot v)^2 + \\ + \{u^2(v\cdot\partial)^2 + v^2(u\cdot\partial)^2\}\alpha_3 + (u\cdot v)(v\cdot\partial)(u\cdot\partial)\alpha_4 + (u\cdot\partial)^2(v\cdot\partial)^2\alpha_5)\big]\,, \tag{3.24}$$

where

$$\begin{cases} \alpha_1 = \frac{1}{3}a_1 a_2 \lambda_t^2 \Delta_F + \frac{1}{2}\lambda_g^2 D_F\,, \\ \alpha_2 = -\,a_1 a_2 \lambda_t^2 \Delta_F - \lambda_g^2 D_F\,, \\ \alpha_3 = -\frac{1}{3}a_1 a_2 (\lambda_t/m)^2 \Delta_F\,, \\ \alpha_4 = -\,6\alpha_3\,, \\ \alpha_5 = \dfrac{2\alpha_3}{m^2}\,. \end{cases} \tag{3.25}$$

3·2. *Structure of $S(x)$ in Euclidean co-ordinate space.* – Let us write (3.24) as

$$S(x) = \frac{1}{\pi^4}\int \mathrm{d}^4u \exp[-u^2]\left\{\int \mathrm{d}^4v \exp[-F(v)]\right\} \tag{3.26}$$

and integrate over v-space first. The integral in square brackets is essentially Gaussian in character and may therefore be evaluated *exactly* by diagonalizing the quadratic form $F(v)$ with the aid of the linear transformation

$$\begin{cases} v_j \to \bar{v}_j\,, & j = 1, \ldots, 4\,, \\ F(v) \to G(\bar{v}) = \sum_{j=1}^{4} \lambda_j \bar{v}_j^2\,; \end{cases} \tag{3.27}$$

λ_j are the eigenvalues of $G(\bar{v})$ which satisfy $\lambda_1\lambda_2\lambda_3\lambda_4 = \mathscr{D}$, where $\mathscr{D}$ is the determinant of the quadratic form $F(v)$. For $\lambda_j > 0$, substitution of (3.27) into (3.26) leads to

$$S(x) = \frac{1}{\pi^4}\int \mathrm{d}^4u \exp[-u^2]\left\{\int_{-\infty}^{+\infty}\mathrm{d}\bar{v}_1 \ldots \int_{-\infty}^{\infty}\mathrm{d}\bar{v}_4 J(v,\bar{v}) \exp\left[-\sum_{j=1}^{4}\lambda_j \bar{v}_j^2\right]\right\},$$

or, as the Jacobian $J(v,\bar{v})$ can be shown to equal unity,

$$S(x) = \frac{1}{\pi^4}\int \mathrm{d}^4u \exp[-u^2]\left\{\prod_{j=1}^{4}(\pi/\lambda_j)^{\frac{1}{2}}\right\},$$

giving

$$S(x) = \frac{1}{\pi^2}\int \frac{\mathrm{d}^4u \exp[-u^2]}{\sqrt{\mathscr{D}}}\,. \tag{3.28}$$

If the λ's are not all strictly positive, each $\bar{v}$-integration must be taken along a contour at 45° to the real axis to ensure convergence.

The expression (3.28) for the massive superpropagator is *exact*: *no approximations* have been made between (3.5) and (3.28). The explicit computation of the determinant $\mathscr{D}$ involves a number of tedious though elementary steps and will not be repeated here. The final form of $\mathscr{D}$ reads

$$\mathscr{D} = A^4 + A^3 E_1 + A^2 E_2 + A E_3 + E_4 \,, \tag{3.29}$$

where

$$A = 1 + \alpha_1 u^2 + (u\cdot\partial)^2 \alpha_3 \,,$$

$$E_1 = \alpha_2 u^2 - (\partial^2 c) + u\cdot g \,,$$

$$E_2 = -\alpha_2\big(u^2(\partial^2 c) - (u\cdot\partial)^2 c\big) + \big((g\cdot\partial)(u\cdot\partial)c\big) - (u\cdot g)(\partial^2 c) + \frac{1}{2}(\partial^2 c)^2 - \frac{1}{2}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) \,,$$

$$\begin{aligned} E_3 = &-\alpha_2\left\{\frac{1}{2}u^2\big((\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) - (\partial^2 c)^2\big) + \big((u\cdot\partial)^2 c\big)(\partial^2 c) - \right.\\ &\left. - \big((u\cdot\partial)(\partial_\mu c)\big)\big((u\cdot\partial)(\partial_\mu c)\big)\right\} - \frac{1}{2}(u\cdot g)\big((\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) - (\partial^2 c)^2\big) - \\ &- \big((g\cdot\partial)(u\cdot\partial)c\big)(\partial^2 c) + \big((u\cdot\partial)(\partial_\mu c)\big)\big((g\cdot\partial)(\partial_\mu c)\big) - \frac{1}{6}(\partial^2 c)^3 + \\ &+ \frac{1}{2}(\partial^2 c)(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) - \frac{1}{3}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\sigma c)(\partial_\nu\partial_\sigma c) \,, \end{aligned}$$

$$\begin{aligned} E_4 = &-\alpha_2\left\{u^2\left(\frac{1}{6}(\partial^2 c)^3 - \frac{1}{2}(\partial^2 c)(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) + \frac{1}{3}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\sigma c)(\partial_\nu\partial_\sigma c)\right) + \right.\\ &+ \frac{1}{2}\big((u\cdot\partial)^2 c\big)\big((\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) - (\partial^2 c)^2\big) + \\ &\left. + \big((u\cdot\partial)(\partial_\mu c)\big)\left[\big((u\cdot\partial)(\partial_\mu c)\big)(\partial^2 c) - \big((u\cdot\partial)(\partial_\nu c)\big)(\partial_\mu\partial_\nu c)\right]\right\} - \\ &- (u\cdot g)\cdot\left(\frac{1}{6}(\partial^2 c)^3 - \frac{1}{2}(\partial^2 c)(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) + \frac{1}{3}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\sigma c)(\partial_\nu\partial_\sigma c)\right) - \\ &- \frac{1}{2}\big((g\cdot\partial)(u\cdot\partial)c\big)\cdot\big((\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c) - (\partial^2 c)^2\big) - \\ &- \big((u\cdot\partial)(\partial_\mu c)\big)\left\{\big((g\cdot\partial)(\partial_\mu c)\big)(\partial^2 c) - \big((g\cdot\partial)(\partial_\nu c)\big)(\partial_\mu\partial_\nu c)\right\} + \\ &+ (\partial^2 c)^2\left(\frac{1}{24}(\partial^2 c)^2 - \frac{1}{4}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c)\right) + \\ &+ (\partial_\mu\partial_\nu c)(\partial_\mu\partial_\sigma c)\left(\frac{1}{3}(\partial^2 c)(\partial_\nu\partial_\sigma c) - \frac{1}{4}(\partial_\nu\partial_\tau c)(\partial_\sigma\partial_\tau c)\right) + \frac{1}{8}(\partial_\mu\partial_\nu c)(\partial_\mu\partial_\nu c)(\partial_\sigma\partial_\tau c)(\partial_\sigma\partial_\tau c) \end{aligned}$$

with

$$c = -\alpha_3 u^2 - (u \cdot \partial)^2 \alpha_5 , \tag{3.30a}$$

$$g_\mu = -6\big(\partial_\mu(u \cdot \partial)\big)\,\alpha_3 . \tag{3.30b}$$

The determinant $\mathscr{D}$ may be cast into a somewhat different form by applying the differential operators $\partial^\mu \equiv \partial/\partial x_\mu$ in A, $E_1, \ldots, E_4$ to the co-ordinate–dependent coefficients $\alpha_1, \ldots, \alpha_5$. Although the latter are generally functions of both Δ_E and D_F, derivatives in $\mathscr{D}$ occur only with respect to Δ_F. Differentiation of (3.18) yields

$$\partial^\mu \Delta_F(x^2, m) = -\frac{m^2}{4\pi^2}\cdot\frac{x^\mu K_2(mr)}{r^2}, \qquad -x^2 = r^2 ,$$

and

$$\partial^\mu \partial^\nu \Delta_F(x^2, m^2) = -\frac{m^2}{4\pi^2}\left\{\delta^{\mu\nu}\,\frac{K_2(mr)}{r^2} - \frac{m x^\mu x^\nu}{r^3}\,K_3(mr)\right\}. \tag{3.31}$$

$\mathscr{D}$ also contains higher derivatives of Δ_F which will not be listed here. We note that the number of ∂^μ-operators occurring in $\mathscr{D}$ is always even.

Substituting expressions like (3.31) into (3.29), we find that the remodelled $\mathscr{D}$, though simpler in form than (3.29), contains a much larger collection of terms (several hundred, in fact). In view of this unwieldy nature of $\mathscr{D}$ and the number of integrations yet to be performed—even after computing the u-space integral, we still have to take the Fourier transform of $S(x)$—we have decided to simplify $\mathscr{D}$ as follows. We shall approximate expressions of type (3.31) and their higher-derivative analogues, by writing

$$x^\mu x^\nu \approx \delta^{\mu\nu} r^2 . \tag{3.32}$$

This approximation is necessary at this stage if we are to gain any detailed insight into the general structure of the massive tensor superpropagator. We note first of all that (3.32) does not alter the damping power of the superpropagator. (We shall see later from (3.55) that $S(x) \sim -r^{12}$ as $r \to 0$, $r^2 = -x^2 > 0$.) This follows from the fact that the determinant (3.29) behaves essentially like r^{-24} as $r \to 0$, whether an approximation is made or not. On the other hand, if we consider only the massive *scalar* superpropagator, then the corresponding determinant behaves like r^{-8} as $r \to 0$. Secondly, the approximation (3.32) does not affect in any way the *massiveness* of $S(x)$.

Finally, (3.32) allows us to retain in our superpropagator at least certain aspects of spin, including those which are characteristic of the propagation of a massless spin-2 field. To see this, we observe that we can rewrite the r.h.s. of (3.13) in the form

$$\langle 0|T[\tilde F^{\mu\nu}(x_1)\,\tilde F^{\alpha\beta}(x_2)]|0\rangle = \tfrac{1}{2}\,(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})\,\Delta_F + H^{\mu\nu\alpha\beta}\,\Delta_F , \tag{3.13a}$$

where the quantity $H^{\mu\nu\alpha\beta}$ contains the differential operators ∂^μ, ∂^ν, ∂^α and ∂^β. Near the origin, the first expression on the r.h.s. of (3.13*a*) corresponds to the propagator for a massless tensor field (we recall that $\Delta_F \approx D_F$ for $r \to 0$), and is seen to remain unaffected by the approximation (3.32).

If we now use the above approximation on the r.h.s. of (3.31), we obtain

$$\partial^\mu \partial^\nu \Delta_F(x^2, m) \simeq -\frac{m^2 \delta^{\mu\nu}}{4\pi^2 r^2}\{K_2(mr) - mrK_3(mr)\}\,. \tag{3.33}$$

In applying this simplification to expressions like $d^{\mu\nu} d^{\alpha\beta} \Delta_F = \partial^\mu \partial^\nu \partial^\alpha \partial^\beta \Delta_F + \dots$, we combine ∂^μ with ∂^ν, and ∂^α with ∂^β (*i.e.* we pair ∂^μ and ∂^ν, because they arise from the same operator $d^{\mu\nu}$, similarly for ∂^α and ∂^β). After a considerable amount of algebra, with the aid of (3.6*c*), the determinant (3.29) reduces to

$$\mathscr{D}_0 = \{1 + A(r, m, \lambda_g)u^2\}^3\{1 + B(r, m, \lambda_g)u^2\}, \tag{3.34}$$

where

$$A(r, m, \lambda_g) \equiv A(r) = -5\lambda_g^2\left\{\frac{4K_0(mr)}{m^2r^2} + \left(1 + \frac{8}{m^2r^2}\right)\frac{K_1(mr)}{mr} - \frac{1}{10}\right\} D_F, \tag{3.35a}$$

$$B(r, m, \lambda_g) \equiv B(r) = +10\lambda_g^2\left\{\frac{4K_0(mr)}{m^2r^2} + \left(1 + \frac{8}{m^2r^2}\right)\frac{K_1(mr)}{mr} - \frac{1}{20}\right\} D_F, \tag{3.35b}$$

$D_F(r)$ being given by (3.22). It is evident from (3.35) that both $A(r)$ and $B(r)$ become negative for certain values of r, although these coefficients are never negative simultaneously. Specifically we find that

$$A(r) < 0\,, \quad B(r) > 0 \quad \text{for} \quad r < R_1\,, \tag{3.36}$$

$$A(r) > 0\,, \quad B(r) > 0 \quad \text{for} \quad R_1 < r < R_2 \tag{3.37}$$

and

$$A(r) > 0\,, \quad B(r) < 0 \quad \text{for} \quad R_2 < r\,, \tag{3.38}$$

where $1.689 < R_1 < 1.690$ and $1.926 < R_2 < 1.927$.

In deriving the expressions for $A(r)$ and $B(r)$ we have made repeated use of the following formulae for the modified Bessel functions ([21]) $K_\nu(z)$, $\nu = 1, 2, 3, \dots$:

$$\left(\frac{1}{z}\frac{\mathrm{d}}{\mathrm{d}z}\right)\frac{K_\nu(z)}{z^\nu} = -\frac{K_{\nu+1}(z)}{z^{\nu+1}} \tag{3.39}$$

([21]) I. S. GRADSHTEYN and I. M. RYZHIK: *Tables of Integrals, Series and Products* (New York, 1965).

and

$$K_{\nu+1}(z) = K_{\nu-1}(z) + \frac{2\nu}{z} K_\nu(z) . \tag{3.40}$$

(In Appendix A, we describe a shorter, but less instructive way of deriving (3.34)-(3.35).) Substituting (3.34) into (3.28) we obtain for the massive superpropagator

$$S(x) = \frac{1}{\pi^2}\int \frac{d^4u \exp[-u^2]}{(1 + A(r)u^2)^{\frac{3}{2}} (1 + B(r)u^2)^{\frac{1}{2}}} . \tag{3.41}$$

In terms of four-dimensional spherical co-ordinates with the volume element

$$d^4u = \varrho^3 \sin\theta \sin^2\psi \, d\varrho \, d\theta \, d\varphi \, d\psi , \qquad |u|^2 = \varrho^2 ,$$

the r.h.s. of (3.41) becomes

$$S(x) = \frac{1}{\pi^2}\int_0^\infty \varrho^3 \exp[-\varrho^2] d\varrho \int_0^\pi \sin\theta \, d\theta \int_0^\pi \sin^2\psi \, d\psi \int_0^{2\pi} d\varphi \frac{1}{(1 + A(r)\varrho^2)^{\frac{3}{2}} (1 + B(r)\varrho^2)^{\frac{1}{2}}} ,$$

or finally

$$S(x) = \int_0^\infty \frac{v \exp[-v] dv}{(1 + A(r)v)^{\frac{3}{2}} (1 + B(r)v)^{\frac{1}{2}}} . \tag{3.42}$$

Although the last integral cannot be evaluated in closed form, we may gain considerable insight into the nature of $S(x)$ by examining the analytic structure of (3.42). It can be seen that the integrand in (3.42) becomes singular when either $A(r) < 0$ or $B(r) < 0$. We shall discuss these two singularities in turn now. (It will be recalled that the possibility $A < 0$ *and* $B < 0$ does not occur.)

Case 1) $A(r) > 0$ *and* $B(r) < 0$. The crucial factor is $(1 + Bv)^{\frac{1}{2}}$ with the onset of a branch cut at the branch point $v = -1/B$. For $0 \leqslant v < -1/B$, $S(x)$ is *real* so that no difficulties arise, but for values of v in the domain $-1/B < v < \infty$ $S(x)$ is seen to become *complex* (purely imaginary, to be precise). To ensure that the superpropagator (3.42) be *real* for all $r^2 = \boldsymbol{x}^2 + x_4^2$, we shall take the average value of $S(x)$ above and below the branch cut [9,22]. To illustrate this procedure, consider Fig. 1, which depicts the contours C and D lying respectively above and below the cut. It may be shown rigorously that

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([22]) B. W. LEE and B. ZUMINO: *Nucl. Phys.*, **13** B, 671 (1969).

the contribution from the branch point $v=-1/B$ is equal to zero for either contour C_1 or D_1. *Above* the cut (contour C_2) the value of the massive super-propagator is

(3.43*a*) $$S(x)=0+iS_I(x)\,,$$

while *below* the cut (contour D_2) it is

(3.43*b*) $$S(x)=0-iS_I(x)\,,$$

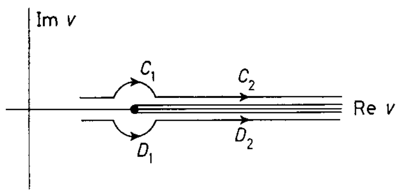

Fig. 1. – Possible contours of integration in the v-plane for the integral (3.42).

where $S_I(x)$ denotes the imaginary part of $S(x)$. Averaging (3.43) we find that the *real part* of $S(x)$ is indeed zero. It is this vanishing of the real part of $S(x)$ for all $v>-1/B$ which permits us to terminate the integral (3.42) effectively at the onset of the branch cut, *i.e.* at $v=-1/B$. Hence (3.42) becomes

(3.44) $$S(x)=\int_0^{-1/B}\frac{v\exp[-v]\,\mathrm{d}v}{(1+Av)^{\frac{3}{2}}(1+Bv)^{\frac{1}{2}}}\,.$$

It is now convenient, for computational reasons, to integrate (3.44) by parts in such a manner that the factor $(1+Bv)^{\frac{1}{2}}$ appears in the numerator of the integrand rather than in the denominator. The final expression

(3.45) $$S(x)=-\frac{2}{B}\int_0^{-1/B}\frac{(1+Bv)^{\frac{1}{2}}}{(1+Av)^{\frac{5}{2}}}\left(1-\left(1+\frac{1}{2}A\right)v-Av^2\right)\exp[-v]\,\mathrm{d}v$$

can be treated numerically with the aid of a computer. The results indicate that $S(x)$ has the *constant* value of 1 within the computational accuracy for all $B<0$. We mention here that $S(x)$ is also constant for $A>0$, $B>0$.

Case 2) $A(r)<0$ *and* $B(r)>0$. In this instance, the integrand in (3.42) contains a branch point at $v=-1/A$, due to the factor $(1+Av)^{-\frac{3}{2}}$, with a branch cut to the right of $v=-1/A$. The integral has a cut in the r-plane between $r=0$ and $r=R_1$. In contrast with Case 1), the $-\frac{3}{2}$ exponent of the $1+Av$ factor implies that the singularity of the integrand at $v=-1/A$

is nonintegrable. We therefore redefine $S(x)$ in the region $0<r<R_1$ (where the singularity is within the range of integration) by analytic continuation.

Let us write the factor $v\exp[-v]/(1+Bv)^{\frac{1}{2}}$ in the integrand in (3.42) as the sum of its value at $v=-1/A$ plus a term which vanishes there:

$$\frac{v\exp[-v]}{(1+Bv)^{\frac{1}{2}}}=C(r)+f(v,r)\,, \tag{3.46}$$

where

$$C(r)=-\frac{(1/A)\exp[1/A]}{(1-B/A)^{\frac{1}{2}}}\,, \tag{3.47}$$

$$f(v,r)=\frac{v\exp[-v]}{(1+Bv)^{\frac{1}{2}}}+\frac{(1/A)\exp[1/A]}{(1-B/A)^{\frac{1}{2}}}\,. \tag{3.48}$$

Then (3.42) becomes

$$S(x)=C(r)I_1(r)+I_2(r) \tag{3.49}$$

with

$$I_1(r)=\int_0^\infty\frac{\mathrm{d}v}{(1+Av)^{\frac{3}{2}}}\,, \tag{3.50}$$

$$I_2(r)=\int_0^\infty\frac{f(v,r)\,\mathrm{d}v}{(1+Av)^{\frac{3}{2}}}\,. \tag{3.51}$$

The integral $I_1(r)$ contains the nonintegrable singularity from $S(x)$. Giving r a small imaginary part, we obtain the well-defined integral

$$I_1(r+i\varepsilon)=\int_0^\infty\frac{\mathrm{d}v}{(1+A(r+i\varepsilon)v)^{\frac{3}{2}}}=\frac{2}{A(r+i\varepsilon)}\,,\qquad \varepsilon>0\,. \tag{3.52}$$

By analytic continuation to the real axis, we may take

$$I_1(r)=\frac{2}{A(r)}\,. \tag{3.53}$$

The singularity structure of $I_2(r)$ depends on the behaviour of $f(v,r)$, which we have chosen to vanish at $v=-1/A$. It is straightforward to show that

$$f(v,r)\sim(1+Av)\qquad\text{as } v\to-1/A\,,$$

so that the integrand in (3.51) behaves like $(1+Av)^{-\frac{1}{2}}$ near $v=-1/A$, the original singularity having been reduced to the type discussed in Case 1). We again take the average value of the integral above and below the cut and then terminate the integration at $v = -1/A$, by analogy with Case 1):

$$I_2(r) = \int_0^{-1/A} \frac{f(v, r)}{(1 + Av)^{\frac{3}{2}}} \,\mathrm{d}v \,. \tag{3.54}$$

This is a convergent integral and can be evaluated on a computer.

The results of the numerical integration of the final expression

$$S(x) = -\frac{(2/A^2)\exp[1/A]}{(1 - B/A)^{\frac{1}{2}}} + \int_0^{-1/A} \frac{f(v, r)\,\mathrm{d}v}{(1 + Av)^{\frac{3}{2}}} \tag{3.55}$$

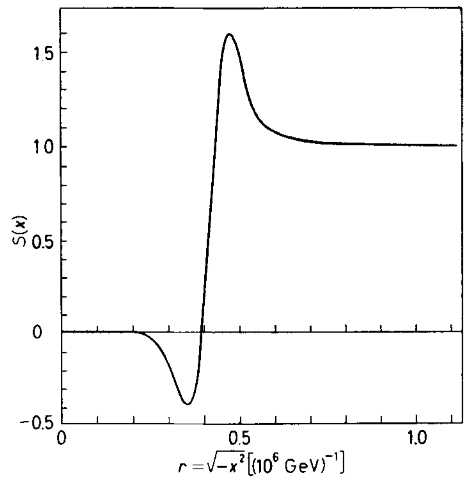

Fig. 2. – Numerical results for $S(x)$, the massive superpropagator for the f-meson–graviton fields, in Euclidean co-ordinate space.

for $0 < r = \sqrt{-x^2} < 10^{-4}$ are shown in Fig. 2. For $10^{-4} \leqslant r < R_1$, we find that $S(x) = 1$ within the computational accuracy. We note that the units of r are inverse GeV, so that for $r = 10^{-4}\,(\mathrm{GeV})^{-1} = 2 \cdot 10^{-18}$ cm, we are already well within the strong-interaction region.

The asymptotic behaviour of $S(x)$ is easily deduced with the help of (3.35).

i) For $r \to \infty$, $A \to 0^+$, while $B \to 0^-$, so that

$$S(x)|_{r\to\infty} = \lim_{r\to\infty} \int_0^{-1/B} \frac{v \exp[-v]\,\mathrm{d}v}{(1 + Av)^{\frac{3}{2}}(1 + Bv)^{\frac{1}{2}}} \to \int_0^{\infty} v \exp[-v]\,\mathrm{d}v = 1 \,. \tag{3.56}$$

This limiting value is the same as for the *massless* superpropagator [12].

ii) For $r \to 0$, $B \to r^{-6}$, while $A \to -r^{-6}$, so that $S(x)$, defined by (3.55), behaves like $-r^{12}$.

It is interesting to compare the basic structure of the f-meson–graviton superpropagator in co-ordinate space with the massive pure f-meson superpropagator, which has been worked out in Appendix B. It is shown there that the coefficient $\bar{A}(r)$—corresponding to $A(r)$ in this Section—is always nonnegative, possibly zero, whereas $\bar{B}(r)$ can become negative for certain r values. It follows that the factor $(1+\bar{A}v)^{-\frac{3}{2}}$ gives rise to no singularities, while the singularities of the term $(1+\bar{B}v)^{-\frac{1}{2}}$ may be treated just as in Case 1) of this Section.

4. – Fourier transform in the Symanzik region.

The Fourier transform of the massive superpropagator $S(x)$ is defined by

$$\tilde{S}(p) = i\int \mathrm{d}^4x\, S(x) \exp[ip\cdot x]\,. \tag{4.1}$$

We have seen in Subsect. 3·2 that for $r \geqslant 10^{-4}\,(\text{GeV})^{-1}$, $S(x)$ has the constant value unity (to the accuracy to which we are able to work computationally), so that we may write

$$S(x) = 1 + S_1(x)\,, \tag{4.2a}$$

$$S_1(x) = 0 \qquad \text{for } r \geqslant 10^{-4}\,(\text{GeV})^{-1}\,, \tag{4.2b}$$

where $r^2 = +\boldsymbol{x}^2 + x_4^2 > 0$. It follows from (4.1) and (4.2) that

$$\tilde{S}(p) = (2\pi)^4\delta^4(p) + \tilde{S}_1(p) \tag{4.3}$$

with

$$\tilde{S}_1(p) = i\int \mathrm{d}^4x S_1(x) \exp[ip\cdot x]\,. \tag{4.4}$$

In the Symanzik region of the external momenta ($p^2 < 0$), $\tilde{S}_1(p)$ assumes the form

$$\tilde{S}_1(p) = \frac{4\pi^2}{q}\int_0^\infty \mathrm{d}r\, r^2 J_1(rq)\, S_1(r)\,, \tag{4.5}$$

where $q^2 = -p^2 > 0$ and $r^2 = +\boldsymbol{x}^2 + x_4^2$. The integral (4.5) may be cut off at $r = 10^{-4}$ since $S_1(x)$ vanishes for larger r. The change of variable

$$r = -\log\left(\frac{1+u}{2}\right) \tag{4.6}$$

in (4.5) leads to

$$(4.7) \qquad \tilde{S}_1(p) = \frac{4\pi^2}{q} \int\limits_{2\exp[-10^{-4}]-1}^{1} \frac{\mathrm{d}u}{1+u} \left(\log\left(\frac{1+u}{2}\right)^2\right) \cdot J_1\left(-q\log\left(\frac{1+u}{2}\right)\right)\left\{S\left(-\log\left(\frac{1+u}{2}\right)\right)-1\right\},$$

which can now be evaluated numerically, by using the form for $S(x)$ in Subsect. **3**·2 and taking its real part (9,22). A plot of the computer results for $\tilde{S}_1(p)$ is shown in Fig. 3.

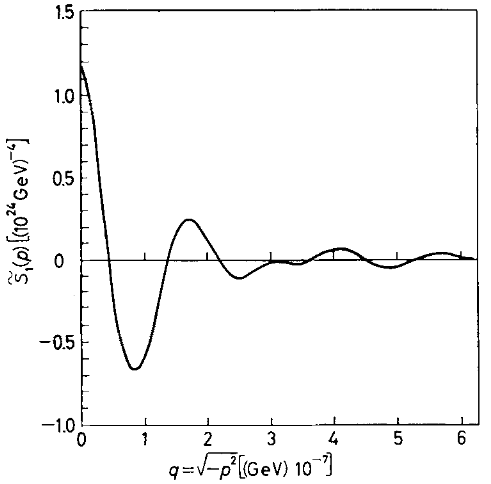

Fig. 3. – Numerical results for the Fouiier transform of $S(x)$ in the Symanzik region.

The value of $\tilde{S}_1(p)$ varies extremely slowly with respect to q for $q \leqslant 10^3$ GeV. The reason for this is that, with the integration cut off at $r = 10^{-4}$, rq is so small for $q \leqslant 10^3$ that $J_1(rq)$ may be approximated by $rq/2$; this cancels the other q-dependence in $\tilde{S}_1(p)$.

For larger values of q, $\tilde{S}_1(p)$ displays an oscillating behaviour. It is also apparent from the form of (4.5) that the magnitude of $\tilde{S}_1(p)$ tends to zero as q tends to infinity. (We note that $J_1(x) \sim \sqrt{2/\pi x}\cos(x - \frac{3}{4}\pi)$ as $|x| \to \infty$.) The absolute value of $\tilde{S}_1(p)$ is less than $10^{-23}\,(\mathrm{GeV})^{-4}$ everywhere; this is due to the smallness of the coupling constant $\lambda_g = 10^{-19}\,(\mathrm{GeV})^{-1}$.

5. – Conclusion.

We have succeeded in obtaining a one-dimensional integral representation for the massive superpropagator both in Euclidean x-space and in momentum space. From the structure of the co-ordinate–space integral we were able to deduce, within the approximation (3.32), the presence of poles and branch cuts in $S(x)$, as well as its asymptotic behaviour for large and small values of r: $S(x)\to 1$ as $r\to\infty$ and $S(x)\sim -r^{12}$ as $r\to 0$. For finite, nonzero r, however, it is hard—in view of the complexity of the coefficients A, $\bar{A}$ and B, $\bar{B}$—to learn anything specific about the behaviour of $S(x)$. In order to fill this gap for nonasymptotic r, and since it is not feasible to integrate (3.42) and (B.8) explicitly, we have computed the one-dimensional integrals numerically In this way we obtain considerable information about the behaviour of *a*) $S(x)$ for general Euclidean x values and *b*) $\tilde{S}(p)$ for general values of q in the Symanzik region ($q^2=-p^2>0$). Needless to say, the presence in $S(x)$ of two disjoint branch cuts makes the numerical computation rather complicated. The co-ordinate representation of the pure f-meson superpropagator, discussed in Appendix B, has only one branch cut and is much easier to handle, because the infinity arising from the factor $(1+Bv)^{-\frac{1}{2}}$, when $B<0$, may conveniently be integrated out. The reality of either superpropagator is guaranteed by taking the average value above and below the cut.

The Fourier transform of $S(x)$ differs from the usual delta-function term by the addition of an oscillating factor which becomes damped as $q\to+\infty$ and whose amplitude is very small $\left(<10^{-23}\,(\text{GeV})^{-4}\right)$. As for its singularity structure in p-space, we note from (4.3) that, apart from the delta-function singularity, $\tilde{S}(p)$ appears to be analytic for $\operatorname{Re}(-p^2)>0$.

We now discuss briefly several features which are not deducible from the present calculations, but which might emerge with further effort.

In the first place, we believe that an exact evaluation of the four-dimensional integral (3.28) would reveal a singularity structure for $S(x)$ more complicated than the present one. Such an exact computation—with all aspects of spin retained—would also tell us the relative importance of *tensor* propagation as compared with the massive *scalar* case. Concerning our calculations in p-space, it should be evident that if a closed expression for the Fourier transform of $[K_1(mr)/r]^z$, z complex, could be found, its use would both improve and simplify the study of the analytic structure of $\tilde{S}(p)$. The structure of $\tilde{S}(p)$ for general values of p could then be obtained by analytic continuation from the Symanzik region.

Finally, we would like to be able to investigate the analytic behaviour of $S(x)$ in the coupling constant λ_g^2. The complexity of the four-dimensional integral (3.28), however, in its present form prevents any conclusive statement in this regard. Such analytic behaviour would, among other things, make more explicit the role of $1/\lambda_g$ as an effective cut-off.

The expression for the massive superpropagator may now be utilized to damp the most virulent infinities in the theory of strong interactions. We expect that this damping effect will be rather large, in view of the extremely rapid fall-off of $S(x)$ in the ultraviolet limit. (We recall from Appendix B that

$|S_t(x)|$, like $|S(x)|$, behaves like r^{12} as $r \to 0$.) We list here some of the physical problems to which our results may be applied:

1) β *decay*: the calculation of this process, using a gravity-modified phenomenological weak-interaction Lagrangian, should provide a good test of the ability of the strong gravity theory to suppress the leading infinities;

2) *further calculation of hadronic mass differences*: such as $m_{\pi^+} - m_{\pi^0}$ [23] and $m_{K_{0L}} - m_{K_{0S}}$;

3) *gravitational collapse of hadronic matter* [9,24]: an investigation of whether the effective force produced by the superpropagator has a short-range repulsive component would throw light on Salam's speculation that « hadrons are (nearly) collapsed objects in the f-gravity field » [24]. The validity of this hypothesis would have far-reaching implications.

* * *

It is a pleasure to thank Prof. A. Salam for suggesting this problem and for his continued interest in our work. We should also like to thank Prof. T. W. B. Kibble for his reading of the manuscript, and Drs. R. Delbourgo and C. J. Isham for several helpful discussions. We are grateful for programming advice from several of our colleagues, especially Mr. P. A. Collins.

One of us (G.L.) wishes to thank Profs. P. T. Matthews and T. W. B. Kibble for their hospitality during his stay in the theoretical physics department at Imperial College. Two of us (D.M.C. and R.M.W.) acknowledge financial support from the Science Research Council.

Appendix A

The result (3.34) may also be obtained by applying the approximation (3.32) directly to the propagator (3.13). Since

$$d^{\mu\nu}\Delta_F(x^2, m) = \left(\delta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{m^2}\right)\Delta_F(x^2, m) = \tag{A.1}$$

$$= \frac{m^2}{4\pi^2}\left(\frac{\delta^{\mu\nu}}{mr} K_1(mr) + \frac{\delta^{\mu\nu}}{m^2 r^2} K_2(mr) - \frac{m^2 x^\mu x^\nu}{m^3 r^3} K_3(mr)\right),$$

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[23] M. J. Duff, J. Huskins and A. Rothery: *Phys. Rev. D*, **4**, 1851 (1971).

[24] A. Salam: Lecture at the *1971 Coral Gables Conference on Fundamental Interactions at High Energy* (Trieste preprint No. IC/71/3).

we find, using (3.32), that

$$(\text{A.2})\qquad d^{\mu\nu}\Delta_F \simeq \frac{m^2\delta^{\mu\nu}}{4\pi^2}\left(\frac{K_1(mr)}{mr}+\frac{K_2(mr)}{(mr)^2}-\frac{K_3(mr)}{mr}\right).$$

Similarly, the expression

$$(\text{A.3})\qquad d^{\mu\nu}d^{\alpha\beta}\Delta_F = \frac{m^2}{4\pi^2}\left\{\frac{\delta^{\mu\nu}\delta^{\alpha\beta}}{mr}\left(K_1(mr)+\frac{K_2(mr)}{mr}\right)-\right.$$
$$\left.-m^2\delta^{\alpha\beta}x^\mu x^\nu\frac{K_3(mr)}{(mr)^3}-\frac{\delta^{\mu\nu}}{m^2}\partial^\alpha\partial^\beta\left(\frac{K_1(mr)}{mr}+\frac{K_2(mr)}{(mr)^2}\right)+\partial^\alpha\partial^\beta\left(\frac{x^\mu x^\nu K_3(mr)}{(mr)^3}\right)\right\}$$

reduces to

$$(\text{A.4})\qquad d^{\mu\nu}d^{\alpha\beta}\Delta_F \simeq \frac{15\delta^{\mu\nu}\delta^{\alpha\beta}}{4\pi^2r^2}\left\{\frac{4K_0(mr)}{(mr)^2}+\left(1+\frac{8}{(mr)^2}\right)\frac{K_1(mr)}{mr}\right\}.$$

In deriving (A.4) we have employed the relation (3.40) for the modified Bessel functions K_ν, $\nu=1, 2, 3, \ldots$. The propagator (3.13) for the $\tilde{F}^{\mu\nu}$ field (recall that $\tilde{F}^{\mu\nu}=\lambda_f^{-1}\tilde{f}^{\mu\nu}$) becomes

$$(\text{A.5})\qquad \langle 0|T(\tilde{F}^{\mu\nu}(x_1)\tilde{F}^{\alpha\beta}(x_2))|0\rangle \simeq \tfrac{1}{2}\left(\delta^{\mu\alpha}\delta^{\nu\beta}+\delta^{\mu\beta}\delta^{\nu\alpha}-\tfrac{2}{3}\delta^{\mu\nu}\delta^{\alpha\beta}\right)C\,,$$

where

$$(\text{A.6})\qquad C=\frac{15}{4\pi^2r^2}\left\{\frac{4K_0(mr)}{(mr)^2}+\left(1+\frac{8}{(mr)^2}\right)\frac{K_1(mr)}{mr}\right\},$$
$$r^2=\boldsymbol{x}^2+x_4^2 \qquad \text{and} \qquad x_\mu\equiv(x_1-x_2)_\mu\,, \qquad \mu=1, 2, 3, 4\,.$$

Substituting (A.5) and (3.20) into the r.h.s. of (3.7), and following the method outlined between (3.23) and (3.28), we obtain the same results as given in the text, namely (3.34) and (3.35).

Appendix B

The pure f-meson massive superpropagator.

It is possible to evaluate the pure f-meson massive superpropagator

$$(\text{B.1})\qquad \left\{\begin{aligned} &S_0^{\mu\nu,\alpha\beta}(x_1-x_2)=\eta^{\mu\nu}\eta^{\alpha\beta}S_f(x_1-x_2)\,,\\ &S_f(x_1-x_2)=\langle 0|T\left(\frac{1}{\sqrt{-\det f^{\mu\nu}(x_1)}}\cdot\frac{1}{\sqrt{-\det f^{\alpha\beta}(x_2)}}\right)|0\rangle\end{aligned}\right.$$

by modifying slightly the method of Sect. **3**. The calculation is in fact simpler since only a single field is involved. In this Appendix, we shall outline the basic steps and state the results.

Using the integral representation (3.4) for determinants, we obtain

$$\text{(B.2)} \qquad S_f(x_1 - x_2) = \\ = \frac{1}{\pi^4}\iint d^4u\, d^4v \langle 0|T\{\exp[-u_\mu u_\nu f^{\mu\nu}(x_1)]\exp[-v_\alpha v_\beta f^{\alpha\beta}(x_2)]\}|0\rangle .$$

The expression (2.7) for $f^{\mu\nu}$ in terms of $F^{\mu\nu}$ may now be substituted into (B.2) to yield

$$\text{(B.3)} \qquad S_f(x_1 - x_2) = \frac{1}{\pi^4}\iint d^4u\; d^4v \exp[-u^2 - v^2]\cdot \\ \cdot \exp\left[\varkappa_f^2 u_\mu u_\nu v_\alpha v_\beta \langle 0|T(F^{\mu\nu}(x_1)F^{\alpha\beta}(x_2))|0\rangle\right] .$$

By analogy with (3.13), the propagator $\langle 0|T(F^{\mu\nu}(x_1)F^{\alpha\beta}(x_2))|0\rangle$ is given by

$$\text{(B.4)} \qquad \langle 0|T(F^{\mu\nu}(x_1)F^{\alpha\beta}(x_2))|0\rangle = \tfrac{1}{2}\left(d^{\mu\alpha}d^{\nu\beta} + d^{\mu\beta}d^{\nu\alpha} - \tfrac{2}{3}\, d^{\mu\nu}d^{\alpha\beta}\right)\Delta_F(x^2 - i\varepsilon, M) ,$$

where M is the mass of the f-meson,

$$x_\mu \equiv (x_1 - x_2)_\mu$$

and

$$d^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{M^2}\frac{\partial^2}{\partial x_\mu \partial x_\nu} .$$

As in Sect. **3**, we use the following single-valued branch of the causal propagator Δ_F which is analytic for $r > 0$:

$$\text{(B.5)} \qquad \Delta_F(r^2, M) = \frac{MK_1(Mr)}{4\pi^2 r} .$$

Substitution of the r.h.s. of (B.4) into (B.3) leads to [17]

$$\text{(B.6)} \qquad S_f(x) = \frac{1}{\pi^4}\int d^4u\, d^4v \exp\Big[-u^2 - v^2 - \\ - \Big(\bar{\alpha}_1 u^2 v^2 + \bar{\alpha}_2 (u\cdot v)^2 + \big(u^2(v\cdot\partial)^2 + v^2(u\cdot\partial)^2\big)\bar{\alpha}_3 + \\ + (u\cdot v)(u\cdot\partial)(v\cdot\partial)\bar{\alpha}_4 + (u\cdot\partial)^2(v\cdot\partial)^2\bar{\alpha}_5\Big)\Big] ,$$

where

(B.7)
$$\begin{cases} \bar{\alpha}_1 = \frac{\varkappa_f^2}{3} \Delta_F\,, \\ \bar{\alpha}_2 = -3\bar{\alpha}_1\,, \\ \bar{\alpha}_3 = -\frac{1}{M^2}\bar{\alpha}_1\,, \\ \bar{\alpha}_4 = \frac{6}{M^2}\bar{\alpha}_1\,, \\ \bar{\alpha}_5 = -\frac{2}{M^4}\bar{\alpha}_1\,. \end{cases}$$

Proceeding now exactly as in Subsect. **3**·2—this includes the approximation (3.22)—we are able to reduce (B.6) to the integral

(B.8)
$$S_f(x) = \int_0^\infty \frac{v \exp[-v]\,dv}{(1+\bar{A}(r)v)^{\frac{5}{2}}(1+\bar{B}(r)v)^{\frac{1}{2}}}$$

with

(B.9a)
$$\bar{A}(r) = \frac{5\varkappa_f^2}{4\pi^2 r^2}\left\{\frac{4K_0(Mr)}{M^2r^2} + \left(1+\frac{8}{M^2r^2}\right)\frac{K_1(Mr)}{Mr}\right\},$$

(B.9b)
$$\bar{B}(r) = -2\bar{A}(r)\,.$$

Before discussing the general features of the integral (B.8), we note (cf. Subsect. **3**·2):

i) As $r\to 0$, $\bar{A}(r)\to+\infty$ and $\bar{B}(r)\to-\infty$ so that $|S_f(x)|\sim r^{12}$. This extremely rapid fall-off in the ultraviolet limit implies a large damping effect by the massive superpropagator. Another surprising feature is the *largeness* of the exponent in r^{12}.

ii) For very large r, both $\bar{A}(r)$ and $\bar{B}(r)$ decrease rapidly so that $S_f(x)\to 1$.

The singularity structure of (B.8) is certainly less complicated than in the case of the mixed f-meson–graviton superpropagator. According to (B.9a), $\bar{A}(r)$ is positive for all r so that *no* singularity arises from the factor $(1+\bar{A}(r)v)^{-\frac{5}{2}}$. The term $(1+\bar{B}(r)v)^{-\frac{1}{2}}$, on the other hand, produces a branch cut in the v-plane, since $\bar{B}(r)$ is negative for all values of r under consideration. We again follow the averaging prescription for obtaining a real-valued superpropagator; the method is equivalent to terminating the integration at $v=-1/B>0$.

A plot of the numerical values obtained for

(B.10)
$$S_f(x) = \int_0^{-1/B} \frac{v \exp[-v]\,dv}{(1+\bar{A}(r)v)^{\frac{5}{2}}(1+\bar{B}(r)v)^{\frac{1}{2}}}$$

is given in Fig. 4. It shows that the region of greatest interest is 0.4 $(\text{GeV})^{-1} \leqslant r \leqslant 1.2\ (\text{GeV})^{-1}$, corresponding to $0.8\cdot10^{-14}\ \text{cm} \leqslant r \leqslant 2.4\cdot10^{-14}\ \text{cm}$.

The Fourier transform of $S_f(x)$ in the Symanzik region ($p^2 < 0$) may now be computed, using the method described in Sect. **4**. We obtain

$$\tilde{S}_f(p) = (2\pi)^4 \delta^4(p) + \tilde{S}_{1f}(p) \tag{B.11}$$

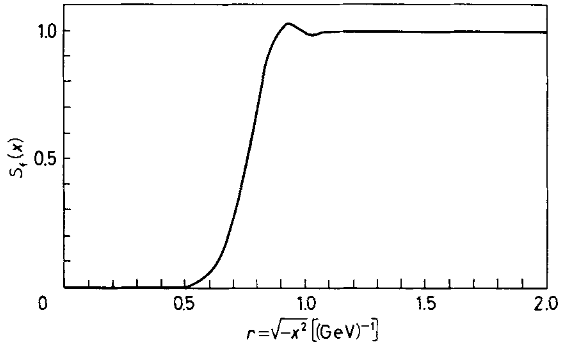

Fig. 4. – Numerical results for $S_f(x)$, the pure f-meson massive superpropagator, in Euclidean co-ordinate space.

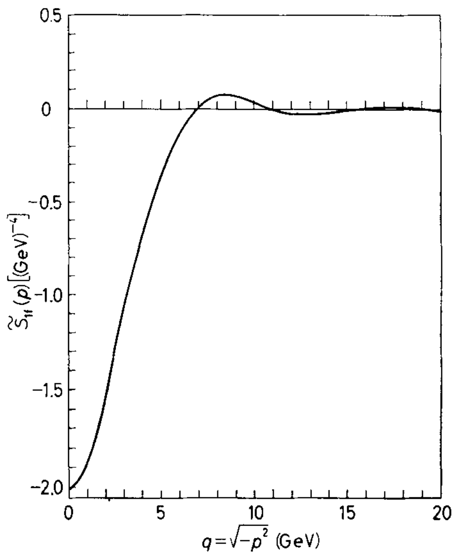

Fig. 5. – Numerical results for the Fourier transform of $S_f(x)$ in the Symanzik region.

with the numerical values shown in Fig. 5. Its large magnitude compared with $\tilde{S}_1(p)$ for the f-g case, follows from the difference in coupling constants $\varkappa_f$ and λ_g ($\varkappa_f/\lambda_g \simeq 10^{19}$).

● RIASSUNTO (*)

Lavorando entro lo schema della teoria della gravità forte usando lagrangiani non polinomiali, si è studiato il superpropagatore *con massa* per un campo *tensoriale* misto consistente nel campo gravitazionale senza massa di Einstein e nel campo della gravità forte del mesone f con massa. L'espressione compatta finale per il superpropagatore con massa nello spazio delle coordinate euclidiano ha la forma di un integrale unidimensionale caratterizzato da poli e tagli di diramazioni. Si è dedotta una rappresentazione integrale simile per il « puro » superpropagatore del mesone f. Che entrambi gli integrali siano reali è garantito da una prescrizione di media. Si sono eseguiti calcoli numerici del superpropagatore con massa sia nello spazio x euclidiano che, per la corrispondente trasformata di Fourier, nella regione di Symanzik degli impulsi esterni.

(*) *Traduzione a cura della Redazione.*

Вычисление массивного суперпропагатора в модели смешивания f-мезон–гравитон.

Резюме (*). — Работая в рамках теории сильной гравитации, используя неполиномиальные Лагранжианы, мы исследовали *массивный* суперпропагатор для смешанного *тензорного* поля, состоящего из гравитационного поля Эйнштейна с нулевой массой и сильного гравитационного поля массивного f-мезона. Окончательное компактное выражение для массивного суперпропагатора в эвклидовом координатном пространстве имеет форму однократного интеграла, характеризующегося полюсами и разрезами ветвлений. Аналогичное интегральное представление было выведено для « чистого » f-мезонного суперпропагатора. Реальность обоих интегралов гарантируется рецептом усреднение. Численные вычисления массивного суперпропагатора были проведены и в эвклидовом x-пространстве и для соответствующего Фурье-преобразования в области Симанзика для внешних импульсов.

(*) *Переведено редакцией.*