Optimization in Castings—An Overview of Relevant Computational Technologies and Future Challenges

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The manufacture of defect-free components at low cost and high productivity is as important to the casting industry today as it was 30 years ago. In the past, experience was gained either by using a ''trial and error'' method or by undertaking expensive experiments. Many ''dos'' and ''don'ts'' have evolved in the casting process over a period of time. However, the important ones that come to mind are so fundamental that they challenge the ''academic mind'' to think all over again. The rules proposed by Professor John Campbell[1] are classic examples. The message is simple: mathematical complexity in computer models needs to go hand in hand with the rules derived from ''first principles.'' In the field of optimization, a variety of methods have been proposed over a period of years. At the start of optimization study, the foundryman's first choice is to use simple but well-established methods such as the use of orthogonal arrays for optimal design of process conditions or the famous ''inscribed'' or Heuvers' circle method[2] for optimal feeding design. The computer simulation software has been based on a variety of computational methods ranging from geometric reasoning techniques (the famous Chvorinov rule and its variants)[11,13,15,29–31] to solving complex partial differential equations using one of the numerical methods. Optimization methods based on solving partial differential methods was an active area of research in the mid-1990s.[6–10,17] This article reviews a variety of optimization methods including—probably for the first time—geometric reasoning methods. The contribution from various computational methodologies is highlighted with particular emphasis on characterizing ''objective functions'' and ''constraints.'' The article also raises some of the challenging issues that the optimization community is facing today for solving casting problems and reports on our recent work on linking geometric reasoning techniques with the finite element method (FEM) and other data mining tools to achieve computationally efficient optimal design of casting processes.

I. INTRODUCTION

ACHIEVING high quality control over solidifying castings is of paramount importance for the manufacturing of critical components that are made up of very expensive metals and superalloys. The design of a casting process often depends on a combination of ''trial and error'' methods based on foundrymen's many years of accumulated nonquantitative experience and intuition. For many decades, researchers have communicated with the foundrymen by developing and prescribing a series of process guidelines, simple heuristics, empirical rules, and criteria. The classic example of such work is reflected in Professor John Campbell's ten rules,[1] which are built on the four rules identified earlier. These incorporate the latest technological developments for producing high quality and reliable castings. Similarly, his six feeding rules[2] that satisfy the heat transfer, volume, feed path, thermal, geometrical, and pressure criteria are another example of work that has led and attracted further research in this field. The evolution of ''dos and don'ts'' in a casting process in the form of simple rules (*e.g.*, part orientation rules, parting plane rules, gate rules, runner rules, sprue rules, and riser rules) is a well-documented example of such communication.[3] Bralla's[4] 12 design rules were also a step forward in this direction. Good casting designs are assessed using these established rules. Engineers have to make a series of design decisions to obtain a defect-free quality casting at low cost. Table I provides an insight into some of the decision-making choices that confront foundrymen at the time of casting.

The casting research fraternities developed these criteria and challenged the simulation community to model, simulate, and implement the criteria-based castings designs on computers. Two parallel streams of research strategies emerged that took on the challenge, using either numerical modeling tools or geometric reasoning techniques, to undertake optimization work in casting processes. In engineering terminology, optimization is understood as a set of strategies to determine process parameters that maximize or minimize some aspect of a process (casting in this case), while ensuring that the process operates within established limits or constraints.[5] Casting process optimization has facilitated foundrymen in making correct choices but still remains one very challenging area that has drawn the attention of many researchers during the last two decades. Most aspects of physics involved in casting are now well established and understood and have been successfully modeled. The numerical modeling community is continuously working to capture and simulate the less understood phenomena of physics, *e.g.*, oxide film, bubble damage, and provision of high quality liquid melt.

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II. OPTIMIZATION TECHNIQUES

This section will now focus on various optimization techniques developed by researchers over the decades and put

Table I. A Partial List of Choices for Different Casting Processes

Design Decisions	Type of Casting				
	Sand Casting	Gravity Die Casting	Pressure Die Casting	Investment Casting	Squeeze Casting
Feeder (number, size, shape, and location)	●	●	●	●	●
Interface conditions (heat transfer coefficients, chills, insulations, and padding)	●	●			
Mold (cooling channels, chills, and heating the mold)	●	●	●		●
Filling and running system	●	●	●	●	●

Table II. Optimization at a Glance

Decisions Method for Optimization	Optimized Design			
	Feeders (Number, Size, and Shape Location)	Interface Conditions (Chills, Insulations, and Die-Coating Thickness)	Mold (Cooling Channels, Chills, and Heating Mold)	Filling and Running System
Modulus method[11]	●	●	●	
Heuvers' circle method[2,11]	●	●	●	
Tiryakioglu *et al*.[14,15,16]	●			
Upadhya and Paul[3]	●			
Dantzigand co-workers,[6–8,10] Chen and Tortorelli[9]	●	●	●	●
Morthland *et al.*[7]	●			
Ransing *et al.*[17,27,28,33]	●	●	●	●

into use in foundry industry. Table II provides an overview of some of the achievements/milestones attained in the field of casting optimization by various researchers. Professor Dantzig and his associates made significant contributions to casting optimization through their pioneering work in the mid-1990s, which is reflected in a series of publications.[6–10] The authors defined the objective functions for optimal feeder/riser design and implemented efficient sensitivity methods based on direct differentiation and adjoint variable technique coupled with the finite element method (FEM). On the other hand, researchers such as Ravi and Srinivasan,[12,13] Tiryakioglu *et al*.,[14,15,16] and Upadhya and Paul[3] based their methods on geometric reasoning techniques to achieve similar objectives in order to produce defect-free castings.

A. *Numerical Optimization*

Table III shows some of the objective functions defined and used by different authors over the years. Haftka and Grandhi[25] resolved problems of structural shape optimization in the mid 1980s and Tortorelli *et al*.[6] went on to apply it to casting. The latter introduced methods for optimizing solidification processes through shape and process parameters modifications. Sensitivity analysis is an important aspect of optimization procedures using gradient-based methods. The authors presented the shape sensitivity analysis for the thermal system and coupled this analysis with nonlinear programming to optimize the design of a sand casting. The geometric modeler used by them allowed shape deformation using a mapping technique, thereby facilitating shape optimization. Morthland *et al*.[7] combined finite element analysis of the solidification process with sensitivity analysis based on the efficient direct differentiation method and numerical optimization to optimize feeder dimensions and volume. Ebrahimi *et al*.[8] extended this approach for the investment casting and their numerical optimization algorithm used the sensitivities to calculate the gradient information and upgrade the design until an optimum was found. Chen and Tortorelli[9] investigated a problem facing unification of computer-aided design (CAD) and FEM packages, *i.e*., relating the finite element nodal coordinates to the CAD solid model dimensions. They developed the critical link between these CAD and FEM data and successfully optimized a three-dimensional connecting rod model to exemplify their method for shape optimization.

Considering that interfacial heat transfer is a critical controlling parameter for gravity die casting, Ransing[26] proposed a regression equation so that the temporal variation of inter- facial heat-transfer coefficients could be optimized to produce a desired solidification path. This equation allowed the temporal variation of heat-transfer coefficients to be used as design variables in the subsequent optimization analysis.[17,27] Later this model was linked with a "thermal stress model"[28] to give realistic initial values for interface heat-transfer coefficients. These values were predicted by calculaing air gap widths at corresponding locations at the interface. Lewis *et al*.[22] proposed an efficient method for interpolating sensitivity values from previous time-steps and used this method to optimally design chill locations for a sand casting process. Recently,

Table III. Objective Functions Used in Optimization Research Using Numerical Methods

Author	Objective Function/Fitness Function/Cost Function	
Tortorelli *et al.*[6]	$G = \frac{1}{2}\left[(T_1 - T_2 + b_{19})^2 + T_2 - T_3 + b_{20})^2 + (T_3 - T_4 + b_{21})^2\right]$	The cost function ensured a positive vertical temperature gradient from casting to feeder to eliminate porosity in the casting.
Morthland *et al.*[7]	$G(b) = \sum_{e=1}^{N_r} \int_{\Omega_{0e}} \lvert J_e \rvert dV_0$	G is the volume of N_r elements in the riser region that is minimized.
Ebrahimi *et al.*[8]	$G = G(T(b),b)$ and $F_i = F_i(T(b),b)$	G represents the riser volume that is minimized while the constraint function F_i enforces directional solidification.
McDavid and Dantzig[10]	$G = \sum_{i=1}^{N_A} \int_0^{tf} f_i dt$	G represents the amount of fluid in contact with runner wall and is maximized, thus minimizing pocket formation.
Ransing and Lewis[17]	$\text{Cost} = \sum_{i=1}^{S_n - 1} p \max[(t_{f_{i+1}} - t_{f_i}), 0]$	Cost function is the deviation from the user-defined feed metal flow path.
Ransing *et al.*[18]	$\begin{cases} \omega = \omega_{\min} & \text{if } 0 \leq \theta \leq \theta_{\text{in}} \\ \omega = \dfrac{k_1}{\arctan\left(\dfrac{k_2}{s}\right)} \cdot \arctan\left(\dfrac{\theta - k_3}{s}\right) + k_4 & \text{if } \theta_{\text{in}} \leq \theta \leq \theta_{\text{fin}} \\ \omega = \omega_{\max} & \text{if } \theta > \theta_{\text{fin}} \end{cases}$	The function describes the relation between the tilting speed ω and the tilting angle θ. This function also allows changing of the shape of the function $\omega = \omega(\theta)$ with only one shape parameters s.
Wolf *et al.*[19]	$FF = w_1$ Mass of Metal 1 w_2 Dir Solid Value 1 w_3 Niyama Value + w_4 Disp Value $FF = \sqrt{\sum_{i,j} \left(T_{i\,\text{calculated}}(t_j) - T_{i\,\text{measured}}(t_j)\right)^2}$	
Singh *et al.*[21]	$F(x) = \sum_{i=1}^{N} \left(t_i^{\text{expt}} - t_i^{\text{model}}\right)^2$ and $F(x) = \sum_{i=1}^{2N} \sum_{j=1}^{M} \left(T_j^{\text{model}} - T_j^{\text{expt}}\right)^2$ $F(x) = (t1\_\text{pro} - t1\_\text{des})^2 + (t2\_\text{pro} - t2\_\text{des})^2$	$F(x)$ represents optimized filling time, temperature, and casting cycle time, respectively.
Lewis *et al.*[22]	$F(X)\{T_A - T_B + C\} + W \times \left\{\dfrac{\text{vol of feeder}}{\text{max vol of feeder}}\right\}$	$F(X)$ included both thermal and volume components.
Lin[23]	Obj $= w_1 \times$ (cooling parameter a) + $w_2 \times$ (cooling parameter b) + $w_3 \times$ (cooling parameter of channel diameter d)	"Obj" optimized the cooling parameters of the injection mold cooling system.
Lin[24]	Obj $= w^*$ (minimum deformation)(cooling system parameters: L,D,R)	"Obj" optimized the gate position for minimum deformation.

Ransing *et al.*[18] numerically modeled the tilt casting phenomenon by keeping the mold stationary and rotating the gravitational force vector. A mathematical expression relating tilting angle with tilting speed was proposed so that an optimal value of tilting speed could be predicted by minimizing splashing effects.

Wolf *et al.*[19] used optimization consisting of inverse analysis of thermomechanical data to fit the results of temperature field calculation to a set of experimental temperature data. The values of interfacial heat-transfer coefficients for a set of ten fixed temperatures were used as optimization parameters and these interpolated values were fed into a finite element based solver for calculation of the temperature field. Their fitness function as given in Table III was built on the difference between calculated and experimental cooling curves. Zabaras *et al.*[20] optimized the boundary heat flux and calculated gradient of the objective function in L_2 space to design optimum mold cooling/heating conditions. Singh *et al.*[21] used optimization procedures to decrease the design cycle time for the casting of a wheel. The authors first optimized the metal flow velocity to match the filing time and then optimized the temperature-dependent heat-transfer coefficient between the metal and mold to match the computed and experimental cooling curves.

Lin[23] proposed a neural network-based approach for optimization of injection-mold cooling parameters and designed the injection mold cavity and cooling system model by comparing the value of error using FEM and neural network prediction. The author used "simulated annealing" techniques to obtain optimal cooling system parameters based on the objective functions given in Table III for the injection mold cooling system[23] and die-casting die.[24]

It is thus gathered from the preceding discussion that research in casting technology has been laboring toward achieving "near optimal" solutions while attempting to keep the computational cost as low as possible. Trial and error methods based on experiments and intuitive and accumulated hands-on experience are quicker and easier to implement, but they do not necessarily always provide the optimal solution. Gradient-based methods provide near optimal designs with higher computational costs. The newly emerging evolutionary computing techniques, however, may lead us toward the optimal designs and solutions but are computationally very expensive (Figure 1). The availability of skilled foundrymen, computing power, relative need for higher quality components, and ease of implementation then influences the decision making on choosing one of these methods.

As illustrated in Figure 2, even though gradient-based methods provide local or global minima, it is advisable to avoid a region of steep gradients of the objective function (A in Figure 2). The aim should rather be to obtain robustness in casting process designs by seeking and opting for a relatively less steep gradient of the objective function that is achieved at (B). This will ensure a robust casting design that is more stable to fluctuations or small changes in design variables or process parameters as the change in the objective function value is minimum (*e.g.*, as in region B).

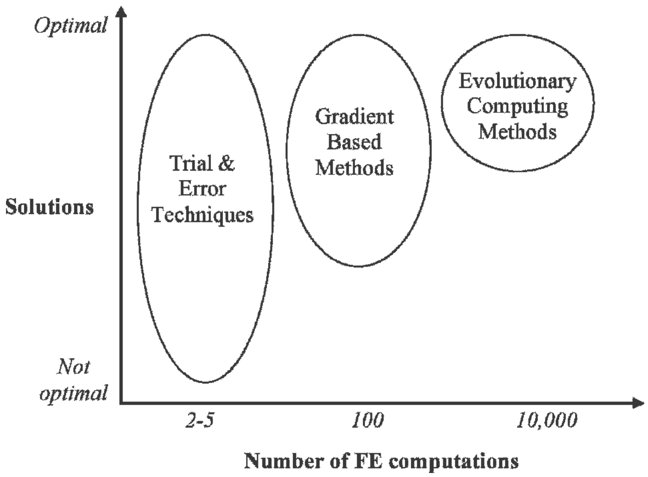

Fig. 1—Performance of various methods.

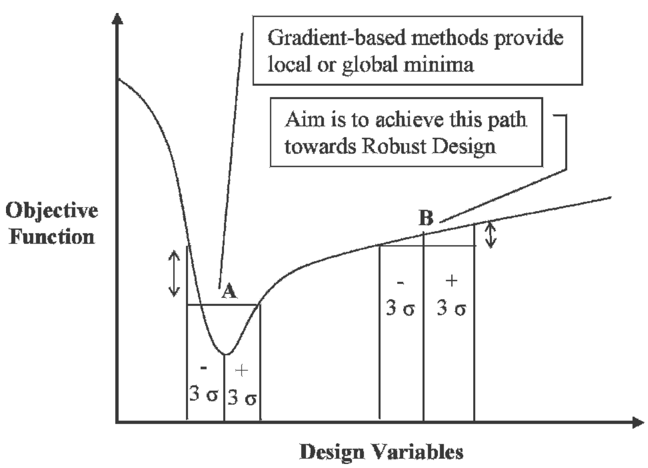

Fig. 2—Aiming for robustness in casting designs.

B. *Geometric Optimization*

In parallel with evolution of numerical optimization methods, some conventional approaches to solidification analysis and optimization have been driven by the casting geometry that essentially influences the sequence of solidification. These approaches linked geometric parameters with the thermal properties of the metal, mold, and heat-transfer systems.[13] One of the earliest optimization efforts was based on the modulus method that had its origin in Chvorinov's classic rule,[11] which related solidification time t_s of a casting to its modulus. The term modulus pertains to the ratio of heat content volume V to the heat-transfer area A of the casting.

$$\text{Solidification time } t_s = k\left(\frac{V}{A}\right)^2 \quad [1]$$

where k is a material constant depending on the cast and mold material. Derivatives of Chvorinov's rule have since been investigated and expanded by many researchers, *e.g.*, Wlodawer,[11] Berry *et al.*,[29] Heine and Uicker,[30] Neises *et al.*,[31] Luby *et al.*,[32] Ravi and Srinivasan,[12,13] and Tiryakioglu *et al.*[14,15,16] Table IV provides a summary of achievements in this stream of optimization research.

Using experimental data, Tiryakioglu *et al.*[15] examined and characterized the effectiveness of nine feeder models and rewrote these models to separate the effects of shape and size of casting. They then determined optimum feeder sizes for a variety of casting volumes and shapes for Al-Si eutectic alloy. The authors then went on to test the validity of the assumption that the casting and feeder solidify simultaneously and analyzed the heat and mass exchange between the feeder and casting during solidification and its effect on the solidification time of castings. Challenging

Table IV. Achievements Made by Researchers Using Geometric Techniques

Author	Proposal	Description of Work
Wlodawer[11]	$M_{CASTING} : M_{NECK} : M_{FEEDER}$ 1 : 11 : 1.2	Used Chvorinov's rule to design the feeders in such a way that the modulus (M) of the feeder is greater than that of the casting and must increase by 10 pct from the casting across the ingate to the feeder for ensuring adequate feeding.
Tiryakioglu *et al.*[14,15,16]	$t = B'k^{1.31}V^{0.67}$ $k = \frac{A_s}{A} = \frac{4.837V^{\frac{2}{3}}}{A}$ $t_t = t_f = at_c$	Pointing to limitations in Chvorinov's rule, authors proved that the modulus includes the effect of both casting shape and size and proposed that the shape factor (k) separate these two independent factors. Using their own superheat model, authors found that the solidification time of optimum-sized feeders in a feeder-casting combination was only fractionally longer than that of the casting (a = 1.046 for Al-12 pct Si alloy and 1.005 for steel castings).
Upadhya and Paul[3]	$PM = \frac{2}{\sum_{i=1}^{N}\frac{1}{d_i}}$	Used Chvorinov's rule to determine a continous modulus for complex shapes discretized in a three-dimensional grid, and the point modulus was then used to calculate the solidification time map.
Ravi and Srinivasan[12]	$M = \sum_i w_i c_i$	Authors developed 30 criteria functions to assess the influence of various features in a casting and then gave this relationship to assess the manufacturability of casting design by a weighted evaluation of all the features using all the criteria.

the conventional mindset of researchers that considered the feeders and castings separately for calculating solidification times and only accounted for mass transfer from feeder to casting, assuming that the transfer takes place isothermally and at pouring temperature, Tiryakioglu *et al*.[16] treated the casting-feeder combination as a single total casting and argued that the thermal center of this combination should be in the feeder. They used the superheat model that is based on the equality of the solidification times of the feeder and total casting. This contribution allowed the geometric reasoning methods to ensure that the thermal center of the total casting is retained in the feeder.

Upadhya and Paul[3] incorporated foundrymen's intuitive skills, accumulated experience, and developed a knowledge-based integrated design system to optimize gating and risering. They applied some empirical heuristics to the discretized solid model and performed a geometric analysis to determine the natural flow path for liquid metal. The risering routine then used a solidification time map to find hotspots and locate feeders at these spots to ensure adequate feeding. Ravi and Srinivasan[12] studied various guidelines and rules on geometric features employed by practicing engineers and came up with a list of 30 castability criteria. The authors then described manufacturability assessment for casting by presenting these criteria as equations in terms of influencing parameters.

C. *The New Combined Approach Based on Linkage of Heuvers' Circle Method to the FEM*

The conventional Heuvers' circle method[2,11] based on the modulus principle is another example of implementing a simple rule to achieve directional solidification. The method consists of inscribing a series of circles, the diameter of which increases in the direction of the feeder head, *e.g.*, from point 1 to point 2 (Figures 3(a) and (b)). This method invited criticism, because all padding additions (Figure 3(c)) have to be removed using expensive dressing operations, and like other geometric methods, it remains a qualitative analysis that is insensitive to material properties and boundary conditions.

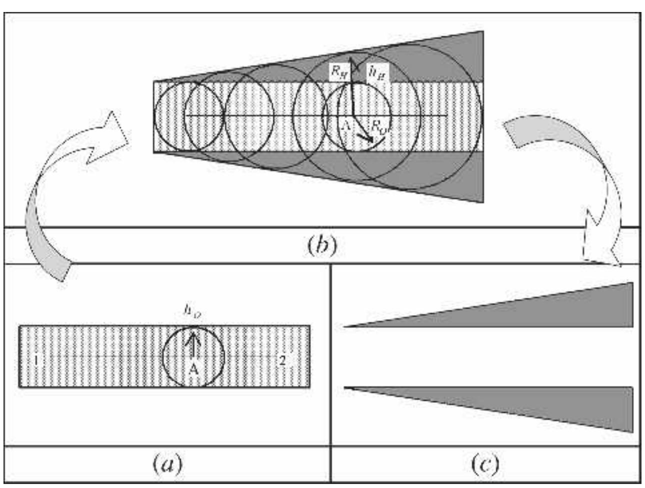

Fig. 3—Heuvers' circle method: (*a*) original casting section, (*b*) the casting with Heuvers' circles, and (*c*) the additional padding required per Heuvers' circles.

Ransing *et al*.[33] proposed a novel method, which inherited the advantages of geometric methods and at the same time derived a relative variation of heat-transfer coefficient values that helped in achieving the desired solidification pattern in castings. As is observed from Figure 3(b), a casting with a series of Heuvers' circles drawn is in fact a new imaginary casting whose modulus is increasing in the direction of the feeder. The authors obtained effective interface boundary conditions using the Heuvers' inscribed circles in such a manner that these boundary conditions helped in achieving the same solidification pattern as would have been provided by the imaginary casting. Their method first used the Heuvers' inscribed circles to predict the hotspot and then related the geometric information (radii of inscribed circles) to obtain effective interface boundary conditions at different locations (*e.g.*, at point A) along the medial axis of casting using equation

$$\therefore h_H = h_O \left(\frac{R_O}{R_H} \right)^2$$

where R_O and h_O are original radius and interfacial heat-transfer coefficient, and R_H and h_H are Heuvers' radius and the modified interfacial heat-transfer coefficient, respectively.

Finally, finite element based numerical analysis was used to accurately simulate the optimized design. Their combined method proposed a geometric optimization technique for ensuring directional solidification and relocating hotspots in the feeder. This geometry-based optimization method outputs initial optimal values of interfacial heat-transfer coefficients, which then can be used in the finite element analysis for further detailed and accurate optimization. When implemented, the method considerably cuts down the number of finite element simulations during the design cycle.

III. FUTURE CHALLENGES: SELF-LEARNING DIAGNOSTIC ALGORITHMS FOR OPTIMAL PROCESS DESIGN

The quality, productivity, and cost of manufactured components are influenced by a large number of process steps and material and design considerations. The grand challenge for the optimization and artificial intelligence community is to develop a computer that not only is capable of optimizing the entire casting process but could also learn from failures, provide corrective actions automatically, and evolve with time. In particular, it is envisaged that the intelligent factory of the future should have the following capabilities: (1) monitor all process control parameters, (2) capture real time process data from a network of distributed sensors, (3) analyze data and suggest feedback/corrective action—if necessary—on its own initiative, (4) keep traceability and control of all and each of the casting/mold produced in the foundry, and (5) allow changes of the process parameters "on demand."

The analysis of "cause and effect" relationships is a challenging task requiring years of experience and specialized knowledge. Such intuitive skills and knowledge gathered over the years from their hands-on experience often lies with experienced foundrymen. For an industry, it is very important to document such experts' knowledge, because when an expert either retires or leaves the job, his expertise is lost forever to the employer. The casting industry is still facing a challenge of storing this knowledge in order to build a computer-based ''institutional memory.''

The representation of the cause and effect relationship on the computer is a key issue in developing such intelligent algorithms. It has been argued by the authors[34] that a first step toward developing intelligent process design algorithms is to encode the entire cause and effect relationship in the form of a ''defect-metacause-remedial action'' format (Figure 4). The ''remedial actions'' are the process, design, or material parameters that can be changed or controlled in the production environment, *e.g.*, pouring temperature, location and size of feeders, vent designs, and cycle time. ''Metacauses'' are the scientific rationale or the underlying physical concepts that relate remedial actions to ''defects,'' *e.g.*, ''improper directional solidification,'' ''quality of melt,'' and ''turbulent flow.''

One of the major limitations of the optimization algorithms developed to this point is that they can only optimize a part of the process. The numerical methods attempt to solve complex mathematical equations that describe the physics identified by one or two metacauses. In the context of numerical methods, the subset of associated remedial actions is represented as material properties, shape or geometry of the domain, and boundary or initial conditions. There are many other process parameters associated with the corresponding metacauses, but they are not accounted for by the numerical simulation software.

The research led by Ransing *et al*.[36,37,38] has focused on developing self-learning algorithms that cannot only learn the entire cause and effect relationships from past examples but can also use this knowledge to optimally design the process by taking into account all process, design, and material parameters. The methodology is summarized in Figure 5.

The research has also resulted in a patented software technology—X1Recall—that has the ability to remember the corrective actions taken by experts to bring a process under control and to also expand its knowledge base to diagnose problems occurring under different circumstances. In the future, we expect such technologies to evolve into developing a next generation manufacturing system—SLEAMS: self-learning, self-evolving autonomous manufacturing system that may meet the grand challenge, as described earlier.

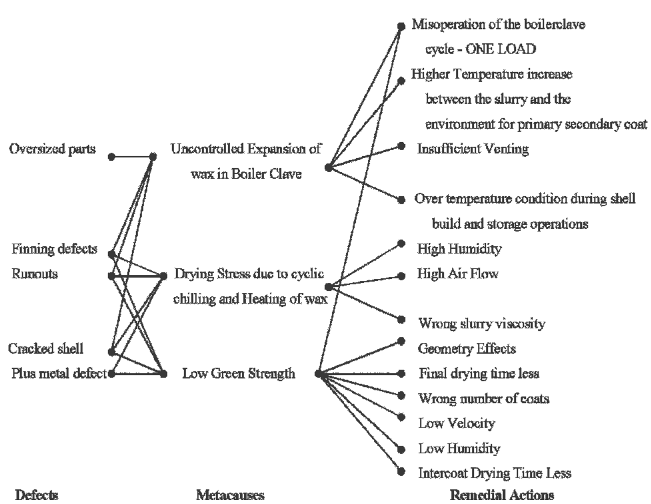

Fig. 4—Knowledge representation of the causal relationship in a defect-metacause-remedial action format.

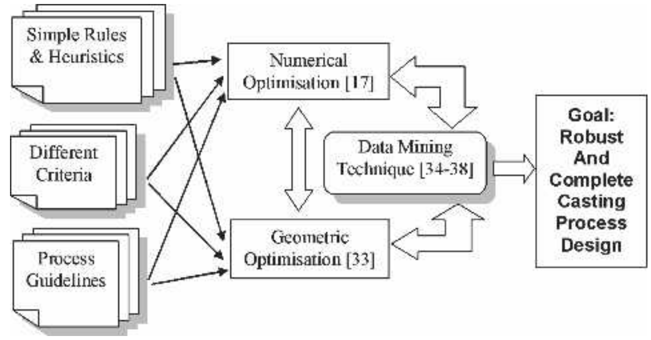

Fig. 5—Optimal design of the entire casting process.

IV. CONCLUSIONS

The ultimate goal of using computers and optimization techniques for the casting process is to design the process optimally, to set optimal process conditions, to learn from its mistakes, and to acquire new knowledge automatically; in short, to exhibit autonomous behavior. The challenge is to achieve the maximum degree of automation with the smallest amount of human intervention. This is demanding, because until recently the casting process was perceived as a ''black art.'' With the scientific rethinking of the process—along the lines suggested by Professor John Campbell—the day is not far when computers can also exhibit some intelligence!

This article has reviewed optimization techniques used over the last three decades covering both numerical and geometric reasoning methods. The past landmarks and milestones have been discussed and future challenges have been identified.

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