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# On the Effect of n-p Tensor Forces in $^3H_\Lambda$ - I (*).

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**Summary.** — Calculations have been done on the hypertriton with a hamiltonian containing a neutron-proton potential with a tensor part. The integral volume of $\Lambda$-$\mathcal{N}$ interaction and the wave function parameters are modified to a certain extent with respect to the results of preceding calculations, performed with a purely central potential. It is concluded that a more refined calculation will be necessary to get a definite answer to the question of the importance of n-p tensor corrections in $^3H_\Lambda$.

## 1. – Introduction.

This paper contains some preliminary results of a work on the effect of tensor forces among nucleons in $\Lambda$-hyperfragments. We have done a calculation on the simplest system containing a $\Lambda^0$, that is the hypertriton. The reason for this choice is that the hypertriton is the hyperfragment with the simplest structure and does not require drastic approximations for its description. The preceding calculations [1] have been performed using a central neutron-proton potential. The parameters of the potential were determined by the prescription to fit the p-p low energy scattering data and the deuteron binding energy. We have introduced into the hamiltonian a neutron-proton tensor term: the aim of this work is to see if the introduction of these corrections influences in some way the prediction of integral volumes of the $\Lambda$-nucleon

(*) This work has been presented at the XLVI Congress of the Italian Physical Society, Naples, 29 September - 5 October 1960.

[1] R. H. DALITZ and B. W. DOWNS: *Phys. Rev.*, **110**, 958 (1958) (this work will be indicated by D.D.I); R. H. DALITZ and B. W. DOWNS: *Phys. Rev.*, **114**, 593 (1959).

potential. At the same time we want to see if there are modifications is the wave function which could influence the calculation of decay probabilities and branching ratios for the various decay channels. Such a calculation will be done in a subsequent paper.

In this work we have restricted ourselves to a preliminary calculation, which has been done in the simplest way compatible with tensor corrections.

## 2. – Construction of trial wave functions.

The possible values of the spin of hypertriton are $\frac{1}{2}$ and $\frac{3}{2}$. We must therefore construct trial functions corresponding to such spin values. To discuss this we shall use the following notation (*):

$$\boldsymbol{L} = \boldsymbol{l}_1 + \boldsymbol{l}_2 , \qquad \boldsymbol{s} = \tfrac{1}{2}(\boldsymbol{\sigma}_p + \boldsymbol{\sigma}_n) , \qquad \boldsymbol{S} = \boldsymbol{s} + \tfrac{1}{2}\boldsymbol{\sigma}_\Lambda ,$$

$$\boldsymbol{L} + \boldsymbol{s} = \boldsymbol{j} , \qquad \boldsymbol{L} + \boldsymbol{S} = \boldsymbol{J} ,$$

where $\boldsymbol{l}_1$ is the neutron-proton relative angular momentum,

$\boldsymbol{l}_2$ is the angular momentum of the $\Lambda$ with respect to neutron-proton center of mass,

$\boldsymbol{\sigma}_p$, $\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_\Lambda$ are the Pauli spin vectors for the proton, neutron and $\Lambda$, respectively.

The selection of trial wave functions is strongly determined by the following consideration: suppose that we are performing a variational calculation for the binding energy of some system and that we would like to use a trial function which is a linear combination of the functions $\Psi$ and $\Psi'$. Suppose further that the matrix element of the hamiltonian between $\Psi$ and $\Psi'$ is zero: then the mean value of the hamiltonian will reach its minimum in correspondence of the choice of only $\Psi$ or only $\Psi'$. In our case this is equivalent to say that we must choose as trial wave functions only those composed of parts which are linked by the neutron-proton central potential plus tensor part. Let us now limit our discussion to the case of spin $\frac{1}{2}$; then the maximum value of $L$ is two. Let us list the spin functions which may constitute our trial function; these are the following (**):

$S_1$) $\qquad L = 0 \qquad s = 0 \qquad l_1 = l_2 = 0$

$S_2$) $\qquad L = 0 \qquad s = 1 \qquad l_1 = l_2 = 0$

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(*) We have chosen units such that $\hbar = c = 1$.

(**) We have omitted the functions $S_{3,4}$: $L=0$, $s=0, 1$, $l_1=l_2=1$ because they are excluded on the same grounds of $D_3$. Note also that the symbol $P$ represents three functions which correspond to different values of $S$ and $s$.

$P$) $\quad L = 1 \quad l_1 = l_2 = 1$

$D_1$) $\quad L = 2 \quad S = \frac{3}{2} \quad l_1 = 2 \quad l_2 = 0$

$D_2$) $\quad L = 2 \quad S = \frac{3}{2} \quad l_1 = 0 \quad l_2 = 2$

$D_3$) $\quad L = 2 \quad S = \frac{3}{2} \quad l_1 = l_2 = 1$

Preceding calculations [1] have used $S_2$. We have chosen a linear combination of $S_2$ and $D_1$ and have excluded the others for the following reasons:

*a*) $S_1$ can be excluded because neither central forces nor tensor forces link spin 1 to spin 0 states.

*b*) $P$ and $D_3$ states are excluded by the fact that neutron-proton forces do not mix proton-neutron states with different orbital parity.

*c*) $D_2$ is excluded by the fact that we assume the $\Lambda$-$\mathcal{N}$ potential does not contain tensor terms.

In the case of spin $\frac{3}{2}$ for the hypertriton, the preceding considerations must be partially modified. It is permitted to consider also the value $L = 3$. On the other side this value is excluded for it would require $l_1 = l_2 = 2$ and such a state would not be linked with the others by the potentials. State $S_1$ cannot be constructed for spin requirements. State $D_1$ is splitted in two states according to the fact that $S$ can assume both the values $\frac{1}{2}$ and $\frac{3}{2}$. We shall call these states $D_1'$ and $D_1''$.

The trial wave functions for spin $\frac{1}{2}$ and $\frac{3}{2}$ are therefore given by:

$$(1) \qquad \begin{cases} \Psi_{\frac{1}{2}} \propto f(r, s, t)\, \Phi_{S_2} + x\, g(s, r, t)\, \Phi_{D_1}\,, \\ \\ \Psi_{\frac{3}{2}} \propto f(r, s, t)\, \Phi_{S_1'} + x\, g(r, s, t)\, \Phi_{D_1'} + y\, h(r, s, t)\, \Phi_{D_1''}\,, \end{cases}$$

where $f$, $g$, $h$ are normalized functions of the triangular co-ordinates $r$, $s$, $t$ ($r$ is the neutron-proton distance, $s(t)$ the $\Lambda$-proton (neutron) distance); $x$ and $y$ are coefficients whose square gives the percentage of $D$ states. The symbols $\Phi$ represent the normalized spin functions corresponding to the states defined by their subscripts. The complete form of functions (1) is given in Appendix A.

For the radial functions $f$, $g$, $h$ we have chosen simple exponentials of the form

$$f \propto \exp\left[-\alpha(s+t) - \beta r\right], \qquad g \text{ and } h \propto r^2 \exp\left[-\gamma(s+t) - \delta r\right],$$

symmetrical in $s$ and $t$ according to charge symmetry of $\Lambda$-nucleon forces. To simplify the calculations we have also chosen $g = h$ in $\Psi_{\frac{3}{2}}$. This is not strictly necessary but results in a considerable reduction of variational parameters.

### 3. – Variational calculation.

The hamiltonian which we shall use in the variational calculation is the following

$$H = K + V[\Lambda] + V_c[np] + V_t[np] ,$$

where $K$ is the three-body kinetic energy and

$$V[\Lambda] = - (U_1 + U_2 \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_\Lambda) \frac{\exp[-\lambda s]}{s^n} - (U_1 + U_2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_\Lambda) \frac{\exp[-\lambda t]}{t^n}$$

($n = 0$, exponential shape; $n = 1$, Yukawa shape),

$$V_c[np] = - (W_1 + W_2 \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_p) \frac{\exp[-kr]}{r} , \tag{2}$$

$$V_t[np] = - W_3 \frac{\exp[-\eta r]}{r} S_{np}(\boldsymbol{r}) , \qquad S_{np}(\boldsymbol{r}) = \frac{1}{r^2} [3(\boldsymbol{\sigma}_n \cdot \boldsymbol{r})(\boldsymbol{r} \cdot \boldsymbol{\sigma}_p) - (\boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_p) r^2] .$$

The variational calculation is then stated in the following way: if $\Psi$ is the trial function then the inequality must hold

$$- (B_d + B_\Lambda) \leqslant \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} , \tag{3}$$

$B_\Lambda$ is defined as the difference between the absolute value of the $^3H_\Lambda$ binding energy and the absolute value of the deuteron binding energy, $B_d$.

In the case of spin $\frac{1}{2}$ the expression on the right side of (3) can be written

$$\frac{\langle \psi_{\frac{1}{2}} | H | \psi_{\frac{1}{2}} \rangle}{\langle \psi_{\frac{1}{2}} | \psi_{\frac{1}{2}} \rangle} = P + Q(U_1 + U_2) - R(U_1 - 2U_2) , \tag{4}$$

$P$, $Q$, $R$ are functions of the variational parameters and of the potential parameters. The whole dependence on $U_1$ and $U_2$ has been shown explicitely. Now $Q$ is, with respect to $P$ and $R$, of order $x^2$, which is presumed to be smaller than 0.05 (*). This is the reason why we have set in place of $(U_1 + U_2)$ in the right-hand side of (4) the value deduced from the results of DALITZ and

(*) To justify this point one might notice that the $^3H_\Lambda$ can be considered as composed by a deuteron with a $\Lambda$ bound to it. According to the small value of $B_\Lambda$, one can presume that the deuteron is not much deformed by the presence of the $\Lambda$ and so the $D$-state percentage in the $^3H_\Lambda$ must be of the order of that in the deuteron.

DOWNS (2). Such an approximation would not change substantially our results. (4) can now be written as

$$\frac{\langle \psi_{\frac{1}{2}} | H | \psi_{\frac{1}{2}} \rangle}{\langle \psi_{\frac{1}{2}} | \psi_{\frac{1}{2}} \rangle} = P' - R' \bar{U}_2 \ (J = \tfrac{1}{2}) ,$$

where $\bar{U}_2(J = \frac{1}{2})$ is the total integral volume for spin $\frac{1}{2}$:

$$\bar{U}_2(J = \tfrac{1}{2}) = \frac{16\pi}{\lambda^3} (2U_2 - U_1) , \quad \text{for exponential shape,}$$

$$= \frac{8\pi}{\lambda^2} (2U_2 - U_1) , \quad \text{for Yukawa shape .}$$

From (3) one gets

$$\bar{U}_2(J = \tfrac{1}{2}) \leqslant \frac{1}{R'} [P' - (B_d + B_\Lambda)]; \tag{5}$$

(5) can be written, showing the $x$-dependence of the right side, in the following form:

$$\bar{U}_2(J = \tfrac{1}{2}) \leqslant G[A + Bx + Cx^2] , \tag{6}$$

where $G$, $A$, $B$, $C$ are functions of $\alpha$, $\beta$, $\gamma$, $\delta$ and of the potential parameters, but $U_1$ and $U_2$.

In the case of spin $\frac{3}{2}$, the expression on the right side of (3) has the form

$$\frac{\langle \psi_{\frac{3}{2}} | H | \psi_{\frac{3}{2}} \rangle}{\langle \psi_{\frac{3}{2}} | \psi_{\frac{3}{2}} \rangle} = P_1 + Q_1(2U_1 - U_2) - R_1(U_1 + U_2) ,$$

$Q_1$ is of order $(x^2 + y^2)$; with the preceding approximation, one can obtain, in conclusion, the analogous of (6) for spin $\frac{3}{2}$

$$\bar{U}_2(J = \tfrac{3}{2}) \leqslant G\left[A + \frac{B}{\sqrt{2}} (x + y) + C(x^2 + y^2)\right] , \tag{7}$$

where $G$, $A$, $B$, $C$ are the same functions as before. $\bar{U}_2(J = \frac{3}{2})$ is given by

$$\bar{U}_2(J = \tfrac{3}{2}) = \frac{16\pi}{\lambda^3} (U_1 + U_2) , \quad \text{for exponential shape,}$$

$$= \frac{8\pi}{\lambda^2} (U_1 + U_2) . \quad \text{for Yukawa shape.}$$

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(2) For exponential shape see: R. H. DALITZ and B. W. DOWNS: *Phys. Rev.*, **111**, 967 (1958). For Yukawa shape see: R. H. DALITZ and B. W. DOWNS: *Phys. Rev.*, **114**, 593 (1959). In the latter case we have used the results for gaussian shape because there is not substantial difference.

The right sides of (6) and (7) are simple functions of the mixing parameters $x$ and $y$ and one could search for the minimum with respect to them. The minimum of the right side of (6) is reached for

$$x = -B/2C\,,$$

which gives

$$\bar{U}_2(J=\tfrac{1}{2}) \leqslant G\left[A - \frac{B^2}{4C}\right]. \tag{8}$$

In (7) one finds that the minimum is obtained if

$$x = y = -\frac{\sqrt{2}B}{4C}\,,$$

and (7) reduces to

$$\bar{U}_2(J=\tfrac{3}{2}) \leqslant G[A + \sqrt{2}Bx + 2Cx^2] = G\left[A - \frac{B^2}{4C}\right],$$

which has the same form of (8). Then there is no difference between the spin $\frac{1}{2}$ and $\frac{3}{2}$ cases except on the different dependence of $\bar{U}_2$ upon $U_1$ and $U_2$. From now on we shall consider only, for definiteness, the case of spin $\frac{1}{2}$ Then (8) represents the basis of our calculations. The detailed form of $G$, $A$, $B$ and $C$ is given in Appendix $B$.

## 4. – Results and discussion.

The parameters of the neutron-proton potential (2) are not rigorously determined by the experimental data. Then the variational calculation has been done for five sets of parameters (see Table I) selected from Table VI of Feshbach and Schwinger's paper ([3]); all the sets give a triplet effective range which agrees with the experimental value. It is worth while to notice that these five potentials give the correct deuteron binding energy. We have used the preceding potentials to make a variational calculation on the deuteron for two reasons:

i) We wanted to have a reasonable starting point for the parameters which describe the deuteron in the wave function of ${}^3H_\Lambda$.

ii) We wanted to check the validity of the approximation of our treatment of the deuteron core of hypertriton.

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([3]) H. FESHBACH and J. SCHWINGER: *Phys. Rev.*, **84**, 194 (1951).

TABLE I. – *Neutron-proton potential parameters.*

| Neutron-proton potential parameters | $W_1+W_2$ (MeV·fermi) | $W_3$ (MeV·fermi) | $k$ (fermi$^{-1}$) | $\eta$ (fermi$^{-1}$) | $\lvert x \rvert$ |
|---|---|---|---|---|---|
| *a*) | 34.46 | 20.99 | 0.7402 | 0.7236 | 0.205 |
| *b*) | 46.92 | 16.88 | 0.8425 | 0.6524 | 0.195 |
| *c*) | 49.82 | 16.92 | 0.9050 | 0.6524 | 0.198 |
| *d*) | 82.46 | 6.19 | 1.0341 | 0.3619 | 0.145 |
| *e*) | 75.11 | 9.32 | 1.0341 | 0.4708 | 0.167 |

The trial function is given in Appendix B. The results of the calculation are listed in Table II. $B_d^*$ represents our estimate of the binding energy of the deuteron.

TABLE II. – *Binding energy $B_d^*$ and wave function parameters of the deuteron for the potentials of Table I.*

| Neutron-proton potential | $B_d^*$ (MeV) | $\beta$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ |
|---|---|---|---|---|
| *a*) | —1.206 | $0.51_5$ | 2.09 | —0.173 |
| *b*) | —1.246 | $0.52_5$ | 2.00 | —0.166 |
| *c*) | —1.216 | $0.53_5$ | $2.02_5$ | —0.167 |
| *d*) | —1.364 | 0.60 | $1.68_5$ | —0.129 |
| *e*) | —1.323 | $0.58_5$ | $1.83_5$ | —0.146 |

The common feature of the results is that the absolute value of the binding energy and of the *S-D* mixing parameter are too small. Such a fact shows that the neutron-proton pair will be poorly described by the ${}^3H_\Lambda$ wave function (1). A better calculation will require an improvement of this part of the ${}^3H_\Lambda$ description: this will be done in a subsequent paper.

As a test of the importance of tensor corrections in hypertriton we have performed a whole set of calculations which could be compared with those

TABLE III. – Λ-*nucleon interaction in* $^3H_\Lambda$. The intrinsic range 0.8411 fermi corresponds to exchange of a K-meson; the value 1.4843 corresponds to exchange of two pions. The numbers marked with an asterisk are the results of D.D.I.

*a*) Yukawa potential shapes.

| $B_d+B_\Lambda$ (MeV) | (i) intrinsic range 0.8411 fermi | | | | | | $B_d+B_\Lambda$ (MeV) | (ii) intrinsic range 1.4843 fermi | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) | | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) |
| 2.226 | $0.66_5$ | $0.59_5$ | $0.64_5$ | $2.49_5$ | −0.177 | 522 | 2.226 | 0.40 | 0.55 | $0.38_5$ | 2.24 | −0.172 | 833 |
| | 0.62* | 0.69* | — | — | — | 489* | | 0.36* | 0.65* | — | — | — | 766* |
| 2.626 | 0.69 | 0.60 | 0.67 | $2.52_5$ | −0.177 | 527 | 2.626 | $0.42_5$ | $0.55_5$ | $0.40_5$ | 2.27 | −0.172 | 853 |
| | 0.64* | 0.71* | — | — | — | 494* | | 0.39* | 0.65* | — | — | — | 786* |
| 3.226 | $0.72_5$ | 0.61 | $0.70_5$ | $2.57_5$ | −0.176 | 533 | 3.226 | 0.46 | $0.56_5$ | 0.44 | $2.31_5$ | −0.172 | 879 |
| | 0.67* | 0.71* | — | — | — | 501* | | 0.43* | 0.66* | — | — | — | 813* |

*b*) Exponential potential shapes.

| $B_d+B_\Lambda$ (MeV) | (i) intrinsic range 0.8411 fermi | | | | | | $B_d+B_\Lambda$ (MeV) | (ii) intrinsic range 1.4843 fermi | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) | | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) |
| 2.226 | $0.60_5$ | 0.59 | $0.57_5$ | 2.45 | −0.175 | 510 | 2.226 | 0.37 | 0.55 | $0.35_5$ | $2.22_5$ | −0.171 | 818 |
| | 0.52* | 0.52* | — | — | — | 492* | | 0.29* | 0.52* | — | — | — | 742* |
| 2.626 | $0.62_5$ | $0.59_5$ | $0.57_5$ | 2.45 | −0.175 | 516 | 2.626 | 0.39 | 0.55 | 0.37 | 2.24 | −0.171 | 841 |
| | 0.55* | 0.53* | — | — | — | 500* | | 0.33* | 0.52* | — | — | — | 772* |
| 3.226 | 0.65 | 0.60 | 0.58 | 2.46 | −0.175 | 523 | 3.226 | $0.41_5$ | 0.56 | $0.39_5$ | $2.30_5$ | −0.171 | 871 |
| | 0.58* | 0.53* | — | — | — | 510* | | 0.37* | 0.52* | — | — | — | 810* |

of D.D.I with a central potential. The results are collected in Table III with the corresponding ones of D.D.I. The latters are marked with an asterisk. All these calculations have been done with the potential of type *b*). Such a potential practically coincides in this case with the more general Hall and Powell's one ([4]), which fits the p-p low-energy scattering data.

Comparing the results of the present work with those of D.D.I, one sees that there is some difference between our wave function parameters and integral volumes and the corresponding quantities of D.D.I. Further the integral volumes increases more than one would expect from the difference between our value for $B_d^*$ and the one of D.D.I. It is doubtful, however, that the potential *b*) is the most suitable to describe the n-p interaction. To improve this point we have done a set of calculations with the five potentials of Table I. These have been done for $B_\Lambda = 0.12$ MeV ([5]). Taking account of the fact that the best value of $B^*$ of D.D.I is 1.6 MeV([*]) and that our $B_d^*$ is smaller of some tenth of MeV (see Table II), these results may be compared with the results of D.D.I for $B_\Lambda = 0.4$ MeV. We have restricted ourselves to the case of a $\Lambda$-$\mathcal{N}$ interaction of Yukawa shape. This results are listed in Table IV.

Inspection of Table IV shows that the best agreement with the results of D.D.I is obtained with the potential of type *d*). However also for this potential the value of the mixing parameter remains low. On the other hand the biggest difference between our integral volumes and the corresponding ones of D.D.I is of about 10 %. Moreover there is a certain difference in the corresponding function parameters. These differences might be relevant on the determination of the $\Lambda$-nucleon well depths. The variation of function parameters might also influence the prediction of decay ratios, but this point ought to be checked by a calculation.

## 5. – Conclusion.

The present work shows that a tensor term in the neutron-proton potential might have some influence in ${}^3H_\Lambda$. There are differences between our results and those obtained by D.D.I with a purely central neutron-proton potential, both in the integral volumes and the wave function parameters. Our results, however, are dependent upon the choice of the neutron-proton potential among the five equivalent potentials of Table I. In order to get a definite answer about the origin of these differences it is necessary to perform

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([4]) H. H. HALL and J. L. POWELL: *Phys. Rev.*, **90**, 912 (1953).

([5]) The value $B_\Lambda = (0.12 \pm 0.26)$ MeV is the one given at the Kiev Meeting of 1959.

([*]) See D.D.I, p. 963.

TABLE IV. – *Results for the various* n-p *potentials of Table I and* $\Lambda$-$\mathcal{N}$ *Yukawa shape.* $B_d + B_\Lambda = 2.346$ MeV.

| i) intrinsic range 0.8411 fermi | | | | | | |
|---|---|---|---|---|---|---|
| Neutron-proton potential | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) |
| *a*) | $0.67_5$ | $0.57_5$ | 0.66 | $2.57_5$ | −0.192 | 528 |
| *b*) | 0.67 | $0.59_5$ | 0.65 | 2.50 | −0.177 | 523 |
| *c*) | 0.67 | 0.61 | 0.65 | $2.52_5$ | −0.177 | 520 |
| *d*) | $0.65_5$ | $0.70_5$ | $0.62_5$ | $2.23_5$ | −0.110 | 497 |
| *e*) | 0.66 | 0.68 | $0.63_5$ | $2.36_5$ | −0.136 | 504 |

| ii) intrinsic range 1.4843 fermi | | | | | | |
|---|---|---|---|---|---|---|
| Neutron-proton potential | $\alpha$ (fermi$^{-1}$) | $\beta$ (fermi$^{-1}$) | $\gamma$ (fermi$^{-1}$) | $\delta$ (fermi$^{-1}$) | $x$ | $\bar{U}_2$ (MeV·fermi$^3$) |
| *a*) | 0.41 | $0.53_5$ | $0.39_5$ | $2.32_5$ | −0.184 | 848 |
| *b*) | 0.41 | 0.55 | $0.39_5$ | $2.24_5$ | −0.172 | 840 |
| *c*) | $0.41_5$ | $0.56_5$ | 0.40 | 2.28 | −0.173 | 835 |
| *d*) | 0.40 | $0.64_5$ | 0.38 | $1.96_5$ | −0.117 | 797 |
| *e*) | 0.40 | $0.62_5$ | 0.38 | 2.10 | −0.140 | 808 |

a detailed calculation with a more flexible wave function. This is necessary to make it possible to decide if the differences we have found can affect the prediction of the $\Lambda$-$\mathcal{N}$ well depths and of the decay ratios.

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## APPENDIX A

**Spin functions.**

The detailed form of the spin functions used in the work is the following:

i) spin $J = \frac{1}{2}$

$$(\Phi_{S_2})^\alpha = \frac{1}{\sqrt{24\pi^2}} \boldsymbol{\sigma}_\mathrm{p} \cdot \boldsymbol{\sigma}_\Lambda \varphi^\alpha ,$$

$$(\Phi_{D_1})^\alpha = \frac{1}{\sqrt{48\pi^2 r^2}} S_{jk} (\boldsymbol{\sigma}_\mathrm{p})_j (\boldsymbol{\sigma}_\Lambda)_k \varphi^\alpha ,$$

ii) spin $J = \frac{3}{2}$

$$(\Phi_{S_1'})^\alpha_j = \frac{1}{6\pi} \left( \boldsymbol{\sigma}_\mathrm{p} + \frac{i}{2} \boldsymbol{\sigma}_\mathrm{p} \times \boldsymbol{\sigma}_\Lambda \right)_j \varphi^\alpha ,$$

$$(\Phi_{D_1'})^\alpha_j = \frac{1}{12\pi r^2} S_{jk} (\boldsymbol{\sigma}_\mathrm{p} - i \boldsymbol{\sigma}_\mathrm{p} \times \boldsymbol{\sigma}_\Lambda)_k \varphi^\alpha ,$$

$$(\Phi_{D_1''})^\alpha_j = \frac{1}{12\pi r^2} \left( S_{jk} (\boldsymbol{\sigma}_\mathrm{p})_k - i \varepsilon_{jkl} (\boldsymbol{\sigma}_\mathrm{p})_k S_{lm} (\boldsymbol{\sigma}_\Lambda)_m \right) \varphi^\alpha ,$$

where

$$\boldsymbol{r} = \boldsymbol{r}_\mathrm{p} - \boldsymbol{r}_\mathrm{n} ,$$

and

$$S_{jk} = 3 x_j x_k - \delta_{jk} r^2 , \qquad (x_1 = x,\ x_2 = y,\ x_3 = z),$$

is the irreducible tensor of angular momentum 2. $\varepsilon_{jkl}$ is the completely antisymmetric tensor of 3rd order. Further

$$\varphi^\alpha = \chi^0 \chi^\alpha(\Lambda) , \qquad \chi^0 = \frac{1}{\sqrt{2}} [\chi^{\frac{1}{2}}(p) \chi^{-\frac{1}{2}}(n) - \chi^{-\frac{1}{2}}(p) \chi^{\frac{1}{2}}(n)] ,$$

where $\chi^s(P)$ represents the Pauli spin function for the particle $P$. $\chi^0$ is the singlet function for the proton-neutron pair. The functions listed before are not, in general, eigenfunctions of the third component of the total spin $J$.

All the functions are normalized. Those for spin $\frac{3}{2}$ are normalized according to

$$\sum_j (\Phi^\alpha_j)^* \Phi^\alpha_j = 1 .$$

## APPENDIX B

**Complete wave functions.**

The trial function used for the variational calculation of $B_d^*$ is the following

$$\Psi_d = \frac{1}{\sqrt{1+x^2}} (C_1 \exp[-\beta r]\sigma_j + x\, C_2 \exp[-\delta r] S_{jk}\sigma_k)\chi^0 ,$$

where $C_1$ and $C_2$ are normalization constants and $\boldsymbol{\sigma}$ represents the spin vector of one of the nucleons.

The trial functions used in the calculations for the ${}^3H_\Lambda$ have the following detailed form:

$$\Psi_{\frac{1}{2}} = \frac{1}{\sqrt{1+x^2}}\left[\frac{1}{\sqrt{I_{111}(2\alpha, 2\alpha, 2\beta)}} \exp[-\alpha(s+t) - \beta r]\Phi_{S_1} + \right.$$
$$\left. + \frac{x}{\sqrt{I_{115}(2\gamma, 2\gamma, 2\delta)}} \exp[-\gamma(s+t) - \delta r]\Phi_{D_1}\right],$$

$$\Psi_{\frac{3}{2}} = \frac{1}{\sqrt{1+x^2+y^2}}\left[\frac{1}{\sqrt{I_{111}(2\alpha, 2\alpha, 2\beta)}} \exp[-\alpha(s+t) - \beta r]\,\Phi_{S_2'} + \right.$$
$$+ \frac{x}{\sqrt{I_{115}(2\gamma, 2\gamma, 2\delta)}} \exp[-\gamma(s+t) - \delta r]\,\Phi_{D_1'} +$$
$$\left. + \frac{y}{\sqrt{I_{115}(2\gamma, 2\gamma, 2\delta)}} \exp[-\gamma(s+t) - \delta r]\Phi_{D_1''}\right].$$

The normalized spin functions $\Phi$ are given in Appendix A. $I_{abc}(\alpha, \beta, \gamma)$ is defined by

$$I_{abc}(\alpha, \beta, \gamma) = (-)^{a+b+c} \frac{\partial^{a+b+c}}{\partial\alpha^a\, \partial\beta^b\, \partial\gamma^c} I_{000}(\alpha, \beta, \gamma) ,$$

where

$$I_{000}(\alpha, \beta, \gamma) = \int \mathrm{d}r\, \mathrm{d}s\, \mathrm{d}t \exp[-\alpha r - \beta s - \gamma t] = \frac{2}{(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)} ,$$

$I_{abc}$ is given explicitely by

$$I_{abc}(\alpha, \beta, \gamma) = I_{000}(\alpha, \beta, \gamma)\, a!\, b!\, c! \sum_{i=0}^{a} \sum_{j=0}^{b} \sum_{k=0}^{c} \binom{a-i+j}{j} \binom{b-j+k}{k} \binom{c-k+i}{i} \cdot$$
$$\cdot \frac{1}{(\alpha+\beta)^{a-i+j} (\beta+\gamma)^{b-j+k} (\gamma+\alpha)^{c-k+i}} .$$

The explicit form of the four functions $G$, $A$, $B$, $C$ (which is the same both for spin $\frac{1}{2}$ and spin $\frac{3}{2}$) is

$$A = \frac{\alpha+\beta}{8\alpha^2+5\alpha\beta+\beta^2}\left[(16\alpha^3+9\alpha^2\beta+4\alpha\beta^2+\beta^3)+\frac{m_\Lambda-m}{m_\Lambda+m}\beta(5\alpha^2+4\alpha\beta+\beta^2)-\right.$$
$$\left.-\frac{8m_\Lambda m}{m_\Lambda+m}(W_1+W_2)\frac{[(2\beta+4\alpha+k)^2+4\alpha^2](\alpha+\beta)^4}{(2\alpha+2\beta+k)^4}\right]+\frac{2m_\Lambda m}{m_\Lambda+m}(B_d+B_\Lambda),$$

$$B = \frac{2^{10}m_\Lambda m}{m_\Lambda+m}\sqrt{\frac{3}{5}}W_3\sqrt{\frac{\alpha^3(\alpha+\beta)^5\gamma^3(\gamma+\delta)^9}{(8\alpha^2+5\alpha\beta+\beta^2)(80\gamma^2+27\gamma\delta+3\delta^2)}}\cdot$$
$$\cdot\frac{35(\alpha+\gamma)^2+18(\alpha+\gamma)(\beta+\delta+\eta)+3(\beta+\delta+\eta)^2}{(\alpha+\gamma)^3(\alpha+\beta+\gamma+\delta+\eta)^6},$$

$$C = \frac{(\gamma+\delta)}{80\gamma^2+27\gamma\delta+3\delta^2}\left[(128\gamma^3+79\gamma^2\delta+24\gamma\delta^2+3\delta^3)+\right.$$
$$+\frac{m_\Lambda-m}{m_\Lambda+m}(32\gamma^3+67\gamma^2\delta+24\gamma\delta^2+3\delta^3)-\frac{128m_\Lambda m}{m_\Lambda+m}(W_1+W_2)\cdot$$
$$\cdot\frac{(\gamma+\delta)^8[84\gamma^2+16\gamma(2\delta+k)+(2\delta+k)^2]}{(2\gamma+2\delta+k)^8}-256M\gamma^3(\gamma+\delta)^8I_{5n1}(2\delta,2\gamma+\lambda,2\gamma)+$$
$$\left.+192\frac{m_\Lambda m}{m_\Lambda+m}W_3[84\gamma^2+16\gamma(2\delta+\eta)+(2\delta+\eta)^2]\left(\frac{\gamma+\delta}{2\gamma+2\delta+\eta}\right)^8\right]+$$
$$+\frac{2m_\Lambda m}{m_\Lambda+m}(B_d+B_\Lambda),$$

where $n=0$ for Yukawa shape of the $\Lambda$-$\mathcal{N}$ potential, $n=1$ for exponential shape. $M$ is a coefficient given in the following table

| | | |
|---|---|---|
| exponential shape | $\lambda = 2.3858$, | $M = 0.13396$, |
| | $\lambda = 4.2102$, | $M = 0.70509$, |
| Yukawa shape | $\lambda = 1.4280$, | $M = 0.0493$, |
| | $\lambda = 2.5200$, | $M = 0.13765$. |

The function $G$ also depends on the shape of the $\Lambda$-$\mathcal{N}$ potential. For Yukawa shape it is given by

$$G = \frac{\pi}{16\lambda^2}\frac{m_\Lambda+m}{m_\Lambda m}\frac{(2\alpha+2\beta+\lambda)^2(4\alpha+\lambda)^2(8\alpha^2+5\alpha\beta+\beta^2)}{\alpha^3(\alpha+\beta)^2[(4\alpha+2\beta+\lambda)^2+4\alpha\beta]}.$$

while for exponential shape it is

$$G = \frac{\pi}{16\lambda^3} \frac{m_\Lambda + m}{m m_\Lambda} \frac{(2\alpha + 2\beta + \lambda)^3 (4\alpha + \lambda)^3 (8\alpha^2 + 5\alpha\beta + \beta^2)}{\alpha^3 (\alpha + \beta)^2 [(4\alpha + 2\beta + \lambda)^3 + 4\alpha\beta(2\alpha + \lambda)]}.$$


## RIASSUNTO

Conti effettuati sul $^3H_\Lambda$ con una hamiltoniana contenente il potenziale neutrone-protone con parte tensoriale mostrano che il volume integrale della interazione $\Lambda$-nucleone ed i parametri della funzione d'onda son modificati rispetto ai risultati di conti precedenti basati su un potenziale totalmente centrale. Si conclude che è necessario un conto più raffinato per avere una risposta definitiva sulla importanza delle correzioni tensoriali nell'ipertritone.