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Study of Nonindependent Random Pulse Trains, with Application to the Barkhausen Noise.

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Summary. — This work covers the calculation of the correlation function and power spectrum of a train of pulses of exponential shape and random amplitude, distributed according to a law of probability experimentally found by H. Sawada in a research on the time interval distribution in the Barkhausen effect. All the features of the experimental power spectra of the Barkhausen noise, found by many authors to be contradictory with the interpretation of statistical independence of pulses, can thus be completely explained. The remarkable physical significance of the introduced distribution gives the results a more general interest.

1. – Introduction.

It is known that many random processes of physical interest can be reduced to trains of overlapping random pulses. The problem can be defined from a statistical standpoint, by giving the distribution of time intervals which describes the shape of the single pulse. This function is generally assumed to be the same for any pulse, except, at the very most, for an amplitude factor, which depends upon the pulse considered. Of course, both the distribution function and the shape of the single pulse depend upon the particular problem under consideration. Studies of trains of overlapping pulses were made by many authors [1-3], but, as a matter of fact, except some results of

(1) S. O. Rice: *Bell. Sys. Tech. Journ.*, **23**, 282 (1944).
(2) D. Middleton: *An Introduction to Statistical Communication Theory* (New York, 1960).
(3) T. W. Lee: *Statistical Theory of Communication* (New York, 1960).

a general nature obtained by S. O. RICE ([1]) on the mean square value of noise, other works generally assume that pulses are statistically independent, and consequently that time intervals are distributed according to Poisson's law of probability, as follows:

$$P(x) = \nu \exp[-\nu x]\,. \tag{1.1}$$

In the above formula, $P(x)$ is the probability density of time interval x between subsequent pulses, and ν is the average number of pulses per unit time.

This is justified by the great importance which this case assumes from a physical standpoint, as many random phenomena, such as shot noise, thermal noise and other, are—or can be considered so as a rough approximation—constituted by independent elementary processes. Nevertheless in some cases the statistical independence of pulses cannot be assumed.

A quite different type of distribution from Poisson's was found by H. SAWADA ([4]) in a research on the time interval distribution in the Barkhausen effect (noise in ferromagnetic materials).

Assuming the symbols as having the same meaning as in eq. (1.1), this distribution can be written as follows:

$$P(x) = 4\nu^2 x \exp[-2\nu x]\,. \tag{1.2}$$

This law of probability, plotted in Fig. 1, has an important physical significance, deduced in the paper by H. SAWADA mentioned before, and derives from the fact that the probability of an occurrence, in this case, becomes a linearly increasing function of time, when pulse repetition frequency tends to zero, instead of becoming constant, as in Poisson's case.

It can be used, at least as an approximation, in practically all the cases in which the correlation between the elementary processes, which enter into the noise formation, is such that each occurrence has an inhibiting effect on the subsequent one which decreases in time.

Of course, for this kind of correlation, other types of distributions can be found, which better fit each particular case. They are, however, more complicated functions of x than eq. (1.2). That makes it almost impossible to calculate the most important statistical functions of noise, as the correlation function and power spectrum.

This paper will calculate the correlation function and power spectrum of a train of overlapping pulses of exponential shape, arbitrary time constant and random amplitude, distributed according to eq. (1.2).

([4]) H. SAWADA: *Journ. Phys. Soc. Japan*, **7**, 575 (1952).

This is chiefly in order to prove that this distribution can completely explain all the features of the experimental power spectra of the Barkhausen effect, which many authors ([5,6]) found to be inconsistent with the statistical

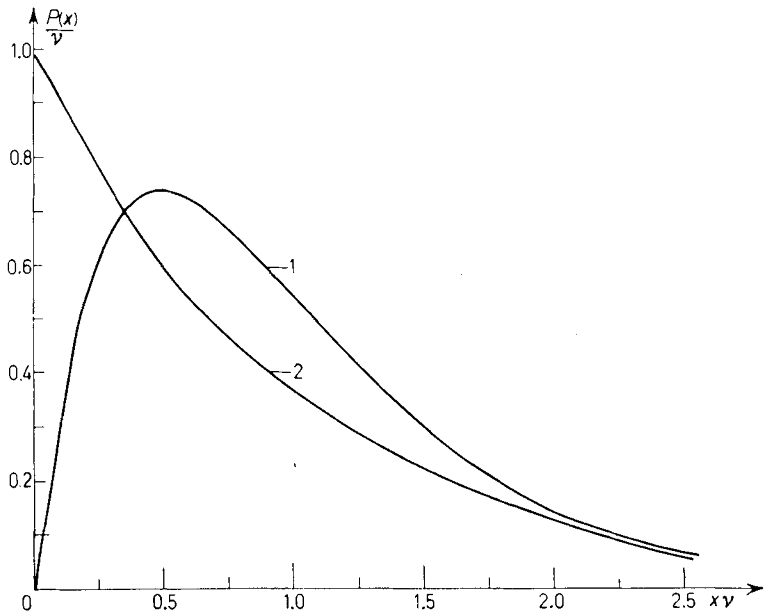

Fig. 1. – Curve 1: shape of the distribution function given by eq. (1.2), plotted on dimensionless co-ordinates; curve 2: shape of Poisson's distribution given by eq. (1.1), plotted on the same co-ordinates.

independence of pulses. The study of this noise is thus based upon new foundations.

The assumed shape of pulses is quite satisfactory in this case ([5,7]). Yet, for the reasons given above, the conclusions of this study have more general interest and can also be extended to trains of pulses having other than an exponential shape.

2. – Calculation of the correlation function and power spectrum.

In order to calculate the correlation function and the power spectrum of a train of overlapping pulses of exponential shape and random amplitude, distributed according to eq. (1.2), a very general expression of the correlation

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([5]) G. Biorci and D. Pescetti: *Journ. Appl. Phys.*, **28**, 777 (1957).

([6]) K. G. Warren: *Electronic Technol.*, **38**, No. 3, 89 (1961).

([7]) R. S. Tebble, I. C. Skidmore and W. D. Corner: *Proc. Phys. Soc. London*, A **63**, 739 (1950).

function shall be used, drawn from a work by G. BIORCI and P. MAZZETTI [8]. This proves that for a train of overlapping pulses, singly described by a function $a_i F(t)$, and distributed according to an arbitrary but normalized function $P(x)$, representing the probability density of the event that two subsequent pulses of the sequence are separated by a time interval x, the correlation function (by definition given by

$$\psi(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} I(t) I(t+\tau)\,\mathrm{d}t\,, \tag{2.1}$$

where $I(t)$ represents the summing function of all the pulses for any value of time) can be expressed as follows:

$$\psi(\tau) = \nu[\overline{a^2}\,\Phi(\tau) + \bar{a}^2(S_1 + S_2)]\,. \tag{2.2}$$

In the above equation, ν is the average number of pulses per unit time, $\overline{a^2}$ and $\bar{a}^2$ are respectively the mean square value and the square of the mean value of the random amplitude factors a_i, $\Phi(\tau)$ is the autocorrelation function of $F(t)$, defined by

$$\Phi(\xi) = \int_{-\infty}^{+\infty} F(t) F(t-\xi)\,\mathrm{d}t\,, \tag{2.3}$$

and calculated for $\xi = \tau$.

Finally, S_1 and S_2 are the following two series of integrals:

$$S_1 = \int_0^\infty P(x)\Phi(x+\tau)\,\mathrm{d}x + \int_0^\infty \mathrm{d}x_1 \int_0^\infty P(x_1)P(x_2)\Phi(x_1+x_2+\tau)\,\mathrm{d}x_2 + \int_0^\infty \mathrm{d}x_1 \int_0^\infty \mathrm{d}x_2 \int_0^\infty P(x_1)P(x_2)P(x_3)\Phi(x_1+x_2+x_3+\tau)\,\mathrm{d}x_3 + \ldots\,, \tag{2.4}$$

$$S_2 = \int_0^\infty P(x)\Phi(x-\tau)\,\mathrm{d}x + \int_0^\infty \mathrm{d}x_1 \int_0^\infty P(x_1)P(x_2)\Phi(x_1+x_2-\tau)\,\mathrm{d}x_2 + \int_0^\infty \mathrm{d}x_1 \int_0^\infty \mathrm{d}x_2 \int_0^\infty P(x_1)P(x_2)P(x_3)\Phi(x_1+x_2+x_3-\tau)\,\mathrm{d}x_3 + \ldots\,, \tag{2.5}$$

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[8] G. BIORCI and P. MAZZETTI: *L'Elettrotecnica*, **48**, 469 (1961), (in English).

where $P(x)$ is the distribution function previously defined, and Φ represents the function defined by eq. (2.3).

Equation (2.2) holds whatever function $F(t)$ is assumed to describe the shape of pulses.

In this specific instance

$$P(x) = k^2 x \exp[-kx] \qquad (k = 2\nu), \tag{1.2}$$

$$F(t) = 1(t) \exp[-\alpha t], \tag{2.6}$$

where $1/\alpha$ is the time constant assumed for the single pulse.

By substituting this expression of $F(t)$ in eq. (2.3), the following is easily obtained:

$$\Phi(\xi) = \frac{\exp[-\alpha|\xi|]}{2\alpha}.$$

The calculation of the sum of the two series of integrals S_1 and S_2 is given in the Appendix, with the following results:

$$S_1 = \frac{k^2 \exp[-\alpha\tau]}{2\alpha^2(\alpha + 2k)}, \tag{2.7}$$

$$S_2 = \frac{k}{8k^2 - 2\alpha^2} \exp[-2k\tau] - \frac{1}{2\alpha}\frac{k^2}{2k\alpha - \alpha^2} \exp[-\alpha\tau] + \frac{k}{2\alpha^2}. \tag{2.8}$$

From eq. (2.2), on the basis of eq. (2.7) and (2.8), the following is obtained:

$$\psi(\tau) = \frac{k}{2}\left[\frac{1}{2\alpha}\left(\overline{a^2} + \frac{2\overline{a}^2 k^2}{\alpha^2 - 4k^2}\right)\exp[-\alpha\tau] + \right. \\ \left. + \overline{a}^2 \frac{k}{8k^2 - 2\alpha^2}\exp[-2k\tau] + \overline{a}^2 \frac{k}{2\alpha^2}\right]. \tag{2.9}$$

This is the correlation function of the pulse train considered herein.

As an indication Fig. 2 shows the development of this function when

$$\overline{a^2} = \overline{a}^2$$

for a particular value of α/k (curve 1).

It can be seen that this function is not a monotonic decreasing function of τ, as in the case where pulses are statistically independent (curve 2).

The power spectrum of the same pulse train can be easily obtained from the correlation function of eq. (2.9), by means of Wiener's transformation:

$$\Phi_p(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(\tau) \exp[j\omega\tau]\,d\tau, \tag{2.10}$$

where, by convention, $\psi(\tau)$ is considered as extended by symmetry to the negative semi-axis of the τ's.

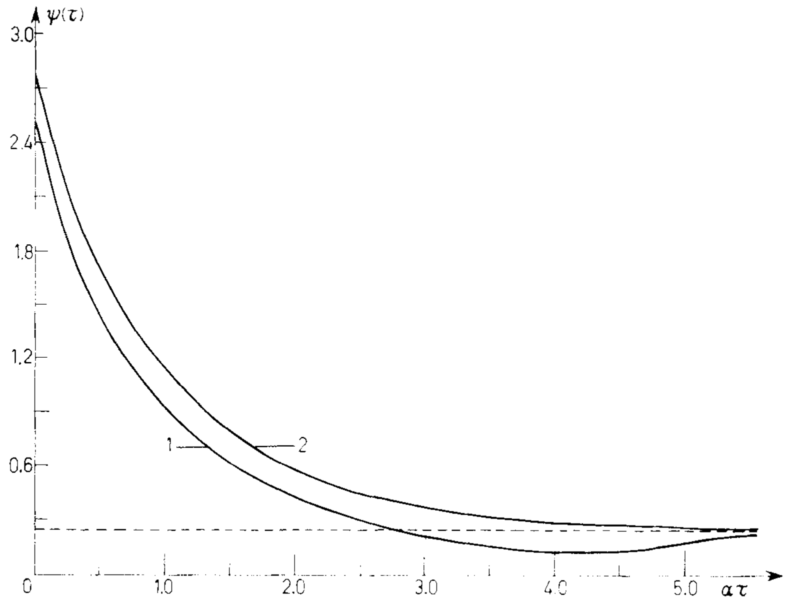

Fig. 2. – Curve 1: correlation function of a train of pulses distributed according to eq. (1.2) and calculated through eq. (2.9) ($\alpha/k = 10$ and $\overline{a^2} = \bar{a}^2 = 1$); curve 2: correlation function of the same pulse train, the pulses being distributed according to Poisson's law, given by eq. (1.1).

After making the integration at the second member of eq. (2.10), and bearing in mind that

$$\int_{-\infty}^{\infty} \exp[j\omega t]\,\mathrm{d}t = 2\pi\,\delta(\omega)\,,$$

where $\delta(\omega)$ is Dirac's impulsive unit function, the following is easily obtained:

$$\Phi_p(\omega) = \frac{\overline{a^2}\nu}{2\pi}\,\frac{1}{\alpha^2+\omega^2} - \frac{4\bar{a}^2\nu^3}{\pi}\,\frac{1}{(\alpha^2+\omega^2)(16\nu^2+\omega^2)} + \frac{\bar{a}^2\nu^2}{\alpha^2}\,\delta(\omega)\,. \tag{2.11}$$

The last term of the second member of this expression represents a line in the origin of the ω's, the area of which is equal to the square of the average value of function $I(t)$, as defined at the beginning of this paragraph.

Evidently this line must be found in the power spectrum of any pulse train, whatever the distribution the pulses are complying with.

Equation (2.11) can be verified by integrating it with regard to ω from $-\infty$ to $+\infty$: the result is the expression of the mean square value of $I(t)$ as given by O. RICE (9), provided calculations are made according to the distribution assumed herein (eq. (1.2)).

3. – Discussion of the results.

Equation (2.11) gives raise to several interesting observations.

Abstracting from the impulsive term in the origin of the ω's, which is not peculiar to the pulse train being studied here, as said before, it can be seen that the first term of the second member of eq. (2.11) represents the spectrum of a train of pulses identical to the pulses of the train to which eq. (2.11) relates, but statistically independent, *i.e.* distributed according to Poisson's law of probability:

$$P(x) = \nu \exp[-\nu x] . \tag{1.1}$$

This can be easily seen by Campbell's theorem, which is one of the most important consequences of Poisson's distribution. For a train of pulses of arbitrary shape it can be written as follows:

$$\Phi_p(\omega) = \nu\overline{a^2}\,|S(\omega)|^2 , \tag{3.1}$$

$S(\omega)$ being the Fourier spectrum of the single pulse, which in the present case is given by

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 1(t) \exp[-\alpha t] \exp[-j\omega t]\, \mathrm{d}t = \frac{1}{\sqrt{2\pi}} \frac{1}{\alpha + j\omega} .$$

However, the second term of the second member of the same expression is peculiar to the introduced distribution of probabilities (1.2), and appears to be a corrective term, taking into account the correlation between the pulses implicitly introduced by eq. (1.2).

It is always negative, and this involves that, in regard to a case of statistically independent pulses, said correlation causes a reduction to the power associated with the noise.

This is obviously a consequence of the fact that the assumed distribution involves a reduction of probability of an almost complete overlapping of subsequent pulses (see Fig. 1).

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(9) S. O. RICE: *Bell Sys. Tech. Journ.*, **23**, 305 (1944) eqs. (1.5)-(9).

It can also be noted that this term assumes its highest value (one half of the value of the first term, when $\overline{a^2} = \overline{a}^2$), for $\omega = 0$, and tends to become negligible in respect to the latter, either with ω tending to infinity or with ν tending to zero. This means that power reduction mostly takes place in the low-frequency area of the spectrum, and increases when the pulse repetition frequency increases.

When $\overline{a^2} = \overline{a}^2$, *i.e.* when the pulses have all the same amplitude a, eq. (2.11), disregarding the impulsive term, can be written as follows:

$$\Phi_p(\omega) = \frac{\nu}{2\pi} \frac{\overline{a}^2}{\alpha^2 + \omega^2} \left[1 - \frac{1}{2 + (1/8)(\omega/\nu)^2}\right], \tag{3.2}$$

whereby it appears that the deviations from the case of statistical independence of pulses become higher than $\sim 5\,\%$ for values of analysis frequency less than twice the pulse repetition frequency ν, with a trend towards $50\,\%$ for very low values in regard to ν.

Thus, frequency $f_0 = 2\nu$ can be taken to divide two areas of the spectrum. In the area of frequencies higher than f_0, Campbell's theorem, given by eq. (3.1), can be considered as being rather effective, while in the other area, it cannot be applied, and the spectrum must be calculated only on the basis of eq. (2.11).

It could also be useful to note that in case $\overline{a^2} \neq \overline{a}^2$, the influence of the term that takes into account the fact that pulses are not statistically independent, is reduced in any case.

4. – The case of the Barkhausen noise.

All the characteristics of the theoretical power spectra outlined in the discussion of eq. (2.11) fully agree with the power spectra of the Barkhausen noise experimentally derived.

This is evident in making a comparison between the theoretical curves of Fig. 3 and Fig. 4, derived directly from eq. (2.11), and the experimental curves of Fig. 5 drawn from a research presently being carried out on the power spectra of the Barkhausen noise.

It can be seen that also the experimental curves agree in the high frequency area with Campbell's theorem—eq. (3.1)—which is a consequence of Poisson's distribution, while in the low-frequency area the power density increases less than linearly with f_m (which is proportional to the average number of pulses per unit time).

The amount of reduction in respect to Campbell's theorem is remarkable in this case, being of the order of $50\,\%$.

Such an effect had already been observed in the experimental power spectra by G. Biorci and D. Pescetti (5), and lately, by K. G. Warren (6), relative to the total power of the noise.

It is important to observe that this feature of the experimental spectra is inconsistent with Poisson's distribution for any shape assumed for the single Barkhausen pulse, and also if a dispersion in the time constants of pulses exists.

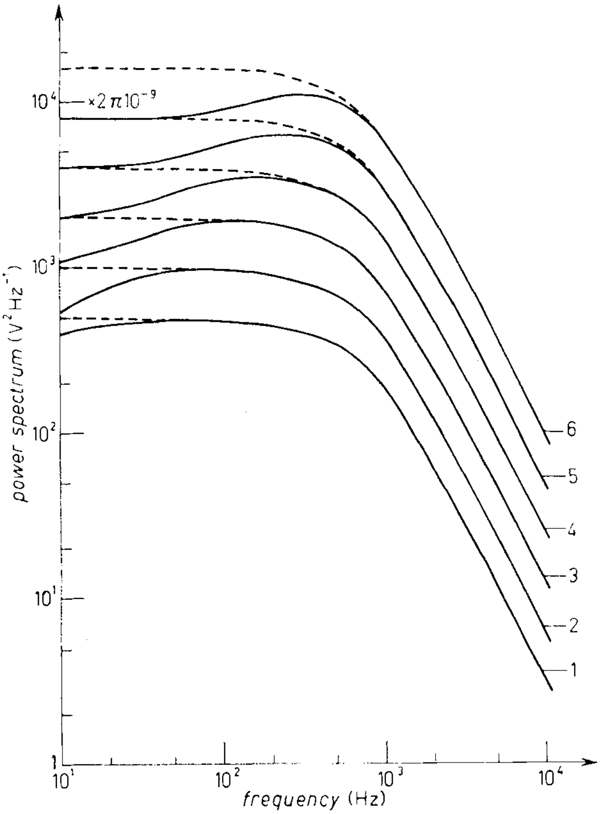

Fig. 3. – Theoretical power spectra calculated through eq. (2.11), corresponding to a train of pulses having a time constant $1/\alpha = 10^{-4}$ s, distributed according to eq. (1.2). The dotted lines represent the spectra of the same pulse train, when the pulses are distributed according to Poisson's law, given by eq. (1.1). It is assumed that $\overline{a^2} = \bar{a}^2 = 1$. The parameter ν represents the average number of pulses per unit time: curve 1: $\nu = 12.5\ s^{-1}$; curve 2: $\nu = 25\ s^{-1}$; curve 3: $\nu = 50\ s^{-1}$; curve 4: $\nu = 100\ s^{-1}$; curve 5: $\nu = 200\ s^{-1}$; curve 6: $\nu = 400\ s^{-1}$.

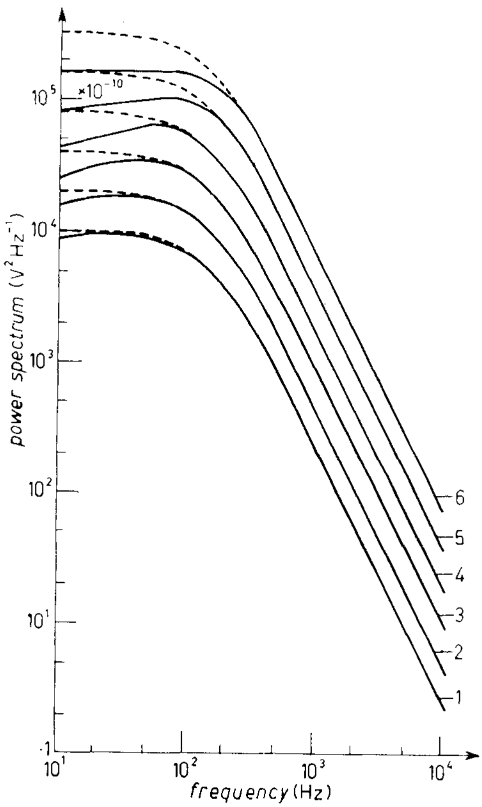

Fig. 4. – The same as Fig. 3, the time constant of the pulses being $1/\alpha = 10^{-3}$ s: curve 1: $\nu = 6.25\ s^{-1}$; curve 2: $\nu = 12.5\ s^{-1}$; curve 3: $\nu = 25\ s^{-1}$; curve 4: $\nu = 50\ s^{-1}$; curve 5: $\nu = 100\ s^{-1}$; curve 6: $\nu = 200\ s^{-1}$.

A quantitative discussion and a physical interpretation of the introduced distribution of pulses in the Barkhausen noise will be given in another work, to be published in the near future.

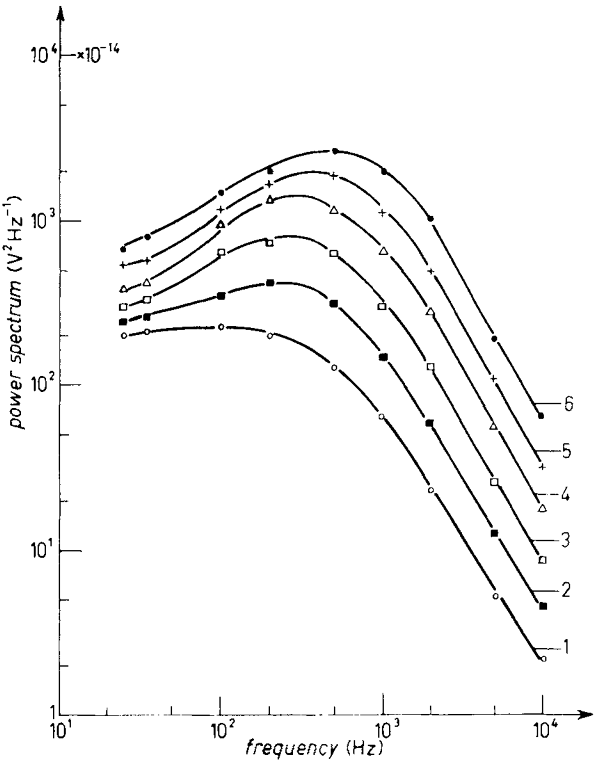

Fig. 5. – Power spectra of the Barkhausen noise detected on a .5 mm diam. and 20 mm long pure iron cylindrical specimen, with a single pickup coil of 1750 turns. f_m is the frequency of the polarizing field, which is proportional to the average number of pulses per unit time: curve 1: $f_m = 0.0125$ Hz; curve 2: $f_m = 0.025$ Hz; curve 3: $f_m = 0.05$ Hz; curve 4: $f_m = 0.1$ Hz; curve 5: $f_m = 0.2$ Hz; curve 6: $f_m = 0.4$ Hz. Integration time on each analysis point is about 15 min.

Here we will remark that in the case of the experimental spectra of the Barkhausen noise the average number of pulses per unit time ν is not constant, but depends upon the instantaneous value of macroscopic magnetization, *i.e.* on time (*).

However this does not change the shape of the power spectra when the

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(*) Averaging must be intended as being made at a certain instant of time in the statistical ensemble, by the ergodic theorem.

frequency of the polarizing field is much smaller than the lowest frequency of analysis, as in the case of the spectra reported in Fig. 5 (*).

5. – Conclusions.

In this paper we have calculated the correlation function and power spectrum of a train of nonindependent pulses of exponential shape and random amplitude, distributed according to eq. (1.2).

The important physical significance of this distribution allows application of the results to a large class of random processes, in particular to the Barkhausen effect, for which the distribution of eq. (1.2) was experimentally found by H. SAWADA.

In this case the agreement with experimental results is quite satisfactory. This allows a simple explanation of many contradictions which arise assuming Poisson's distribution for the pulses.

Many results drawn from the discussion of eq. (2.11) (as for instance the fact that deviations from Campbell's theorem occur only in the low frequency area of the spectrum and always cause a reduction to the power associated with the noise) are characteristic of the introduced type of distribution and do not depend, within large limits, upon the particular shape assumed for the single pulse. Consequently they have more general interest.

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The author wishes to thank Professors G. BIORCI, G. MONTALENTI, and R. SARTORI for helpful discussions.

APPENDIX

1. – Calculation of the sum of the series S_1 of eq. (2.2).

Let us evaluate the general term of this series:

$$J_n = \int_0^\infty \mathrm{d}x_1 \int_0^\infty \mathrm{d}x_2 \dots \int_0^\infty P(x_1)P(x_2) \dots P(x_n)\Phi(x_1 + x_2 + \dots + x_n + \tau)\,\mathrm{d}x_n\,,$$

(*) This can be proved on the basis of an expression given by O. RICE [10]. For the modifications of the power spectra in the region of the analysis frequencies near the polarizing frequency, due to the periodic fluctuation of ν, see [11].

[10] S. O. RICE: *Bell Sys. Tech. Journ.*, **23**, 324 eq. (2.6-11), (1944).

[11] F. V. BUNKIN: *G. T. F.*, **26**, 1782 (SSSR, 1956).

where

$$P(x) = k^2 x \exp[-kx]\,, \tag{1.2}$$

$$\Phi(\xi) = \frac{\exp[-\alpha|\xi|]}{2\alpha}\,. \tag{2.3}$$

Because $x_1, x_2, \dots x_n$ and τ are variables always positive, the argument of the function Φ, in the above written expression of J_n, is always positive, and eq. (2.3) holds without the signs of absolute value.

From this easily follows

$$J_n = \frac{1}{2\alpha}\varphi(\tau)\left(\int_0^\infty P(x)\varphi(x)\,\mathrm{d}x\right)^n,$$

where it is assumed

$$\varphi(\xi) = \exp[-\alpha\xi]\,. \tag{A.1}$$

Thus the series S_1 becomes a geometric series and can be easily summed up.

By putting

$$q = \int_0^\infty P(x)\varphi(x)\,\mathrm{d}x\,,$$

it results (*)

$$S_1 = \frac{1}{2\alpha}\varphi(\tau)\frac{q}{1-q}\,.$$

The value of q can be calculated through eqs. (1.2) and (A.1), and the above written expression of S_1 becomes

$$S_1 = \frac{k^2\exp[-\alpha\tau]}{2\alpha(\alpha+2k)}\,. \tag{A.2}$$

2. – Calculation of the sum of the series S_2 of. eq. (2.2).

Let us evaluate the general term I_n of this series:

$$I_n = \int_0^\infty \mathrm{d}x_1 \int_0^\infty \mathrm{d}x_2 \dots \int_0^\infty P(x_1)P(x_2)\dots P(x_n)\Phi(x_1+x_2+\dots+x_n-\tau)\,\mathrm{d}x_n\,.$$

(*) Obviously it is

$$q = \int_0^\infty P(x)\varphi(x)\,\mathrm{d}x < \int_0^\infty P(x)\,\mathrm{d}x = 1\,,$$

being

$$\varphi(x) \leqslant 1$$

for any value of x.

Let D_1 be the region of the domain of the integration in each point of which the condition

$$x_1 + x_2 + \ldots + x_n - \tau \leqslant 0$$

is satisfied, and D_2 the remaining part of it, in each point of which:

$$x_1 + x_2 + \ldots + x_n - \tau > 0 \, .$$

From eqs. (2.3) and (A.1) one obtains

$$I_n = \frac{\varphi(\tau)}{2\alpha} \int_{D_1} P(x_1) P(x_2) \ldots P(x_n) \varphi(-x_1 - x_2 \ldots - x_n) \, dx_1 \, dx_2 \ldots dx_n + \\ + \frac{\varphi(-\tau)}{2\alpha} \int_{D_2} P(x_1) P(x_2) \ldots P(x_n) \varphi(x_1 + x_2 + \ldots + x_n) \, dx_1 \, dx_2 \ldots dx_n \, .$$

In the n-dimensional orthogonal cartesian reference system $\{x_1, x_2, \ldots x_n\}$, D_1 and D_2 are the two regions in which the plane

$$\text{(A.3)} \qquad x_1 + x_2 + \ldots + x_n = \tau$$

divides the positive generalized quadrant of the reference system itself.

It is convenient to perform the following linear transformation:

$$\begin{cases} x_1 + x_2 + \ldots + x_n = y_n \, , \\ x_1 = y_1 \, , \\ x_2 = y_2 \, , \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ x_{n-1} = y_{n-1} \, . \end{cases}$$

In the new reference system $\{y_1, y_2, \ldots y_n\}$ the plane of eq. (A.3) is represented by the equation

$$y_n = \tau \, ,$$

and becomes parallel to the co-ordinated plane

$$y_n = 0 \, .$$

The whole domain of integration is now the positive region of the n-dimensional pyramid limited by the planes:

$$\begin{cases} y_{n-1} = 0 \, , \\ y_{n-2} = 0 \, , \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ y_1 \quad = 0 \, , \\ y_n - y_1 - y_2 - \ldots - y_{n-1} = 0 \, . \end{cases}$$

The value of the Jacobian of this transformation is 1, and the general term of the series S_2 becomes

$$(A.4)\qquad I_n = \frac{\varphi(\tau)}{2\alpha}\int\limits_{D_1} P(y_1)P(y_2)\ldots P(y_n-y_1-y_2-\ldots-y_{n-1})\varphi(-y_n)\,\mathrm{d}y_1\ldots\mathrm{d}y_n + \\ + \frac{\varphi(-\tau)}{2a}\int\limits_{D_2} P(y_1)P(y_2)\ldots P(y_n-y_1-y_2-\ldots-y_{n-1})\varphi(y_n)\,\mathrm{d}y_1\ldots\mathrm{d}y_n\,.$$

The expression of the probability distribution $P(x)$ given by eq. (1.2) becomes

$$P(y_n-y_1-y_2-\ldots-y_{n-1}) = \\ = k^2(y_n-y_1-y_2-\ldots-y_{n-1})\exp[-k(y_n-y_1-y_2-\ldots-y_{n-1}) = \\ = k^2y_n\quad \exp[-ky_n]\exp[ky_1]\exp[ky_2]\ldots\exp[ky_{n-1}]- \\ -k^2y_1\quad \exp[-ky_n]\exp[ky_1]\exp[ky_2]\ldots\exp[ky_{n-1}]- \\ -\ldots\ldots- \\ -k^2y_{n-1}\exp[-ky_n]\exp[ky_1]\exp[ky_2]\ldots\exp[ky_{n-1}]\,.$$

Then eq. (A.4) can be written as follows:

$$I_n = \frac{\varphi(\tau)}{2a}k^{2n}\Bigg[\int_0^\tau \varphi(-y_n)y_n\exp[ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}\,\mathrm{d}y_{n-1}- \\ -\int_0^\tau \varphi(-y_n)\exp[ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1^2\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}\,\mathrm{d}y_{n-1}- \\ -\int_0^\tau \varphi(-y_n)[\exp[-ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2^2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}\,\mathrm{d}y_{n-1}- \\ -\ldots\ldots- \\ -\int_0^\tau \varphi(-y_n)\exp[-ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}^2\,\mathrm{d}y_{n-1}\Bigg] + \\ + \frac{\varphi(-\tau)}{2\alpha}k^{2n}\Bigg[\int_\tau^\infty \varphi(y_n)y_n\exp[-ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}\,\mathrm{d}y_{n-1}- \\ -\int_\tau^\infty \varphi(y_n)\exp[-ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1^2\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}\,\mathrm{d}_{n-1}- \\ -\ldots\ldots- \\ -\int_\tau^\infty \varphi(y_n)\exp[-ky_n]\,\mathrm{d}y_n\int_0^{y_n} y_1\,\mathrm{d}y_1\int_0^{y_n-y_1} y_2\,\mathrm{d}y_2\;\ldots\int_0^{y_n-y_1-y_2-\ldots-y_{n-2}} y_{n-1}^2\,\mathrm{d}y_{n-1}\Bigg|\,.$$

By putting:

$$T_n(y_n)=\int_0^{y_n} y_1\,dy_1\int_0^{y_n-y_1} y_2\,dy_2\int_0^{y_n-y_1-y_2} y_3\,dy_3\ \dots\int_0^{y_n-y_1-y_2-\dots-y_{n-1}} y_n\,dy_n\,,$$

$$R_n(y_n)=\int_0^{y_n} y_1^2\,dy_1\int_0^{y_n-y_1} y_2\,dy_2\int_0^{y_n-y_1-y_2} y_3\,dy_3\ \dots\int_0^{y_n-y_1-y_2-\dots-y_{n-1}} y_n\,dy_n=$$

$$=\int_0^{y_n} y_1\,dy_1\int_0^{y_n-y_1} y_2^2\,dy_2\int_0^{y_n-y_1-y_2} y_3\,dy_3\ \dots\int_0^{y_n-y_1-y_2-\dots-y_{n-1}} y_n\,dy_n=$$

$$=\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .=$$

$$=\int_0^{y_n} y_1\,dy_1\int_0^{y_n-y_1} y_2\,dy_2\int_0^{y_n-y_1-y_2} y_3\,dy_3^2\ \dots\int_0^{y_n-y_1-y_2-\dots-y_{n-1}} y_n^2\,dy_n\ (^*)\,,$$

(*) The fact that these integrals are equal can be proved by an inversion of the order of integration. In fact

$$R_n=\int_0^{y_n} y_1\,dy_1,\ \dots\int_0^{y_n-y_1-\dots-y_{\varrho-2}} y_{\varrho-1}\,dy_{\varrho-1}\int_0^{y_n-y_1-\dots-y_{\varrho-1}} y_\varrho^2\,dy_\varrho,\ \dots\int_0^{y_n-y_1-\dots-y_{n-1}} y_n\,dy_n=$$

$$=\int_0^{y_n} y_1\,dy_1,\ \dots\int_0^{X} y_{\varrho-1}\,dy_{\varrho-1}\int_0^{X-y_{\varrho-1}} y_\varrho^2 f(y_\varrho+y_{\varrho-1}+X)\,dy_\varrho\,,$$

where

$$X=y_n-y_1-\dots-y_{\varrho-2}\,,$$

$$f(\xi)=\int_0^{\xi} y_{\varrho+1}\,dy_{\varrho+1}\int_0^{y_n-y_1-\dots-y_{\varrho+1}} y_{\varrho+2}\,dy_{\varrho+2},\ \dots\int_0^{y_n-y_1-\dots-y_{n-1}} y_n\,dy_n\,.$$

Finally by inverting the order of integration in the two last integrals one obtains

$$R_n=\int_0^{y_n} y_1\,dy_1,\ \dots\int_0^{X} y_\varrho^2\,dy_\varrho\int_0^{X-y_{\varrho-1}} y_{\varrho-1} f(y_\varrho+y_{\varrho-1}+X)\,dy_{\varrho-1}=$$

$$=\int_0^{y_n} y_1\,dy_1,\ \dots\int_0^{y_n-y_1-\dots-y_{\varrho-2}} y_{\varrho-1}^2\,dy_{\varrho-1}\int_0^{y_n-y_1-\dots-y_{\varrho-1}} y_\varrho\,dy_\varrho,\ \dots\int_0^{y_n-y_1-\dots-y_{n-1}} y_n\,dy_n\,,$$

by changing ϱ in $\varrho-1$ and $\varrho-1$ in ϱ.

the expression of I_n can be written

$$I_n = \frac{\varphi(\tau)}{2a} k^{2n}\left[\int_0^\tau \varphi(-y_n)y_n \exp[-ky_n]T_{n-1}(y_n)\,\mathrm{d}y_n - \right.$$

$$\left. -(n-1)\int_0^\tau \varphi(-y_n)\exp[-ky_n]R_{n-1}(y_n)\,\mathrm{d}y_n\right] +$$

$$+\frac{\varphi(-\tau)}{2\alpha}k^{2n}\left[\int_\tau^\infty \varphi(y_n)y_n\exp[-ky_n]T_{n-1}(y_n)\,\mathrm{d}y_n - \right.$$

$$\left. -(n-1)\int_\tau^\infty \varphi(y_n)\exp[-ky_n]R_{n-1}(y_n)\,\mathrm{d}y_n\,.\right.$$

Now the integrals T_n and R_n can be evaluated, and it turns out (see the note at the end of this Appendix)

$$T_n = \frac{y_n^{2n}}{(2n)!}\,,$$

$$R_n = 2\,\frac{y_n^{2n+1}}{(2n+1)!}\,.$$

Introducing these expressions of T_n and R_n in the last expression of I_n and remembering eq. (A.1), one gets

$$I_n = k^{2n}\left[\frac{\exp[-\alpha\tau]}{2\alpha}\int_0^\tau \exp[(\alpha-k)y_n]y_n\frac{y_n^{2n-2}}{(2n-2)!}\,\mathrm{d}y_n + \frac{\exp[\alpha\tau]}{2\alpha}\int_\tau^\infty \exp[-(\alpha+k)y_n]\cdot\right.$$

$$\left.\cdot y_n\frac{y^{2n-2}}{(2n-2)!}\,\mathrm{d}y_n\right] - 2(n-1)k^{2n}\left[\frac{\exp[-\alpha\tau]}{2\alpha}\int_0^\tau \exp[(\alpha-k)y_n]\frac{y^{2n-1}}{(2n-1)!}\,\mathrm{d}y_n + \right.$$

$$\left. + \frac{\exp[\alpha\tau]}{2\alpha}\int_\tau^\infty \exp[-(\alpha+k)y_n]\cdot\frac{y^{2n-1}}{(2n-1)!}\,\mathrm{d}y_n\right].$$

Hence, by carrying out the elementary integrations,

$$I_n = \frac{k^{2n}}{2\alpha(2n-1)!}\left[\exp[-\alpha\tau]\left\{\exp[(\alpha-k)\tau]\left[\frac{\tau^{2n-1}}{\alpha-k} - \frac{(2n-1)\tau^{2n-2}}{(a-k)^2} + \right.\right.\right.$$

$$\left. + \frac{(2n-1)(2n-2)\tau^{2n-3}}{(\alpha-k)^3} - \ldots + \frac{(2n-1)!}{(\alpha-k)^{2n-1}} - \frac{(2n-1)!}{(\alpha-k)^{2n}}\right] +$$

$$\left. + \frac{(2n-1)!}{(\alpha-k)^{2n}}\right\} + \exp[-k\tau]\left\{\frac{\tau^{2n-1}}{\alpha+k} + \frac{(2n-1)\tau^{2n-2}}{(\alpha+k)^2} + \right.$$

$$\left.\left. + \frac{(2n-1)(2n-2)\tau^{2n-3}}{(\alpha+k)^3} + \ldots + \frac{(2n-1)!}{(\alpha+k)^{2n}}\right\}\right].$$

This expression can also be written in the following form:

$$I_n = -\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}\left[\frac{(k-\alpha)^{2n-1}\tau^{2n-1}}{(2n-1)!}+\frac{(k-\alpha)^{2n-2}\tau^{2n-2}}{(2n-2)!}+\ldots+1\right]+$$
$$+\frac{\exp[-\alpha\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}+\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}\cdot$$
$$\cdot\left[\frac{(k+\alpha)^{2n-1}\tau^{2n-1}}{(2n-1)!}+\frac{(k+\alpha)^{2n-2}\tau^{2n-2}}{(2n-2)!}+\ldots+1\right].$$

By adding and subtracting the terms

$$\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}\exp[(k-\alpha)\tau]=\frac{\exp[-\alpha\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}$$
$$\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}\exp[(k+\alpha)\tau]=\frac{\exp[\alpha\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}},$$

this expression of I_n becomes

$$I_n=\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}\left[\exp[(k-\alpha)\tau]-\frac{(k-\alpha)^{2n-1}\tau^{2n-1}}{(2n-1)!}-\right.$$
$$\left.-\frac{(k-\alpha)^{2n-2}\tau^{2n-2}}{(2n-2)!}-\ldots-1\right]+\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}\cdot$$
$$\cdot\left[-\exp[(k+\alpha)\tau]+\frac{(k+\alpha)^{2n-1}\tau^{2n-1}}{(2n-1)!}+\frac{(k+\alpha)^{2n-2}\tau^{2n-2}}{(2n-2)!}+\ldots+1\right]+$$
$$+\frac{\exp[\alpha\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}.$$

Hence, by expansion of $\exp[(k-\alpha)\tau]$ and $\exp[(k+\alpha)\tau]$ in power series of τ

$$I_n=\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha-k)^{2n}}\sum_{m=0}^{\infty}\frac{(k-\alpha)^{2n+m}\tau^{2n+m}}{(2n+m)!}-$$
$$-\frac{\exp[-k\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}\sum_{m=0}^{\infty}\frac{(k+\alpha)^{2n+m}\tau^{2n+m}}{(2n+m)!}+\frac{\exp[\alpha\tau]}{2\alpha}\frac{k^{2n}}{(\alpha+k)^{2n}}.$$

Now summing over all values of n from 1 to ∞, one gets

$$\text{(A.5)}\qquad \sum_{n=1}^{\infty}I_n=\frac{\exp[-k\tau]}{2\alpha}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{k^{2n}}{(\alpha-k)^{2n}}\cdot\frac{(k-\alpha)^{2n+m}\tau^{2n+m}}{(2n+m)!}-$$
$$-\frac{\exp[-k\tau]}{2\alpha}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{k^{2n}}{(\alpha+k)^{2n}}\cdot\frac{(k+\alpha)^{2n+m}\tau^{2n+m}}{(2n+m)!}+\frac{\exp[\alpha\tau]}{2\alpha}\sum_{n=1}^{\infty}\frac{k^{2n}}{(\alpha+k)^{2n}}.$$

The double series in this relationship are, as we shall show, absolutely convergent (the second series has always positive terms), so that it is possible to change the order of the terms.

It is easy to verify that the expression above written can be rearranged in the form

$$(A.6)\qquad \sum_{n=1}^{\infty} I_n = \frac{\exp[-k\tau]}{2\alpha}\Big[\sum_{m=1}^{\infty}(c_{2m}+c_{2m+1})\sum_{n=1}^{m}a^{2n} - \sum_{m=1}^{\infty}(d_{2m}+d_{2m+1})\sum_{n=1}^{m}b^{2n}\Big] + \frac{\exp[\alpha\tau]}{2\alpha}\sum_{n=1}^{\infty}b^{2n},$$

where

$$a = \frac{k}{\alpha - k},$$

$$b = \frac{k}{\alpha + k},$$

$$c_s = \frac{(k-\alpha)^s\tau^s}{s!},$$

$$d_s = \frac{(k+\alpha)^s\tau^s}{s!}.$$

This way of writing eq. (A.5) corresponds to sum diagonally the terms in the table which represents each double series, by taking the terms two by two in each row.

By noting that

$$\sum_{n=1}^{m} a^{2n} = \frac{a^{2m+2} - a^2}{a^2 - 1},$$

$$\sum_{m=1}^{\infty} b^{2n} = \frac{b^2}{1 - b^2} \quad \text{(absolute convergence)},$$

eq. (A.6) can be written

$$(A.7)\qquad \sum_{n=1}^{\infty} I_n = \frac{\exp[-k\tau]}{2\alpha}\left|\sum_{m=1}^{\infty}\frac{a^{2m+2}-a^2}{a^2-1}(c_{2m}+c_{2m+1}) - \sum_{m=1}^{\infty}\frac{b^{2m+2}-b^2}{b^2-1}(d_{2m}+d_{2m+1})\right| + \frac{\exp[\alpha\tau]}{2\alpha}\frac{b^2}{1-b^2} =$$

$$= \frac{\exp[-k\tau]}{2\alpha}\cdot\frac{1}{a^2-1}\Big[\sum_{m=1}^{\infty}a^{2m+2}(c_{2m}+c_{2m+1}) - a^2\sum_{m=2}^{\infty}c_m\Big] - \frac{\exp[-k\tau]}{2\alpha}\frac{1}{b^2-1}\Big[\sum_{m=1}^{\infty}b^{2m+2}(d_{2m}+d_{2m+1}) - b^2\sum_{m=2}^{\infty}d_m\Big] + \frac{\exp[\alpha\tau]}{2\alpha}\cdot\frac{b^2}{1-b^2}.$$

From the definition of a, b, c_s, d_s, follows

$$a^s c_s = (-1)^s \frac{k^s \tau^s}{s!},$$

$$b^s d_s = \frac{k^s \tau^s}{s!}.$$

By these relationships we get

$$\sum_{n=1}^{\infty} I_n = \frac{\exp[-k\tau]}{2\alpha}\left[\frac{a^2}{a^2-1}\sum_{m=1}^{\infty}\frac{k^{2m}\tau^{2m}}{2m!} - \frac{a}{a^2-1}\sum_{m=1}^{\infty}\frac{k^{2m+1}\tau^{2m+1}}{(2m+1)!} - \right.$$
$$- \frac{a^2}{a^2-1}\sum_{m=1}^{\infty} c_m - \frac{b^2}{b^2-1}\sum_{m=1}^{\infty}\frac{k^{2m}\tau^{2m}}{2m!} - \frac{b}{b^2-1}\sum_{m=1}^{\infty}\frac{k^{2m+1}\tau^{2m+1}}{(2m+1)!} +$$
$$\left. + \frac{b^2}{b^2-1}\sum_{m=2}^{\infty} d_m\right] + \frac{\exp[\alpha\tau]}{2\alpha}\frac{b^2}{1-b^2}.$$

The series which appear in this expression are absolutely convergent.

By remembering the expansions in power series of $\cosh x$, $\sinh x$ and e^x, one easily deduces

$$\text{(A.8)} \quad \sum_{n=1}^{\infty} I_n = \frac{\exp[-k\tau]}{2\alpha}\frac{a^2}{a^2-1}(\cosh k\tau - 1) - \frac{\exp[-k\tau]}{2\alpha}\frac{a}{a^2-1}(\sinh k\tau - k\tau) -$$
$$- \frac{\exp[-k\tau]}{2\alpha}\frac{a^2}{a^2-1}(\exp[(k-\alpha)\tau] - 1 - (k-\alpha)\tau) -$$
$$- \frac{\exp[-k\tau]}{2\alpha}\frac{b^2}{b^2-1}(\cosh k\tau - 1) - \frac{\exp[-k\tau]}{2\alpha}\frac{b}{b^2-1}(\sinh k\tau - k\tau) +$$
$$+ \frac{\exp[-k\tau]}{2\alpha}\frac{b^2}{b^2-1}(\exp[(k+\alpha)\tau] - 1 - (k+\alpha)\tau) + \frac{\exp[\alpha\tau]}{2\alpha}\frac{b^2}{1-b^2} =$$
$$= \frac{\exp[-k\tau]}{2\alpha}\left[\left(\frac{a^2}{a^2-1} - \frac{b^2}{b^2-1}\right)\cosh k\tau + \left(\frac{a}{1-a^2} + \frac{b}{1-b^2}\right)\sinh k\tau\right] -$$
$$- \frac{\exp[-\alpha\tau]}{2\alpha}\frac{a^2}{a^2-1} = \frac{k}{8k^2-2\alpha^2}\cdot\exp[-2k\tau] - \frac{k^2}{2k\alpha-\alpha^2}\frac{\exp[-\alpha\tau]}{2\alpha} + \frac{k}{2\alpha^2}.$$

This expression gives the sum of the series S_2.

Note:

a) Evaluation of the integral:

$$T_n = \int_0^{\tau} y_1\,dy_1 \int_0^{\tau-y_1} y_2\,dy_2 \int_0^{\tau-y_1-y_2} y_3\,dy_3 \ldots \int_0^{\tau-y_1-y_2-\ldots-y_{n-1}} y_n\,dy_n.$$

Let us define the auxiliary function

$$A_n(x) = \int_0^x y \, \frac{(x-y)^n}{n} \, \mathrm{d}y = \frac{1}{n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} x^{n-k} \frac{x^{k+2}}{k+2} =$$

$$= \frac{x^{n+2}}{n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{1}{k+2} = \frac{x^{n+2}}{n\cdot(n+1)(n+2)} .$$

In fact it is possible to verify that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{1}{k+2} = \frac{1}{(n+1)\cdot(n+2)} .$$

Hence

$$T_1 = \int_0^\tau y_1 \mathrm{d}y_1 = \frac{\tau^2}{2} ,$$

$$T_2 = \int_0^\tau y_1 \, \mathrm{d}y_1 \int_0^{\tau-y_1} y_2 \mathrm{d}y_2 = \int_0^\tau y_1 \, \frac{(\tau-y_1)^2}{2} \, \mathrm{d}y_1 = A_2(\tau) ,$$

$$T_3 = \int_0^\tau y_1 \, \mathrm{d}y_1 \int_0^{\tau-y_1} y_2 \, \mathrm{d}y_2 \int_0^{\tau-y_1-y_2} y_3 \, \mathrm{d}y_3 = \int_0^\tau y_1 A_2(\tau-y_1) \, \mathrm{d}y_1 = \frac{1}{2\,3} \int_0^\tau y_1 \, \frac{(\tau-y_1)^4}{4} \, \mathrm{d}y_1 = \frac{1}{2\cdot 3} A(\tau) .$$

In a like way one gets

$$T = \frac{1}{2\cdot3\cdot4\cdot5} \int_0^\tau y_1 \, \frac{(\tau-y_1)^6}{6} \, \mathrm{d}y_1 = \frac{1}{5!} A_6(\tau) ,$$

consequently

$$T_n = \frac{1}{(2n-3)!} A_{2n-2}(\tau) = \frac{\tau^{2n}}{(2n)!} .$$

b) Evaluation of the integral:

$$R_n = \int_0^\tau y_1^2 \, \mathrm{d}y_1 \int_0^{\tau-y_1} y_2 \, \mathrm{d}y_2 \ldots \int_0^{\tau-y_1-y_2-\ldots-y_{n-1}} y_n \, \mathrm{d}y_n = \int_0^\tau y_1 \, \mathrm{d}y_1 \int_0^{\tau-y_1} y_2^2 \, \mathrm{d}y_2 \ldots \int_0^{\tau-y_1-\ldots-y_{n-1}} y_n \, \mathrm{d}y_n =$$

$$= \int_0^\tau y_1 \, \mathrm{d}y \int_0^{\tau-y_1} y_2 \, \mathrm{d}y_2 \ldots \int_0^{\tau-y_1-\ldots-y_{n-1}} y_n^2 \, \mathrm{d}y_n .$$

It is convenient to start by the last expression of R_n. Proceeding in the way shown above, one easily obtains

$$R_n = 2 \, \frac{1}{(2n-2)!} A_{2n-1}(\tau) = 2 \, \frac{\tau^{2n+1}}{(2n+1)!} .$$

RIASSUNTO (*)

Questo lavoro riguarda il calcolo della funzione di correlazione e dello spettro di potenza di un treno di impulsi di forma esponenziale e di ampiezza casuale, distribuiti secondo una legge di probabilità, trovata sperimentalmente da H. SAWADA durante la ricerca della distribuzione degli intervalli di tempo nell'effetto Barkhausen. Si possono così spiegare completamente tutte le caratteristiche degli spettri di potenza sperimentali del rumore di Barkhausen, che molti autori trovavano in contraddizione con l'interpretazione dell'indipendenza statistica degli impulsi. Il notevole significato fisico della distribuzione che si introduce attribuisce ai risultati un interesse più generale.

(*) *Traduzione a cura della Redazione.*