Gravitational Motion and Radiation - III (*).

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Summary. — It is shown that the secular change of the total gravitational mass of a system of two bodies, uniformly rotating around each other, is exactly equal to minus the radiated energy (independently computed in the previous part of this paper).

In the previous part of this paper (1) we found the equations of motion of a system of two masses uniformly rotating around each other, and showed that the total (Newtonian) energy undergoes a secular change.

The same result can be obtained by computing the seventh order correction to the effective gravitational mass M. One has

$$-\sum \underset{7}{\overset{*}{m}} = \sum \underset{5}{\overset{*}{m}}\, \overline{\underset{3}{F}{}^{0}_{00}} + \sum m v^k v^l\, \overline{\underset{6}{\overset{*}{F}}{}^{0}_{kl}} + 2\sum m v^k\, \overline{\underset{7}{\overset{*}{F}}{}^{0}_{0k}} + \sum m\overline{\underset{8}{\overset{*}{F}}{}^{0}_{00}}\,,$$

with

$$\left\{\begin{aligned}
\underset{6}{\overset{*}{F}}{}^{0}_{kl} &= \tfrac{1}{4}\delta_{kl}(\underset{5}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,0} - \underset{5}{\mathfrak{g}}{}^{mm}{}_{,0}) + \tfrac{1}{2}\underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0} + \tfrac{1}{2}(\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,l} + \underset{6}{\overset{*}{\mathfrak{g}}}{}^{0l}{}_{,k})\,,\\
\underset{7}{\overset{*}{F}}{}^{0}_{0k} &= -\tfrac{1}{2}(\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,k} + \underset{5}{\overset{*}{\mathfrak{g}}}{}^{00}\underset{2}{\mathfrak{g}}{}^{00}{}_{,k})\,,\\
\underset{8}{\overset{*}{F}}{}^{0}_{00} &= \tfrac{1}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll}{}_{,0} - \tfrac{3}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,0} + \tfrac{1}{4}\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}\underset{2}{\mathfrak{g}}{}^{00}{}_{,k} - \tfrac{3}{4}\underset{(5)}{(\mathfrak{g}^{00}\overset{*}{\mathfrak{g}}_{00}\mathfrak{g}_{00})}\underset{2}{\mathfrak{g}}{}^{00}{}_{,0}\,.
\end{aligned}\right.$$

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(1) A. PERES: *Nuovo Cimento* **11**, 644 (1959), hereafter refered to as II. All notations thoughout the present paper are those of II.

As the motion is circular, the kinetic and potential energies are separately constant (in the Newtonian approximation) and one has $\underset{5}{\overset{*}{\mathfrak{m}}} = \underset{5}{\overset{*}{\mathfrak{g}}}{}^{00} = \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kk} = 0$. Moreover

$$\overline{\left(\frac{1}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll}{}_{,0} - \frac{3}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,0}\right)} = \overline{\frac{d}{dt}\left(\frac{1}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll} - \frac{3}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}\right) - v^k\left(\frac{1}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll}{}_{,k} - \frac{3}{4}\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,k}\right)}\,.$$

The first term in the right-hand member can be neglected, as its time average vanishes. There remains

$$(1) \qquad -\sum \underset{7}{\overset{*}{\mathfrak{m}}} = \tfrac{1}{2}\sum m v^k v^l(\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,l} + \underset{6}{\overset{*}{\mathfrak{g}}}{}^{0l}{}_{,k} + \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0}) - \tfrac{1}{4}\sum m v^k(\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,k} + \underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll}{}_{,k}) +$$

$$+ \tfrac{1}{4}\sum m \overline{\underset{2}{\mathfrak{g}}{}^{00}{}_{,k}\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}}\,.$$

However

$$\tfrac{1}{4}\sum m \underset{2}{\mathfrak{g}}{}^{00}{}_{,k}\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k} = \sum m a^k \underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}\,,$$

$$= \frac{d}{dt}\left(\sum m v^k\, \overline{\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}}\right) - \sum m v^k \frac{d}{dt}\overline{\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}}\,.$$

Once more, the first term may be neglected, and there remains

$$-\sum m v^k \frac{d}{dt}\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k} = -\sum m v^k\, \overline{\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,0}} - \sum m v^k v^l\, \overline{\underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,l}}\,.$$

The last term of this equation cancels the first and second terms in (1), and there remains

$$-\sum \underset{7}{\overset{*}{\mathfrak{m}}} = -\sum m v^k\overline{[-\tfrac{1}{2}v^l\underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0} + \tfrac{1}{4}(\underset{7}{\overset{*}{\mathfrak{g}}}{}^{00}{}_{,k} + \underset{7}{\overset{*}{\mathfrak{g}}}{}^{ll}{}_{,k}) + \underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,0}]}\,.$$

Adding to the right-hand member the expression (having a null time-average)

$$\frac{1}{2}\frac{d}{dt}\sum m v^k v^l\, \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl} = \sum m v^k\left(\frac{1}{2}v^l\, \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0} + a^l\, \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}\right),$$

$$= \sum m v^k(\tfrac{1}{2}v^l\underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0} + \tfrac{1}{4}\underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}\, \underset{2}{\mathfrak{g}}{}^{00}{}_{,l})\,,$$

one gets

$$-\sum \underset{7}{\overset{*}{\dot{m}}} = -\sum mv^k[-v^l \underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}{}_{,0} + \tfrac{1}{4}(\underset{7}{\mathfrak{g}}{}^{00}{}_{,k} + \underset{7}{\mathfrak{g}}{}^{ll}{}_{,k}) - \tfrac{1}{4}\underset{5}{\overset{*}{\mathfrak{g}}}{}^{kl}\underset{2}{\mathfrak{g}}{}^{00}{}_{,l} + \underset{6}{\overset{*}{\mathfrak{g}}}{}^{0k}{}_{,0}]\,,$$

$$= -\sum mv^k \underset{5}{\overset{*}{\ddot{a}}}{}^k\,.$$

This is exactly equal to the radiated energy that was computed in II.

RIASSUNTO (*)

Si dimostra che la variazione secolare della massa gravitazionale totale di un sistema di due corpi rotanti uniformemente uno attorno all'altro è esattamente uguale all'energia irradiata cambiata di segno (calcolata indipendentemente nella prima parte del presente lavoro).

(*) *Traduzione a cura della Redazione.*