Does the Lifetime of an Unstable System Depend on the Measuring Apparatus? (*)

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Summary. — Within the description of the experimental determination of the decay law recently proposed we investigate the possibility that the experimentally determined lifetime τ be different from the theoretical lifetime $1/\gamma$ for undisturbed evolution of the unstable quantum system. It is shown that in some specific examples the deviation of τ from $1/\gamma$ is competitive with the accuracy of the experiment. It is made plausible that such a difference, together with a variation of τ with the different experimental set-ups, could be revealed by properly choosing some specific unstable systems and properly devised measuring apparatuses.

1. – Introduction.

In a recent paper (1) it has been shown that a critical analysis of the actual experimental situation leads to the conclusion that, in the determination of the decay law of an unstable system, one cannot consider the system as evolving undisturbed but that on the contrary it is unavoidably subjected to measure-

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(1) L. FONDA, G. C. GHIRARDI, A. RIMINI and T. WEBER: *Nuovo Cimento*, **15** A, 689 (1973).

ment processes, occurring at random during the decay time. These processes ascertain whether the system has decayed or not. The recognition of this fact has, for the theoretical description of decay processes, far-reaching consequences. In particular it implies that, even though the nondecay probability $P(t)$ when the system evolves without measurements

$$P(t) = |(\psi_{\text{unstable}}, \exp[-iHt]\psi_{\text{unstable}})|^2 \tag{1.1}$$

is not purely exponential for all times, the actual law $P_M(t)$ which takes into account the random measurements turns out to be a pure exponential

$$P_M(t) = \exp[-t/\tau] . \tag{1.2}$$

The lifetime τ of (1.2) is determined by the functional equation

$$\lambda\int_0^\infty \exp[-(\lambda - 1/\tau)t]P(t)\,\mathrm{d}t = 1 , \tag{1.3}$$

where λ is the mean frequency of the measurements, so that $\lambda\,\mathrm{d}t$ gives the probability that the system suffers a measurement in the time interval $\mathrm{d}t$, and $P(t)$ is given by (1.1).

From conventional quantum mechanics, $P(t)$ turns out to be exponential for a large time interval (*)

$$P(t) \simeq \exp[-\gamma t] . \tag{1.4}$$

These considerations naturally lead to the following problems which we want to discuss and solve in this paper.

i) What is, in practical cases, the difference between the « experimental » lifetime τ and the « theoretical » lifetime for undisturbed evolution $1/\gamma$?

ii) If τ turns out to be different from $1/\gamma$, since this difference is related to the measurement processes whose frequency can vary from experiment to experiment, should one expect to get experimentally a different lifetime according to the different experimental apparatus that one uses?

Let us remark that the mean frequency of the measurements is quite high, so that the values of $P(t)$ which are relevant belong to small times. As is well known, at small times $P(t)$ exhibits deviations from the exponential behav-

(*) γ would be the width of the resonance accompanying the unstable system and appearing in the scattering of the decay products. In practical cases, when the unstable system lives long enough γ is too small to be detected experimentally. For example, if $\tau \simeq 10^{-10}$ s, $\gamma \simeq 10^{-5}$ eV.

iour (1.4). In particular it can be shown completely in general that [2]

$$\left.\frac{\mathrm{d}P(t)}{\mathrm{d}t}\right|_{t=0} = 0\,, \tag{1.5}$$

so that at least in a neighbourhood of $t = 0$, $P(t) > \exp[-\gamma t]$. Actually, FLEMING [3] has shown that for any wave packet

$$\cos tw < P(t) < 1 \qquad \text{for } t \ll \frac{1}{w}, \tag{1.6}$$

w being the energy spread of the wave packet. Moreover, some explicit calculations in the framework of specific models performed by WINTER and by NEWTON [4] show that relevant deviations from the purely exponential law can be present even up to tenths of $1/\gamma$. If this were the case the obtained decay law would have a lifetime τ completely different from $1/\gamma$. In fact it is easily seen that the time interval where 98 % of the measurements occurs is

$$10^{-2}/\lambda \leqslant t \leqslant 5/\lambda \tag{1.7}$$

and, as we shall see in Sect. **4**, drastic lower and upper bounds on λ give from (1.7) $10^{-19}\,\text{s} \leqslant t \leqslant 10^{-11}\,\text{s}$. Therefore, for all practical cases there is a very large number of measurements in a lifetime. Moreover, by changing λ one would correspondingly obtain different values of τ. Incidentally, we remark that this variation of the lifetime with the experimental set-ups would constitute the simplest way of revealing the deviations from the pure exponential shape, which $P(t)$ exhibits for small times. In fact it is not possible, in the framework of a given experiment, to detect experimentally these deviations by studying the experimental decay law at small times. According to the discussion of ref. [1], the experimental decay law is in fact given by eq. (1.2) and *is exponential at all times.*

2. – Dependence of the lifetime on the experimental apparatus.

To study how τ is influenced by the presence of the measurements with mean frequency λ and by the existing deviations of $P(t)$ from the pure exponential $\exp[-\gamma t]$ at small times, let us introduce the function $\Delta(\gamma t)$ and the

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([2]) See, for example, J. LUKIERSKI: ICTP, Trieste preprint IC/72/128.
([3]) G. N. FLEMING: *Nuovo Cimento* (in press).
([4]) R. G. NEWTON: *Scattering Theory of Waves and Particles*, Chap. 19 (New York, N. Y., 1965); R. G. WINTER: *Phys. Rev.*, **123**, 1503 (1961).

quantity $\delta\gamma/\gamma$ according to

$$P(t) = \exp[-\gamma t][1 + \Delta(\gamma t)] , \tag{2.1}$$

$$\frac{1/\tau}{\gamma} = 1 + \frac{\delta\gamma}{\gamma} , \tag{2.2}$$

where in the definition (2.1) we have taken into account that the time scale which is relevant to our problem is given by the « theoretical » lifetime $1/\gamma$. The function $\Delta(x)$ gives the percent deviation of $P(t)$ from the pure exponential. Since $P(0) = 1$, we have that $\Delta(0) = 0$. On the other hand, from (1.5) we get $\mathrm{d}\Delta(x)/\mathrm{d}x|_{x=0} = 1$. The parameter $\delta\gamma/\gamma$ is the percent deviation of the decay rate $1/\tau$ (when measurements occur) from the decay rate γ (with no measurements). Note that for $\delta\gamma/\gamma = -\frac{1}{2}$ the experimental lifetime τ is twice the « theoretical » $1/\gamma$, while for $\delta\gamma/\gamma = 1$ it is one-half of $1/\gamma$. When $\delta\gamma/\gamma \to -1$, $\tau \to \infty$ and the system becomes stable.

In eq. (1.3) a crucial role is played by the parameter λ, *i.e.* the mean frequency of the measurements. In order that the statistical averaging effect leading to a pure exponential decay law take place, as discussed in ref. [1], a large number of reductions has to take place in a « theoretical » lifetime $1/\gamma$, so that $\lambda \gg \gamma$. Let us introduce the adimensional parameter

$$n = \frac{\lambda}{\gamma} \gg 1 , \tag{2.3}$$

which represents the mean number of measurements suffered by the system in a « theoretical » lifetime $1/\gamma$. Since, for experimental reasons, $1/\tau < \lambda$, we get from (2.2)

$$n - \frac{\delta\gamma}{\gamma} > 1 . \tag{2.4}$$

If we use the definitions (2.1), (2.2) and (2.3), eq. (1.3) becomes

$$\int_0^\infty \exp\left[-\left(n - \frac{\delta\gamma}{\gamma}\right)x\right]\Delta(x)\mathrm{d}x = -\frac{\delta\gamma/\gamma}{n(n - \delta\gamma/\gamma)} . \tag{2.5}$$

Equation (2.5) has a clear physical meaning. Suppose $\Delta(x) \geqslant 0$ for all x. The l.h.s. of (2.5) is then positive; since $n - \delta\gamma/\gamma > 0$ from (2.4) we get $\delta\gamma/\gamma < 0$. Consequently, the « experimental » lifetime τ resulting from the presence of measurement processes is greater that the « theoretical » $1/\gamma$ characterizing the evolution without measurements.

Let us try to understand how the deviations $\Delta(x)$ from the exponential law influence $\delta\gamma/\gamma$. Let us define the mean deviation $\langle\Delta_n\rangle$ weighted with

$\exp[-\lambda t]$ giving the distribution of the measurements:

$$(2.6)\qquad \langle\Delta_n\rangle = \frac{\int_0^\infty \mathrm{d}t \exp[-\lambda t]\,\Delta(\gamma t)}{\int_0^\infty \mathrm{d}t \exp[-\lambda t]} = n\int_0^\infty \mathrm{d}x \exp[-nx]\,\Delta(x)\,.$$

Equation (2.6) defines a statistical average of the deviation $\Delta(\gamma t)$ which is particularly suitable to characterize the relevant deviations occurring at early times of the decay process, since the weight function $\exp[-\lambda t]$ strongly cuts the contributions coming from times greater than about $5/\lambda$.

We can rewrite (2.5) as

$$(2.7)\qquad \langle\Delta_{n-\delta\gamma/\gamma}\rangle = -\frac{\delta\gamma/\gamma}{n}\,,$$

or as

$$(2.8)\qquad \langle\Delta_n\rangle = \frac{-\delta\gamma/\gamma}{n+\delta\gamma/\gamma}\,.$$

Equation (2.8) yields the functional dependence of $\delta\gamma/\gamma$ on the mean number n of measurements in a « theoretical » lifetime $1/\gamma$ and on the mean deviation $\langle\Delta_n\rangle$ from the purely exponential decay law $\exp[-\gamma t]$. Note that in (2.8) all quantities are known once one has a theory giving $P(t)$ and knows the explicit structure of the apparatus, *i.e.* the mean frequency of the reductions. On can therefore evaluate $\delta\gamma/\gamma$ and learn which will be the difference between the measured lifetime τ and the theoretical one $1/\gamma$. Equation (2.8) will then constitute the basis of our discussion.

From the definition (2.6) we immediately get

$$(2.9)\qquad \langle\Delta_n\rangle = \int_0^\infty \mathrm{d}y \exp[-y]\,\Delta(y/n)\,.$$

For fixed γ, and therefore taking into consideration a particular unstable particle, let us see the limit of $\langle\Delta_n\rangle$ for $n\to\infty$, *i.e.* $\lambda\to\infty$. Since from (1.5)

$$(2.10)\qquad \Delta(x) \underset{x\to 0}{\simeq} x\,,$$

from (2.9) we get

$$(2.11)\qquad \langle\Delta_n\rangle \underset{n\to\infty}{\simeq} \frac{1}{n}\,.$$

Equation (2.11), through (2.8), implies

$$\lim_{\lambda\to\infty}\frac{\delta\gamma}{\gamma}=-1\,,\qquad i.e.\qquad \lim_{\lambda\to\infty}\tau=\infty\,, \tag{2.12}$$

which means that our unstable particle has become stable. Therefore we conclude that for any given unstable particle its lifetime τ can be made to increase abnormally by increasing the frequency λ of the measurements. This consideration gives rise to the possibility that for a given unstable particle its measured lifetime depends on the experimental apparatus.

3. – Approximate expression for $\langle\Delta_n\rangle$.

As discussed in ref. (1), the measurements which ascertain whether the system has decayed or not essentially establish whether the decay products are well separated in space or not, let us say whether they are within or outside the range of the forces which brought about the formation of the unstable system. Practically this means that the mean time $1/\lambda$ between two measurements cannot be so small that the decay products have not travelled outside the localization distance R:

$$\frac{1}{\lambda}>\frac{R}{v_+}\,, \tag{3.1}$$

where v_+ is the velocity of the fastest of the decay products.

Let us consider in greater detail the measurement process. For simplicity we shall use the framework of two-body potential scattering, but one can easily convince oneself that analogous considerations hold in general. As shown in ref. (1) if one writes the unstable state as

$$\psi_{\text{unstable}}=\sum_{lm}\int \mathrm{d}^3k\,\frac{u_l(k)}{k}\,y_{lm}(\hat{k})\,\psi_{\boldsymbol{k}}^{(+)}\,, \tag{3.2}$$

where $\psi_{\boldsymbol{k}}^{(+)}$ is the outgoing wave scattering state of the total Hamiltonian H and $\boldsymbol{k}$ is the relative momentum, then the identification of the measurement with a localization procedure tells us that

$$u_l(k)=\delta_{ll_R}\frac{J_l(k,k_R)}{f_l(k)}\,, \tag{3.3}$$

where l_R and k_R are the relative angular and linear momenta at the resonance, $f_l(k)$ is the Jost function and $J_l(k, k_R)$ is a function of k having a slow energy dependence in the resonance region. Using (3.3) and (3.2) in (1.1) we get

$$P(t) = \left|(2l_R + 1)\int_0^\infty \mathrm{d}k \frac{|J_{l_R}(k, k_R)|^2 \exp[-iE_k t]}{|f_l(k)|^2}\right|^2 ; \tag{3.4}$$

we see that $P(t)$ is related to the Fourier transform of a function which is a Breit-Wigner resonance formula, due to the zero of the Jost function, times the factor $|J_{l_R}(k, k_R)|^2$ whose role is that of cutting the integral for energies too far away from the resonance region. As discussed in ref. [5], $J_{l_R}(k, k_R)$ is practically constant over a region centred at k_R and of width

$$\Delta k \cong \frac{1}{R}, \tag{3.5}$$

R being the radius of the localization region. We can then use for $P(t)$ the approximate expression

$$P(t) \cong N \left|\int_{E_R - w/2}^{E_R + w/2} \mathrm{d}E \frac{\exp[-iEt]}{(E - E_R)^2 + \gamma^2/4}\right|^2, \tag{3.6}$$

where N is a normalization factor and w the energy spread of $J_{l_R}(k, k_R)$. w is related to R by

$$w \cong v \Delta k \cong \frac{v}{R}, \tag{3.7}$$

where $v = |\boldsymbol{v}_1 - \boldsymbol{v}_2|$ is the relative velocity of the two decay products which in this example constitute the structure of the unstable system. Using (3.1) together with the fact that $1 \leqslant v/v_+ \leqslant 2$, we get

$$\frac{\gamma}{w} < \frac{1}{n}, \quad i.e. \quad \frac{\lambda}{w} < 1 . \tag{3.8}$$

In the Appendix we have evaluated explicitly $\langle \Delta_n \rangle$ using the form (3.6) for $P(t)$. It turns out that

$$\langle \Delta_n \rangle = \frac{4}{\pi}\left(\frac{\gamma}{w}\right) + \frac{4}{\pi^2}\left(\frac{\gamma}{w}\right)^2 + \frac{8}{\pi}\left(\frac{1}{\pi^2} - \frac{1}{6} + \frac{n}{6} - \frac{n^2}{2}\right)\left(\frac{\gamma}{w}\right)^3 + O\left(\frac{\gamma^4}{w^4}\right). \tag{3.9}$$

([5]) L. Fonda and G. C. Ghirardi: *Nuovo Cimento*, **6** A, 553 (1971).

Equation (3.9) allows us to draw the following conclusions:

a) If $n\gamma/w = \lambda/w \ll 1$, then

$$\langle \Delta_n \rangle \simeq \frac{\gamma}{w}, \tag{3.10}$$

which, through (2.8), implies

$$\frac{\delta\gamma}{\gamma} \simeq -\frac{n\gamma}{w} = -\frac{\lambda}{w}, \tag{3.11}$$

and therefore $\tau \simeq 1/\gamma$ within the experimental errors.

b) When γ/w is comparable to n, *i.e.* $\lambda/w \simeq 1$, then $\delta\gamma/\gamma$ is appreciably different from zero and therefore we have $\tau \neq 1/\gamma$. Moreover, we are in a region where by varying λ we correspondingly get different values for τ.

c) When $n\gamma/w \simeq \lambda/w \gg 1$, then from the considerations leading to (2.12) we get $\delta\gamma/\gamma \simeq -1$ and we have an abnormal stability.

This last case *c*), being inconsistent with the basic equation (3.1), would however make ambiguous the interpretation of the experimental data and would require a reconsideration of our description of the experimental determination of the decay law.

Although the above conclusions are based on the simple model (3.6), they give good estimates of the orders of magnitude of the quantities which are relevant for our problem. We shall see in Sect. **4** that specific calculations made for some physical cases show that we are close to situation *a*) above. This could seem rather surprising since, as we have already stated, authors like NEWTON [4] give relevant deviations from the exponential law, extending up to times of the order of tenths of $1/\gamma$, while we know that the measurements take place in a very small fraction of a lifetime. However, in ref. [4] such deviations are obtained by multiplying the Breit-Wigner resonance times a Gaussian function whose width is of the order of γ or even smaller. For the choice made for the parameters, this means that the background wave packet has an energy spread of the order of 10^{-7} eV which is completely unrealistic.

Before concluding this Section we want to single out the relevant parameters which govern the main results, so that we can identify the changes in the experimental set-ups which should be made in order to obtain variations of τ. The relation (3.11) tells us that when

$$\frac{\lambda}{w} \ll 1$$

the deviation of τ from $1/\gamma$ is practically undetectable. To violate this relation we can then either:

a) increase drastically the number of reductions within one lifetime, *i.e.* increase λ, which, for example, in bubble chamber experiments means to increase the density within the chamber, a thing which does not seem practically feasible;

b) decrease w, which through (3.7) means to increase R, *i.e.* to work with an apparatus yielding a worse localization of the decay products causing therefore a smaller energy spread of the wave packet, or to decrease v, *i.e.* to consider resonances close to threshold.

We point out that the lifetime $1/\gamma$ is not a relevant parameter for our problem. We stress again that if (3.1) is violated all the above considerations are no longer applicable, so that the only possibility to detect a difference between τ and $1/\gamma$ is to make (3.8) almost hold with the equality sign. Actually it is sufficient that $\gamma/w \simeq 10^{-1} \div 10^{-2}$ to obtain a $\delta\gamma/\gamma$ of the same order of magnitude and therefore experimentally detectable. Such values of λ and w can occur in some decay processes as we shall see now.

4. – Estimates for some practical cases.

In this Section we shall list a certain number of specific examples of unstable systems, to see whether the condition $\gamma/w \ll 1$ is satisfied for them or not. In order to do this one has first of all to get an idea of the possible values of λ. Let us first consider bubble-chamber–type experiments. If one remembers that $1/\lambda$ is related to the mean free path of the unstable system within the chamber and one assumes that whenever the system interacts electromagnetically with the environment one has a measurement, one can relate λ to the cross-section σ as follows:

$$\lambda = \sigma \varrho u \,, \tag{4.1}$$

where ϱ is the density of scatterers within the chamber and u the velocity of of the unstable system. In the case, for instance, of a relativistic particle a rough estimate of the order of magnitude of λ from (4.1) gives

$$\lambda \simeq 10^{16}\ \text{s}^{-1} \,. \tag{4.2}$$

We point out however that the assumption that each electromagnetic interaction corresponds to a measurement is rather strong. In general we can consider (4.1) and (4.2) as giving an upper bound for λ. The opposite attitude would be to assume that only the interactions producing bubbles correspond to measurements. Taking into account that only a very small fraction of the ionization processes give rise to the formation of bubbles, while in (4.1) we have used a cross-section σ which takes into account all electromagnetic interactions, we get a λ of the order of

$$\lambda \simeq 10^{11}\,\text{s}^{-1}\,. \tag{4.3}$$

Since, when a bubble is seen, a measurement has taken place for sure, (4.3) is for sure a lower bound for λ. It seems then reasonable to assume that the relevant range of values for λ for bubble-chamber–type experiments is

$$10^{11}\,\text{s}^{-1} < \lambda < 10^{16}\,\text{s}^{-1} \qquad \text{in the bubble chamber.} \tag{4.4}$$

Let us now consider the second type of decay processes, *i.e.* those peculiar to radioactive materials. Also in this case the estimate of λ is approximate. Speaking semi-classically, one would say that λ should be related to the frequency of revolution of the electrons of the internal shells of the considered atom whose nucleus is performing radioactive decay. This gives for λ a range of values of the following order of magnitude:

$$10^{15}\,\text{s}^{-1} < \lambda < 10^{17}\,\text{s}^{-1} \qquad \text{for radioactive materials}\,. \tag{4.5}$$

From our point of view, a completely ionized atom would present a shorter lifetime than the neutral atom.

The other quantity which plays an important role in our analysis is the parameter w, which is related to the reduction distance R, via (3.7). To have an idea of the values of w we have then to guess the value of R. Speaking of nuclear-type particles, and bubble chamber experiments, one must assume that R is greater than the range of the nuclear forces in order that the measurement not disturb the inner structure of the unstable system. On the other hand, since atomic phenomena are involved in the measurement, R cannot exceed the atomic dimensions. It seems therefore reasonable to assume that

$$10^{-12}\,\text{cm} < R < 10^{-8}\,\text{cm} \qquad \text{in the bubble chamber}\,. \tag{4.6}$$

For radioactive materials, if one assumes as already done above that the inner electrons are responsible for the measurements, one must correspondingly assume that the localization radius is the radius of these orbits, *i.e.*

$$10^{-10}\,\text{cm} < R < 10^{-8}\,\text{cm} \qquad \text{for radioactive materials}\,. \tag{4.7}$$

With these assumptions, we can now consider some specific cases, which are summarized in Table I. We see that for radioactive materials we get into a region of values of λ/w which are rather large, of the order of $10^{-2}\div 1$. We suspect therefore that for some of these types of decay processes the experimental life-

TABLE I ([a]).

Decay process	v	w, eq. (3.7)	λ/w ([b])
$\pi^\pm \to \mu^\pm + \nu$	$\approx c$	$\approx 10^{18}$ for $R = 10^{-8}$	$\approx 10^{-12}$ for $\lambda = 10^{11}$, $R = 10^{-12}$
$K^\pm \to 2\pi$			$\approx 10^{-8}$ for $\lambda = 10^{11}$, $R = 10^{-8}$
$\Lambda \to p + \pi^-$		$\approx 10^{22}$ for $R = 10^{-12}$	$\approx 10^{-7}$ for $\lambda = 10^{16}$, $R = 10^{-12}$
$\Xi^- \to \Lambda + \pi^-$			$\approx 10^{-3}$ for $\lambda = 10^{16}$, $R = 10^{-8}$
${}^{212}Po \to {}^{208}Pb + \alpha$	$(10^{-1} \div 10^{-2})\, c$	$\approx 10^{17}$ for $R = 10^{-8}$	$\approx 10^{-4}$ for $\lambda = 10^{15}$, $R = 10^{-10}$
			$\approx 10^{-2}$ for $\lambda = 10^{15}$, $R = 10^{-8}$
${}^{144}Nd \to {}^{140}Ce + \alpha$		$\approx 10^{19}$ for $R = 10^{-10}$	$\approx 10^{-2}$ for $\lambda = 10^{17}$, $R = 10^{-10}$
			≈ 1 for $\lambda = 10^{17}$, $R = 10^{-8}$

(a) c = velocity of light, w and λ given in s^{-1}, R given in cm.
(b) When $\lambda/w \ll 1$, it coincides with $\delta\gamma/\gamma$.

time τ is likely to deviate from the theoretical $1/\gamma$. We see from Table I that for bubble chamber experiments we get at most values $\lambda/w \approx \delta\gamma/\gamma \approx 10^{-3}$ for $\lambda = 10^{16}$, $R = 10^{-8}$ (*). The difference between the two types of decay experiments is due both to the fact that probably λ is greater for radioactive materials and to the fact that v (and therefore w) is greater for elementary particles.

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APPENDIX

Evaluation of $\langle \Delta_n \rangle$.

In the simple model described in Sect. **3**, eq. (3.6) for $P(t)$ can be written as

$$P(t) = [\varphi(\gamma t)]^2 , \tag{A.1}$$

(*) We note that the tests of CPT for $\pi^\pm$ and $K^\pm$ decays yield values 0.05% and 0.1%, respectively.

where, in terms of the parameter $\alpha = w/2\gamma$

$$\varphi(x) = \frac{2}{\operatorname{arctg} 2\alpha} \int_0^\alpha dy \, \frac{\cos xy}{1+4y^2} \,. \tag{A.2}$$

We notice that the probability function $P(t)$ does not depend on the energy of the resonance but only on the parameter α, while its dependence on time enters only through the variable $x = \gamma t$. Of course, in the limit $\gamma \to 0$ one gets $P(t) = 1$ corresponding to the evolution of a bound state and in the unphysical limit $w \to \infty$ the pure exponential law $P(t) = \exp[-\gamma t]$ is recovered.

In order to discuss the large-α behaviour $P(t)$ it is useful to introduce the following function:

$$A(\alpha, x) = \frac{\exp[x/2]}{\pi\alpha} \int_1^\infty dy \, \frac{\cos(\alpha x y)}{y^2 + 1/(4\alpha^2)} \,, \tag{A.3}$$

in terms of which eq. (A.2) is rewritten as

$$\varphi(x) = \frac{\pi \exp[-x/2]}{2 \operatorname{arctg} 2\alpha} [1 - A(\alpha, x)] \,. \tag{A.4}$$

Substituting now the expressions (A.4) and (A.1) into the definition (2.1), we obtain

$$\Delta(x) = \frac{1}{(1-(2/\pi)\operatorname{arctg}(1/2\alpha))^2} \left\{ \frac{4}{\pi} \operatorname{arctg}\left(1 - \frac{1}{\pi} \operatorname{arctg} \frac{1}{2\alpha}\right) - \right.$$
$$\left. - 2A(\alpha, x) + [A(\alpha, x)]^2 \right\} . \tag{A.5}$$

Averaging then this last expression according to the definition (2.6), we have for the mean deviation the following integral representation:

$$\langle \Delta_n \rangle = \left(1 - \frac{2}{\pi} \operatorname{arctg} \frac{1}{2\alpha}\right)^{-2} \cdot \left\{ \frac{4}{\pi} \operatorname{arctg} \frac{1}{2\alpha} \cdot \left(1 - \frac{1}{\pi} \operatorname{arctg} \frac{1}{2\alpha}\right) - \right.$$
$$- \frac{2}{\pi} \frac{2n-1}{n-1} \left[\operatorname{arctg} \frac{1}{2\alpha} - \frac{1}{2n-1} \operatorname{arctg} \frac{2n-1}{2\alpha} \right] +$$
$$+ \frac{n(n-1)}{2\pi^2\alpha^4} \int_1^\infty dy_1 \int_1^\infty dy_2 \left\{ \left(y_1^2 + \frac{1}{4\alpha^2}\right)\left(y_2^2 + \frac{1}{4\alpha^2}\right)\left[(y_1+y_2)^2 + \left(\frac{n-1}{\alpha}\right)^2\right] \right\}^{-1} +$$
$$\left. + \frac{n(n-1)}{2\pi^2\alpha^4} \int_1^\infty dy_1 \int_1^\infty dy_2 \left\{ \left(y_1^2 + \frac{1}{4\alpha^2}\right)\left(y_2^2 + \frac{1}{4\alpha^2}\right)\left[(y_1-y_2)^2 + \left(\frac{n-1}{\alpha}\right)^2\right] \right\}^{-1} \right\} . \tag{A.6}$$

Without entering into the details of the evaluation of the double integrals appearing in this expression, we summarize our results on the large-α behaviour of $\langle \Delta_n \rangle$ by giving the power series expansion

$$\langle \Delta_n \rangle = \sum_{k=1}^{\infty} d_k(n)\, \alpha^{-k} + \ln \alpha \sum_{k=6}^{\infty} \bar{d}_k(n)\, \alpha^{-k}\,, \tag{A.7}$$

where the logarithmic dependence on α originates only from the last double integral appearing in eq. (A.6). However, since the series multiplying the logarithmic terms starts from the power α^{-6}, it can be disregarded when $\alpha \ll 1$ as it must be in order that our approach be sensible. The first power series in eq. (A.7) has a radius of convergence equal to $1/(n-\frac{1}{2})$ while the radius of convergence of the second one is $1/(n-1)$. However, since we are interested in values of n which are much greater than one, the expression (A.7) can be used for practical evaluations of the mean deviation $\langle \Delta_n \rangle$ only when $1/\alpha < 1/n$. The first three coefficients of the first power series are found to be

$$d_1(n) = \frac{2}{\pi}\,, \qquad d_2(n) = \frac{1}{\pi^2}\,, \qquad d_3(n) = \frac{1}{\pi}\left(\frac{1}{\pi^2} - \frac{1}{6} + \frac{n}{6} - \frac{n^2}{2}\right). \tag{A.8}$$

Note that the quadratic dependence on n of the third coefficient $d_3(n)$ implies that the third term of the expansion is of the same order of magnitude as the first one when $n \simeq 2\alpha^2$, as was to be expected from the previous considerations on the radius of convergence of the series. In conclusion, when $w/\gamma \gg n$ we can take only the first term in (A.7), getting

$$\langle \Delta_n \rangle = \frac{4}{\pi}\frac{\gamma}{w} + O\left(\frac{\gamma^2}{w^2}\right). \tag{A.9}$$

Under these conditions, from (2.8) we then have

$$\frac{\delta\gamma}{\gamma} = -\frac{4}{\pi}\frac{n\gamma}{w} + O\left(\frac{\gamma^2}{w^2}\right). \tag{A.10}$$

● RIASSUNTO

Seguendo lo schema teorico recentemente proposto per la descrizione della determinazione sperimentale della legge di decadimento, si investiga la possibilità che la vita media τ determinata sperimentalmente risulti diversa da quella teorica $1/\gamma$ ottenuta supponendo che il sistema instabile evolva indisturbato. Si mostra che in alcuni casi specifici la deviazione di τ da $1/\gamma$ è dello stesso ordine della precisione sperimentale. Diventa pertanto plausibile che una siffatta deviazione, come pure una variazione di τ al mutare degli apparati sperimentali, possa essere rivelata scegliendo opportuni sistemi instabili e utilizzando apparati di misura adatti.

Зависит ли время жизни нестабильной системы от измерительной аппаратуры?

Резюме (*). — В рамках недавно предложенного описания экспериментального определения закона распада мы исследуем возможность, что экспериментально определенное время жизни τ отличается от теоретического времени жизни $1/\gamma$ по причине невозмущенной эволюции нестабильной квантовой системы. Показывается, что в некоторых специальных случаях отклонение τ от $1/\gamma$ конкурирует с точностью эксперимента. Оказывается правдоподобным, что такая разница вместе с изменением τ в зависимости от различной экспериментальной аппаратуры может быть обнаружена с помощью соответствующего выбора некоторых специальных нестабильных систем и соответствующей измерительной аппаратуры.

(*) *Переведено редакцией.*