A Detailed Study of Entropy Jump Across Shock Waves in Relativistic Fluid Dynamics.

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Summary. — A detailed study of the function η which characterizes entropy jump across shock waves is carried out for relativistic hydrodynamics at thermal equilibrium. It is shown that the function η is defined only if the normal velocity of the shock waves does not exceed the speed of light in vacuo, consistently with the claims of relativity; moreover, the entropy jump goes to infinity as soon as the shock speed approaches the speed of light and γ is lower than 2, while, for $\gamma = 2$, the lightlike shock vanishes.

PACS. 47.75. – Relativistic fluid dynamics.
PACS. 47.35. – Hydrodynamic waves.
PACS. 05.70. – Thermodynamics.

1. – The function η in relativistic hydrodynamics.

A theory of shock wave behaviour, weak solutions to generalized conservative systems of covariant quasi-linear hyperbolic partial differential equations of type

$$\nabla_\alpha \boldsymbol{F}^\alpha(\boldsymbol{U}) = \boldsymbol{f}(\boldsymbol{U}) \tag{1.1}$$

consistent with a scalar supplementary conservation law

$$\nabla_\alpha h^\alpha(\boldsymbol{U}) = g(\boldsymbol{U}) \tag{1.2}$$

is developed in ref. ([1]). Thus we shall relate to ([1]) as a starting point which will be presupposed in the following; also the notations employed in the present paper will be the same as in ([1]).

The relativistic Rankine-Hugoniot matching conditions governing shocks, arising as weak solutions to (1.1), are given by ([2])

$$\varphi_\alpha[\boldsymbol{F}^\alpha] = 0 \,, \tag{1.3}$$

while the shock-generating function η (sometimes named also generalized entropy) will be ([1])

$$\eta = \varphi_\alpha[h^\alpha] \,, \tag{1.4}$$

which is generally nonvanishing, unless the shock becomes characteristic.

Previous studies of the shock-generating function have been developed in the literature, concerning nonrelativistic polytropic fluid ([3]) and nonrelativistic gas mixtures ([4]).

Here we shall develop an investigation concerning the function η of a polytropic relativistic fluid.

We recall that the equations of relativistic hydrodynamics assume the form (1.1) as soon as we choose

$$\boldsymbol{F}^\alpha \equiv \begin{pmatrix} T^{\alpha\beta} \\ ru^\alpha \end{pmatrix}, \qquad \boldsymbol{f} = 0 \,, \tag{1.5}$$

where the energy-momentum tensor is

$$T^{\alpha\beta} = rfu^\alpha u^\beta - pg^{\alpha\beta} \,, \tag{1.6}$$

r being proper matter density, f the fluid index, p the specific pressure and u the four-velocity of the fluid particle; as usual, the speed of light is taken equal to unity.

We suppose, according to ref. ([1]), the contribution of the fluid mass to the global gravitational field to be irrelevant, so that the metric g can be considered

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([1]) T. RUGGERI and A. STRUMIA: *Ann. Inst. Henri Poincaré A*, **34**, 65 (1981); A. STRUMIA: *Ann. Inst. Henri Poincaré A*, **38**, 113 (1983). The first results obtained in noncovariant formalism were given by G. BOILLAT: *C. R. Acad. Sci, Ser. A*, **278**, 909 (1974); G. BOILLAT and T. RUGGERI: *C. R. Acad. Sci., Ser. A*, **289**, 257 (1979). On shocks in relativistic astrophysics see, *e.g.*, E. P. T. LIANG: *Astrophys. J.*, **211**, 361 (1977).

([2]) A. H. TAUB: *Phys. Rev.*, **74**, 328 (1948).

([3]) D. FUSCO: *Rend. Semin. Mat. Modena*, **28**, 223 (1979).

([4]) N. VIRGOPIA and F. FERRAIOLI: *Il Nuovo Cimento B*, **81**, 197 (1984).

as assigned and we do not need to include Einstein equations into our system.

Then it results

$$h^\alpha = - rSu^\alpha\,, \qquad g = 0\,, \tag{1.7}$$

S being the entropy per mass unit. It follows that

$$\varphi_\alpha[rfu^\alpha u^\beta - pg^{\alpha\beta}] = 0\,, \tag{1.8}$$

$$\varphi_\alpha[ru^\alpha] = 0 \tag{1.9}$$

and

$$\eta = \varphi_\alpha[- rSu^\alpha]\,, \tag{1.10}$$

where

$$[w] = w - w_* \qquad \forall w \tag{1.11}$$

denotes the jump across the shock.

Taking account of (1.9) and introducing

$$\sigma_* = - u_*^\alpha \varphi_\alpha\,, \tag{1.12}$$

it results

$$\eta = \sigma_* r_* (S - S_*)\,. \tag{1.13}$$

The dependence of the state function S on the two independent variables p and V is easily obtained from Gibbs relation

$$\theta\, \mathrm{d}S = \mathrm{d}e + p\, \mathrm{d}V\,, \tag{1.14}$$

where θ is the absolute thermodynamic themperature, e the specific internal energy and

$$V = 1/r \tag{1.15}$$

the volume of the mass unit, thanks to the state equation

$$pV = K\theta \tag{1.16}$$

and the constitutive relation

$$e = C_v \theta = pV/(\gamma - 1) \tag{1.17}$$

characterizing the polytropic fluid (the symbols are well known).

Even if a more realistic relativistic approach would involve the more complicated state equation (5)

$$e = 3K\theta + mK_1/rK_2\,, \tag{1.18}$$

as a first approximation, usual in the literature (6), we shall work with the simpler equation (1.17).

The present approach, in any case, includes also the ultrarelativistic limit which is governed by the state equation

$$e = 3pV \tag{1.19}$$

and results form (1.16) and (1.17), as is well known, when one chooses

$$\gamma = 4/3 \qquad \text{(photonic gas).} \tag{1.20}$$

After integration it follows

$$S - S_* = C_v \log\{(p/p_*)(V/V_*)^\gamma\} \tag{1.21}$$

and, by substituting into (1.13),

$$\eta = C_v(\sigma_*/V_*) \log\{(p/p_*)(V/V_*)^\gamma\}\,. \tag{1.22}$$

In order to analyse the behaviour of η, we need to solve the relativistic hydrodynamic shocks, *i.e.* we must express p, V in terms of p_*, V_*, σ_*.

In the present paper, we examine quite general k-shocks (1) in the rest frame of the unperturbed fluid, developing a complete qualitative analysis of the function η. In (7) the authors examine the one-space dimensional problem with special emphasis on numerical tests.

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(5) See, *e.g.*, R. SYNGE: *The Relativistic Gas* (North Holland, Amsterdam, 1957).

(6) A. LICHNEROWICZ: *Ondes des choc, ondes infinitesimales et rayons en hydrodynamique et magnetohydrodynamique relativistes*, in *Relativistic Fluid Dynamics*, Corso C.I.M.E. 1970 (ed. Cremonese, Roma, 1971), p. 87.

(7) N. VIRGOPIA and F. FERRAIOLI: *Il Nuovo Cimento D*, 7, 151 (1986).

2. – Relativistic hydrodynamic shocks.

Following LICHNEROWICZ [7,8] we solve the Rankine-Hugoniot equations by introducing one scalar and one vector invariants across the shock:

$$J_1 = r u^\alpha \varphi_\alpha \,, \tag{2.1}$$

$$j^\beta = T^{\alpha\beta}\varphi_\alpha = J_1 f u^\beta - p\varphi^\beta \,. \tag{2.2}$$

Then the Rankine-Hugoniot equations (1.8), (1.9) become simply

$$J_1 = J_1^* \,, \tag{2.3}$$

$$j^\beta = j^\beta_* \,. \tag{2.4}$$

Now the decomposition of (2.4) along the normal to the shock surface and, respectively, onto the platform tangent to the shock manifold enables us to introduce two further scalar invariants:

$$J_2 = j^\beta \varphi_\beta \,, \tag{2.5}$$

the invariance of which comes out from contraction of (2.4) with φ_β.

The last invariant introduced is

$$J_3 = G j^{\parallel}_\beta j^\beta_\parallel / J_1^2 \tag{2.6}$$

with

$$j^\beta_\parallel = j^\beta - J_1 \varphi^\beta / G \tag{2.7}$$

and

$$G = \varphi_\beta \varphi^\beta \,. \tag{2.8}$$

We point out that $G \neq 0$ for the fluid, since lightlike shock manifolds do not occur. Moreover, since we are interested only in noncharacteristic shocks when studying η (η vanishes for characteristic shocks), also J_1 will be nonvanishing.

In fact, $J_1 = 0$ corresponds to the contact shock which is characteristic.

Now, on introducing the space-time decomposition

$$\varphi_\alpha = -\sigma_* u^*_\alpha + n^*_\alpha \,, \tag{2.9}$$

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(8) A. LICHNEROWICZ: *J. Math. Phys.* (*N. Y.*), **17**, 213 (1976).

where it is not restrictive to assume

$$n^*_\beta n^\beta_* = -1\,, \tag{2.10}$$

it results

$$G = \sigma_*^2 - 1\,. \tag{2.11}$$

Then σ_* is the shock speed relative to the rest fluid frame.

Taking account of the constitutive relation

$$f = 1 + \alpha p V\,, \qquad \alpha = \gamma/(\gamma - 1)\,, \tag{2.12}$$

holding for a polytropic fluid, we can express the invariance conditions of J_2, J_3 in terms of the variables f, V, f_*, V_*, σ_*:

$$\sigma_*^2 fV/V_*^2 + (1-\sigma_*^2)(f-1)/\alpha V = \sigma_*^2 f_*/V_* + (1-\sigma_*^2)(f_* - 1)/\alpha V\,, \tag{2.13}$$

$$f^2(1-\sigma_*^2 + \sigma_*^2 V^2/V_*^2) = f_*^2\,. \tag{2.14}$$

Summarizing, the invariance of J_2, J_3 allows us to solve the jump of the two thermodynamic variables f, V, while (2.3), (2.4) yield the jump of the four-velocity

$$u^\beta = u^\beta_* f_*/f - \varphi^\beta(1 - V/V_*)/\{(1-\sigma_*^2)f\}\,. \tag{2.15}$$

The study of η does not involve the velocity as follows from (1.22).

From (2.14) one is able to reach

$$f = \frac{f_*}{\sqrt{1-\sigma_*^2}\sqrt{1 + \left((\sigma_*/\sqrt{1-\sigma_*^2})\right)(V/V_*))^2}}\,. \tag{2.16}$$

Before introducing (2.16) into (2.13), it is convenient to define the auxiliary variable ψ as

$$\operatorname{tg}\psi = \frac{\sigma_*}{\sqrt{1-\sigma_*^2}}\frac{V}{V_*}\,, \tag{2.17}$$

where, thanks to periodicity, the angle ψ can be chosen in the first quadrant $(\psi > 0)$ and, respectively, in the fourth one $(\psi < 0)$.

It follows

$$f = \frac{f^* \cos\psi}{\sqrt{1-\sigma_*^2}}\,, \tag{2.18}$$

which greatly simplifies calculations.

Substitution into (2.13) yields, through the usual trigonometric relations,

$$\alpha f_* \sin^2\psi + f_* \cos^2\psi - \beta \sin\psi - \sqrt{1-\sigma_*^2}\cos\psi = 0 , \tag{2.19}$$

where

$$\beta = \alpha J_2 V_* / \sigma_* \tag{2.20}$$

has been introduced in the r.h.s. of (2.13). Solving eq. (2.19) gives all information concerning the shock waves. Equation (2.19) can be reduced to algebraic form by setting

$$y = \sin\psi . \tag{2.21}$$

Then

$$\sqrt{1-y^2} = \cos\psi \tag{2.22}$$

since ψ is in the first or the fourth quadrant.

Substituting (2.21) and (2.22) into (2.19) and squaring we arrive at a fourth-degree equation

$$A_0 y^4 + A_1 y^3 + A_2 y^2 + A_3 y + A_4 = 0 , \tag{2.23}$$

the coefficients of which are

$$A_0 = f_*^2(\alpha - 1)^2 , \tag{2.24}$$

$$A_1 = -2(\alpha - 1) f_* \beta , \tag{2.25}$$

$$A_2 = \beta^2 + 2(\alpha - 1) f_*^2 + 1 - \sigma_*^2 , \tag{2.26}$$

$$A_3 = -2 f_* \beta , \tag{2.27}$$

$$A_4 = f_*^2 - 1 + \sigma_*^2 . \tag{2.28}$$

We must point out that one of the four solutions to (2.23) is easily determined, since the Rankine-Hugoniot equations must be fulfilled by the trivial solution (vanishing shock). That occurs when $[U] = 0$, for any field variable. Then in (2.17) it follows that

$$V = V_* \qquad \text{(vanishing shock)}, \tag{2.29}$$

$$\operatorname{tg}\psi = \frac{\sigma_*}{\sqrt{1-\sigma_*^2}} \tag{2.30}$$

from which

$$y = \sin\psi = \sigma_* , \qquad \cos\psi = \sqrt{1-\sigma_*^2} . \tag{2.31}$$

We may remove the trivial shock solution from (2.23) to reach the cubic equation

$$a_0 y^3 + a_1 y^2 + a_2 y + a_3 = 0\,, \tag{2.32}$$

with the coefficients

$$a_0 = (\alpha - 1)^2 f_*^2\,, \tag{2.33}$$

$$a_1 = (\alpha - 1) f_* \{(\alpha - 1) f_* \sigma_* - 2\beta\}\,, \tag{2.34}$$

$$a_2 = \sigma_* a_1 + \beta^2 + 2(\alpha - 1) f_*^2 + 1 - \sigma_*^2\,, \tag{2.35}$$

$$a_3 = -\sigma_* + (1 - f_*^2)/\sigma_*\,. \tag{2.36}$$

Equation (2.32) can be solved exactly by employing the resolution formulae of the cubic equation (see *e.g.* ([9])).

Finally from (1.22), (2.12) and (2.18) one is able to evaluate

$$\eta = C_\nu \frac{\sigma_*}{V_*} \log \left\{ \frac{f_* \sqrt{1 - y^2} - \sqrt{1 - \sigma_*^2}}{(f_* - 1)\sqrt{1 - \sigma_*^2}} \left(\frac{\sqrt{1 - \sigma_*^2}\, y}{\sigma_* \sqrt{1 - y^2}} \right)^{\gamma - 1} \right\}. \tag{2.37}$$

Before starting the study of the function η defined through the parametric equation (2.32), we must make some further comments on the shock problem. It must be emphasized that from a physical standpoint an unique nonvanishing solution to (2.32) is expected (physically meaningful shock). Therefore, we need to determine how many real solutions to (2.32) exist.

A geometrical approach will be convenient.

On introducing a second variable

$$x = \sqrt{1 - y^2} = \cos\psi \geqslant 0\,, \tag{2.38}$$

eq. (2.32) will result equivalent to the system of equations

$$f_* x^2 + \alpha f_* y^2 - \sqrt{1 - \sigma_*^2}\, x - \beta y = 0\,, \tag{2.39}$$

$$x^2 + y^2 = 1\,, \qquad x \geqslant 0, \tag{2.40}$$

the solutions of which provide the intersections of an ellipse with the unitary circumference centred into the origin of axes. Since $x > 0$, only nonnegative x-valued solutions will be acceptable.

The ellipse crosses the origin and has principal axes parallel to the coordinate ones.

([9]) G. CIMMINO: *Elementi di analisi algebrica* (ed. Pàtron, Bologna, 1953).

The centre co-ordinates are given by

$$x_c = \frac{\sqrt{1-\sigma_*}}{2f_*} \geqslant 0 , \qquad y_c = \beta/2\alpha f_* . \tag{2.41}$$

i) $-1 < \sigma_* < 1$: *physical speed values.* The physical values of the shock speed in relativistic fluid dynamics are bounded by the speed of light. Inside this range, one real solution (intersection) always exists representing the vanishing shock. But another real solution must occur, since the system (2.39), (2.40) is of fourth degree. The remaining two solutions can be either real or complex conjugate.

In any case, since the ratio of the principal axes of the ellipse is

$$a/b = \sqrt{\alpha} > 1 , \tag{2.42}$$

α being always greater than unity because of the thermodynamic condition $\gamma > 1$, only two real intersections fall in the half-plain $x \geqslant 0$ (see fig. 1: *a*) $\gamma = 5/3$, monoatomic gas; *b*) $\gamma = 4/3$, photonic gas).

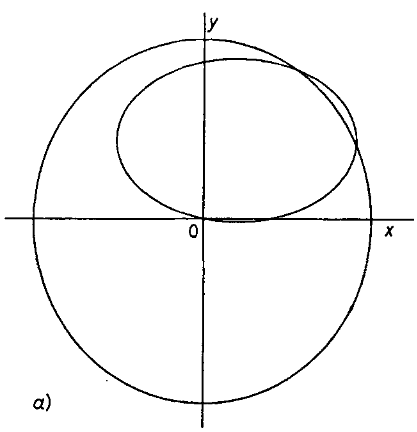

Fig. 1. – Plots of y *vs.* x: $f = 1.5$, $C_v/V_* = 1000$, $\sigma_* = 0.80$: *a*) $\gamma = 1.66$, monoatomic gas; *b*) $\gamma = 1.33$, photonic gas.

In any case the real solution to the cubic equation (2.32), which determines the intersection of physical interest, can be picked up as (9)

$$y = \sqrt[3]{z} - \frac{P}{3\sqrt[3]{z}} - a_1/a_0 , \tag{2.43}$$

where it is intended that the cubic root which must be taken is the real-valued one among the three roots that can be evaluated in the complex field.

The quantity z is given by

$$z = -Q/2 + \sqrt{Q^2/4 + P^3/27} \tag{2.44}$$

and

$$P = a_2/a_0 - a_1^2/3a_0^2\,, \tag{2.45}$$

$$Q = a_3/a_0 - a_1a_2/3a_0^2 + 2a_1^3/27a_0^3\,. \tag{2.46}$$

ii) $\sigma_* \to \pm 1$: *limiting cases.* When σ_* approaches to the speed of light (± 1), the jumps of the field variables are not defined unless the shock is vanishing; see, *e.g.*, (2.18).

Therefore, those speed values must be analysed as limiting cases.

In order to study the limiting shocks, we can introduce the speed values ± 1 into eq. (2.19) which is continuous, for such values, with respect to the parameter σ_*.

The substitution yields

$$(1-\alpha)\sin^2\psi \pm \alpha \sin\psi - 1 = 0 \tag{2.47}$$

from which

$$\sin\psi = \begin{cases} \pm(\gamma-1)\,, \\ \pm 1 \end{cases} \qquad \text{(vanishing shocks)}\,. \tag{2.48}$$

Of course, since for the vanishing shock (2.31) hold, it follows that the physical limiting shock is characterized by

$$\sin\psi = \pm(\gamma - 1)\,, \tag{2.49}$$

which is defined only if

$$\gamma \leqslant 2\,. \tag{2.50}$$

For $\gamma = 2$ all the real solutions identify with the vanishing shock. The multiplicity of the intersection, or, respectively, of the vanishing shock solution to (2.23) is four, and consequently it is three for the solution to the cubic equation (2.32).

For $\gamma > 2$ physical shock cannot occur since no real angle exists the sine of which is greater than unity.

A physical explanation of such a mathematical constraint is related to the relativistic bound of sound speed across perturbed fluid. In fact, the sound speed is given, for a polytropic gas, by

$$c_s^2 = \gamma p/rf = (\gamma - 1)(f-1)/f\,. \tag{2.51}$$

Taking account of (2.18), we have

$$c_s^2 = (\gamma - 1)\,(f_* \cos\psi - \sqrt{1 - \sigma_*^2})/(f_* \cos\psi)\,, \tag{2.52}$$

from which

$$\{c_s^2 - (\gamma - 1)\}\, f_* \cos\psi = (\gamma - 1)\sqrt{1 - \sigma_*^2}\,. \tag{2.53}$$

Since ψ belongs to the first or to the fourth quadrant so that $\cos\psi \geqslant 0$ and $\gamma > 1$ according to thermodynamics, eq. (2.53) is consistent only if it results

$$c_s^2 \geqslant \gamma - 1\,. \tag{2.54}$$

Now, on requiring the relativistic bound

$$c_s^2 \leqslant 1\,, \tag{2.55}$$

it follows

$$\gamma \leqslant 2\,. \tag{2.56}$$

From an energetic point of view the present result is reasonable: in fact, γ increases with the number of the degrees of freedom of the gas molecule. Since relativistic velocities mean high kinetic energies which break complex molecules into simpler ones, the number of degrees of freedom cannot become too large and then γ is forbidden to reach high values. It is easily shown that condition (2.56) implies that also the sound speed across unperturbed fluid is always bounded by the speed of light.

3. – Behaviour of the function η.

Let us analyse the qualitative behaviour of the function η (see fig. 2).

i) *Range.* The function is defined only if $-1 < \sigma_* < 1$, since, for $\sigma_* = \pm 1$, the rational argument is not defined and $\sigma_*^2 > 1$ makes the function complex.

ii) *Zeroes.* The function vanishes in correspondence to the characteristic shocks which are given by

$$\sigma_* = 0 \qquad \text{(contact shock)}$$

and

$$\sigma_* = \pm c_s^* \qquad \text{(sonic shocks)},$$

$$c_s^* = \sqrt{(\gamma - 1)(f_* - 1)/f_*}$$

being the sound speed.

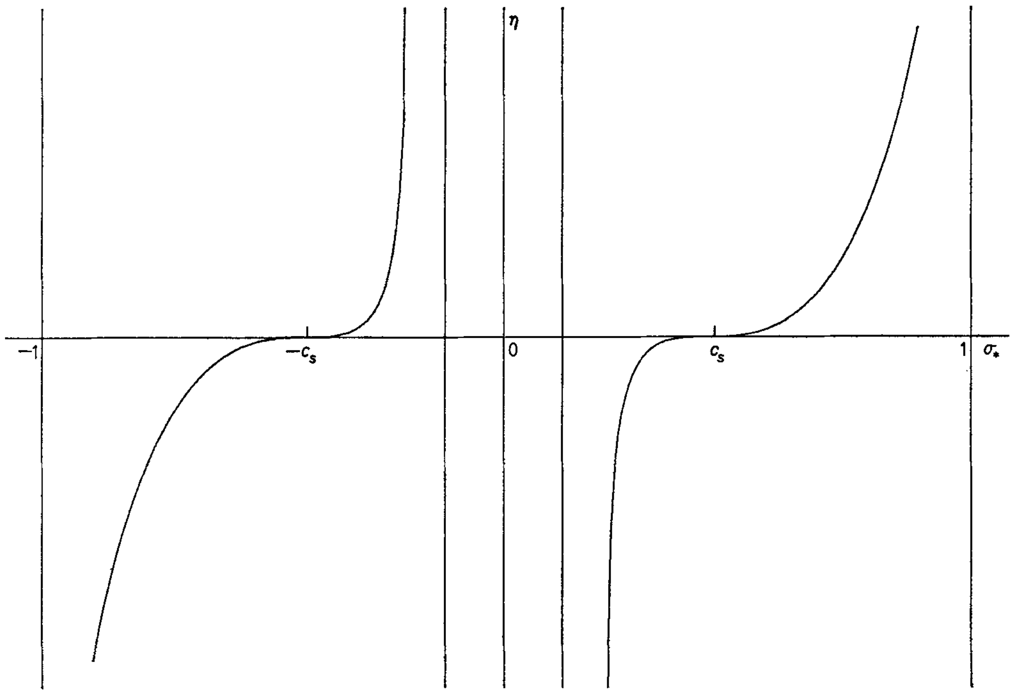

Fig. 2. – Plot of η *vs.* σ_*: $f = 1.5$, $C_v/V_* = 1000$, $\gamma = 1.66$, monoatomic gas.

iii) *Asymptotes.* Two cases are possible:

a) $y \to \pm 1$ (thermodynamic asymptotes) from which it follows in (2.19)

$$\sigma_* = \begin{cases} \pm \dfrac{(\gamma - 1)(f_* - 1)}{\gamma - 1 + f_*} & \text{(asymptotes)}, \\ \pm 1 & \text{(vanishing shocks)}. \end{cases} \tag{3.1}$$

Such asymptotes arise also in nonrelativistic fluid dynamics of shocks and are of thermodynamic nature.

b) $\sigma_* \to \pm 1$ (relativistic asymptotes). In order to evaluate the limit values of the function η when the shock speed approaches to the speed of light, we introduce the following factorization:

$$F(\sigma_*) = G(\sigma_*) H(\sigma_*), \tag{3.2}$$

where

$$G(\sigma_*) = \frac{f_* \sqrt{1 - y^2} - \sqrt{1 - \sigma_*^2}}{f_* - 1} \left(\frac{y}{\sigma_* \sqrt{1 - y_*^2}} \right)^{\gamma - 1}, \tag{3.3}$$

$$H(\sigma_*) = (1 - \sigma_*^2)^{\gamma/2 - 1}. \tag{3.4}$$

Now, taking account that

$$\lim_{\sigma_* \to \pm 1} y = \pm (\gamma - 1) \tag{3.5}$$

and that $\gamma < 2$ because of the relativistic bound of the shock speed, it follows

$$\lim_{\sigma_* \to \pm 1} G(\sigma_*) = G_0 < + \infty . \tag{3.6}$$

Moreover, since $2 - \gamma > 0$, it results

$$\lim_{\sigma_* \to \pm 1} H(\sigma_*) = + \infty . \tag{3.7}$$

Then

$$\lim_{\sigma_* \to \pm 1} F(\sigma_*) = + \infty \tag{3.8}$$

and finally

$$\lim_{\sigma_* \to \pm 1} \eta = \pm \infty . \tag{3.9}$$

For $\gamma = 2$ all lightlike shocks vanish and $\eta = 0$.

4. – Lax conditions and conclusion.

Lax conditions ([10]), when written in covariant form ([11]), are

$$\begin{cases} \lambda_*^{(1)} < \lambda_*^{(2)} \ldots < \lambda_*^{(k)} \leqslant \sigma_* \leqslant \lambda_*^{(k+m)} < \ldots < \lambda_*^{(N)} , \qquad k = 1, 2, \ldots, N , \\ \lambda^{(1)} < \lambda^{(2)} < \ldots < \lambda^{(k-m)} \leqslant \sigma_* \leqslant \lambda^{(k)} < \ldots < \lambda^{(N)} , \end{cases} \tag{4.1}$$

where m is the multiplicity of the k-th characteristic velocity.

In relativistic hydrodynamics the characteristic velocities with respect to the fluid rest frame in a four-dimensional space-time are given by

$$\begin{cases} \lambda^{(1)} = - c_s , \quad m = 1 , \\ \lambda^{(2)} = 0 , \quad m = 3 , \\ \lambda^{(3)} = c_s , \quad m = 1 . \end{cases} \tag{4.2}$$

([10]) P. D. LAX: *Shock waves and entropy*, in *Contributions to Nonlinear Functional Analysis*, edited by E. H. ZARANTONELLO (Academic Press, New York, N. Y., 1971), p. 603.

([11]) A. STRUMIA: *Rend. Circolo Mat. Palermo*, ser. II, **31**, 68 (1982).

It follows that Lax conditions are fulfilled for

i) $k = 1$ if

$$- c_s^* \leqslant \sigma_* \leqslant - c_s \,, \tag{4.3}$$

i.e. when the shock is supersonic across unperturbed fluid and subsonic with respect to the perturbed one;

ii) $k = 2, 3, 4$ for

$$\sigma_* = 0 \qquad \text{(contact shock);} \tag{4.4}$$

iii) $k = 5$ if

$$c_s^* \leqslant \sigma_* \leqslant c_s \,, \tag{4.5}$$

result which is symmetric to case i).

When the strict inequalities in (4.3) or (4.5) hold, entropy increases across the shock (noncharacteristic shocks are thermodynamically irreversible), while, when σ_* assumes one of the characteristic values, entropy jump vanishes (reversible shocks).

We conclude pointing out the remarkable result that the function η related to the behaviour of thermodynamic entropy jump across k-shocks in relativistic fluid dynamics is characterized by the occurrence of two new asymptotes in correspondence to the lightlike shock manifolds. Those asymptotes do not appear in nonrelativistic hydrodynamics and are manifestly a relativistic effect. It is remarkable that such a relativistic breakdown of the function η for superluminal shock speeds arise naturally from the theory and must not be imposed as an *ad hoc* condition. The result provides a further proof of consistency of shock wave theory with the claims of relativity.

● RIASSUNTO

È sviluppato uno studio dettagliato della funzione η, che caratterizza il salto dell'entropia attraverso le onde d'urto, in fluidodinamica relativistica, all'equilibrio termico. Si mostra che la funzione η è definita solo quando la velocità normale delle onde d'urto non supera la velocità della luce nel vuoto, compatibilmente con la teoria della relatività; inoltre si fa vedere che il salto dell'entropia tende all'infinito al tendere della velocità dell'urto a quella della luce, quando γ è minore di 2, mentre, per $\gamma = 2$, l'urto sul cono-luce si annulla.

Подробное исследование скачка энтропии поперек ударных волн в релятивистской газодиунамике.

Резюме (*). — Проводится подробное исследование функции η, которая характеризует скачок энтропии поперек ударных волн в релятивистской гидродинамике в состоянии теплового равновесия. Показывается, что функуия η определяется только при условии, если нормальная скорость ударных волн не превышает скорости света в вакууме, в соответствии с требованиями теории относительности. Более того, скачок энтропии стремится к бесконечности, когда скорость волны приближается к скорости света и γ меньше двух, тогда как для $\gamma = 2$ светоподобная унарная волна обращается в нуль.

(*) *Переведено редакцией.*