Automodellism in the Large-Angle Elastic Scattering and Structure of Hadrons.

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1. – In studying the processes of elastic hadron collisions one of the important problems now is the investigation of reactions with large momentum transfers. The corresponding kinematic region is $-t$, $s \to \infty$ at t/s fixed, or, due to the formula $\cos\theta = 1 + 2t/s$, the region of large-angle scattering in the centre-of-mass system. Interest in these processes is due to the present experimental possibilities of getting large momentum transfers on new accelerators [1]. On the other hand, there are some theoretical arguments to expect that the interaction mechanism at large momentum transfers differs essentially from that one which determines the region of small momentum transfers.

The usual opinion is that the elastic collisions with small t are determined by a global structure of hadrons like, for example, the effective radius of interaction of the order of 1 fm which is related to the slope parameter of the differential cross-section.

It is natural to expect that in elastic collisions with extremely large momentum transfers an inner local structuie of hadrons, which as is presently assumed has a « hard » or « pointlike » character, becomes more important.

In this paper we should like to point to the possibility of revealing a local or pointlike structure of hadrons in pure hadronic reactions of elastic scattering at large momentum transfers. In the framework of the quark model by using the principle of automodellism [2] it is shown that in the limit $-t$, $s \to \infty$ at t/s fixed the automodel asymptotic relations for the differential cross-sections of elastic hadron-hadron and electron-hadron reactions of Table I hold.

In the general case of two-particle collisions the differential cross-section of large-angle elastic scattering has the asymptotics

$$\frac{d\sigma}{dt}(ab \to ab) \xrightarrow[t/s \text{ fixed}]{s\to\infty} \frac{1}{s^{2(n_a+n_b-1)}} f_{ab}(t/s) . \tag{1}$$

(1) J. V. ALLABY, G. COCCONI, A. N. DIDDENS, A. KLOVNING, G. MATTHIAE, E. J. SACHARIDIS and A. M. WETHERELL: *Phys. Lett.*, **25** B, 156 (1967); **34** B, 431 (1971).
(2) V. A. MATVEEV, R. M. MURADYAN and A. N. TAVKHELIDZE: *Lett. Nuovo Cimento*, **5**, 907 (1972).

TABLE I.

Reaction $a+b\to a+b$	$\mathrm{d}\sigma/\mathrm{d}t(a+b\to a+b)$
$pp\to pp$	$(1/s^{10})f_{pp}(t/s)$
$\pi p\to\pi p$	$(1/s^{8})f_{\pi p}(t/s)$
$\pi\pi\to\pi\pi$	$(1/s^{6})f_{\pi\pi}(t/s)$
$ep\to ep$	$(1/s^{6})f_{ep}(t/s)$
$e\pi\to e\pi$	$(1/s^{4})f_{e\pi}(t/s)$
$ee\to ee$	$(1/s^{2})f_{ee}(t/s)$

Here n_a and n_b are the numbers of constituent quarks of a and b particles, *e.g.* for proton $n_p=3$ and for pion $n_\pi=2$. Considering an electron as a structureless particle we should put $n_e=1$ in formula (1) for reactions with electrons.

Attempts to the derivation of the power character of the asymptotic behaviour of differential cross-sections at large angles have been done in a number of recent works [3] from the various model assumptions.

2. – We proceed now to the derivation of formula (1) which is the main result of this work. As was mentioned above in particle collisions at high energies with bounded momentum transfers

$$q_\perp\leqslant R^{-1},\qquad R\sim 1\ \text{fm}\,,$$

the essential dimensional parameter which determines the behaviour of observable quantities is the effective radius of strong interactions.

In processes with extremely large momentum transfers

$$q_\perp\sim q_\parallel\sim E\,,\qquad E\to\infty\,,$$

the radius of strong interactions is not very important and the asymptotic behaviour is determined by the dynamics of interactions at small distances and small time intervals, *i.e.* by an inner local structure of hadrons.

Consider the general case of elastic scattering of two arbitrary particles $a+b\to a+b$.

Let us assume that in collisions with extremely large momentum transfers a particle behaves as a composite system with n_a constituent quarks and with a state vector of the form

$$|a\rangle=\mathcal{N}_a^{\frac{1}{2}}\cdot|n_a\ \text{quarks}\rangle\,. \tag{2}$$

Since the dimension of one particle state vector in the relativistic normalization is equal to

$$[|1\rangle]=m^{-1}\,,$$

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[3] D. CLINE, F. HALZEN and M. WALDROP: University of Wisconsin preprint; D. HORN and M. MOSHE: *Nucl. Phys.*, **48** B, 557 (1972); W. R. THEIS: DESY preprint 72/35 (1972); A. V. EFREMOV: JINR, E2-6612 (1972).

one can easily determine the dimension of the structure factor

$$[\mathcal{N}_a] = m^{2(n_a-1)}. \tag{3}$$

The differential cross-section is related to the transition matrix element in the usual way:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(ab \to ab) = \left|\frac{\langle ab|T|ab\rangle}{s}\right|^2 . \tag{4}$$

Substituting relation (2) for the state vectors of two colliding particles a and b in formula (4) for the differential cross-section, we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(ab \to ab) = (\mathcal{N}_a\mathcal{N}_b)^2 F_{ab}(s, t) ,$$

where

$$F_{ab}(s, t) = \frac{1}{s^2}\left|\langle n_a, n_b|T|n_a, n_b\rangle\right|^2. \tag{5}$$

Using the dimensions of the factors $\mathcal{N}_a$ and $\mathcal{N}_b$ one finds the dimension of the quantity

$$[F_{ab}] = m^{-4(n_a+n_b-1)}. \tag{6}$$

We formulate now the automodellism hypothesis for the processes of large-angle scattering at high energies. *At large s and $-t$ for fixed ratios t/s all the essential dimensional constants are contained in the factors $\mathcal{N}_a$ and $\mathcal{N}_b$.*

According to this hypothesis the function $F_{ab}(s, t)$ depends only on the kinematical variables s and t and under the momentum scale transformations

$$\begin{cases} p_i \to \lambda p_i , & i = a, b, \\ s \to \lambda^2 s , \\ t \to \lambda^2 t , \end{cases} \tag{7}$$

has to transform as a homogeneous function of the corresponding physical dimension.

One gets from (6) that when the particle momenta undergo the scale transformations (7) the following relation has to hold:

$$F_{ab}(\lambda^2 s, \lambda^2 t) = \lambda^{-4(n_a+n_b-1)} \cdot F_{ab}(s, t) , \tag{8}$$

or

$$F_{ab}(s, t) = \frac{1}{s^{2(n_a+n_b-1)}} f_{ab}(t/s) . \tag{9}$$

From this we obtain formula (1).

3. – The following circumstances should be mentioned here:

First of all, as is seen from eq. (1), the asymptotic behaviour of the differential cross-sections at large angles depends essentially on the number of constituents, *i.e.* on the « degree of complexity » of particles.

It is assumed that the electron behaves as a really elementary particle with a number of constituents $n_e = 1$. For what concerns hadrons, in the framework of the quark model there exist, in general, side by side with the basic quark configuration, the additional quark-antiquark pairs with appropriate probabilities. So when we say that the proton consists of three quarks, this means that the decomposition of the proton state vector in the Fock space begins from the three-quark state

$$|p\rangle = \int \mathrm{d}\boldsymbol{k}_1 \mathrm{d}\boldsymbol{k}_2 \mathrm{d}\boldsymbol{k}_3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 - \boldsymbol{p}) C(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) |\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3\rangle . \tag{10}$$

An account of the contribution of the higher configurations gives the corrections to the differential cross-section of the relative order $s^{-4n_{q\bar{q}}}$, where $n_{q\bar{q}}$ is a number of the additional quark-antiquark pairs.

Thus the main contribution to the differential cross-section of large-angle elastic scattering at high energies is due to the basic quark configuration.

In the Feynman's parton-type models, where the number of constituents varies within wide limits and can be bounded from above by the kinematically allowed number of secondary particles, *i.e.*

$$n_a + n_b \to n_{\text{partons}} < \frac{1}{\mu}\sqrt{s} , \tag{11}$$

our consideration leads naturally to the lower estimate for the differential cross-section of the large-angle elastic scattering:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} > C(\theta) \exp\left[-\frac{2}{\mu}\sqrt{s}\ln s\right], \qquad s \to \infty,\ \theta \text{ fixed}. \tag{12}$$

This result does not contradict the Cerelus-Martin [4] and Logunov-Mestvirishvili [5] asymptotic bound, which follows from the general principles of quantum field theory.

Secondly, the analysis discussed here does not contradict the automodellism principle for hadron collisions considered in analogy with the « plain » explosion and which are characterized by the bounded values of the transverse momenta [2].

It is quite interesting to extent this consideration to the interactions of relativistic nuclei as was done in ref. [6] in considering the « cumulating » effect.

The processes with large momentum transfers are very rare as compared to the background of events with bounded momentum transfers and are determined by an interaction region the size of which is much smaller than the effective cross-section of the particles:

$$l_0 \sim \frac{1}{E} \ll R \sim 1\,\text{fm} .$$

Thus, the processes of elastic collisions with large momentum transfers are deter-

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(4) F. CERULUS and A. MARTIN: *Phys. Lett.*, **8**, 80 (1964).

(5) A. A. LOGUNOV and M. A. MESTVIRISHVILI: *Phys. Lett.*, **24** B, 583 (1967).

(6) A. M. BALDIN: JINR, P7-5808 (1971), Physical Institute of the Academy of Sciences of the USSR, preprint N1 (1971).

mined mainly by the local structure of the hadron, which is considered at high energies as an infinitely thin disc. The transverse size plays no essential role.

4. – The approach proposed above can be extended to the consideration of the asymptotic behaviour of the electromagnetic form factors in the limit of large momentum transfers.

We assume that at large momentum transfers the form factors can be represented by the formula

$$F_a(t) = \mathcal{N}_a f_a(t)\,, \tag{13}$$

where $f_a(t)$ does not contain any dependence on the fixed dimensional constants. The dimension of $\mathcal{N}_a$ is given by (3). It is easy to see that the dimension of $f_a(t)$ is

$$[f_a(t)] = m^{-2(n_a-1)}. \tag{14}$$

Therefore, according to the principle of automodellism, the function $f_a(t)$ transforms like a homogeneous function of t, *i.e.*

$$f_a(t) \sim \frac{1}{t^{n_a-1}} \tag{15}$$

under the scale transformations (7).

Thus, for the electron (quark), meson and nucleon the asymptotic behaviours of the form factors are respectively

electron:	$F_e(t) \sim 1$	(pointlike),
meson:	$F_\pi(t) \sim 1/t$	(pole),
nucleon:	$F_p(t) \sim 1/t^2$	(dipole).

It is easy to see that such a behaviour is in complete agreement with the results which can be obtained by the direct application of the dimensional analysis to electron-hadron and electron-electron scattering (Table I).

We note, in conclusion, that the asymptotics of the vertex functions have been considered in ref. [7,8] from the point of view of the anomalous dimensions.

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(7) D. V. Shirkov: JINR, P2-6938 (1973).
(8) A. A. Migdal: *Phys. Lett.*, **37** B, 98 (1971).