Vacuum Electromagnetics Derived Exclusively from the Properties of an Ideal Fluid.

E. M. KELLY

Department of Physics
Californial State Polytechnic University - Pomona, Cal. 91768

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Summary. — If a return to ether concepts becomes necessary or desirable, an event not as unlikely as it seemed a short time ago, the medium dealt with here may be of interest. Maxwell's vacuum equations are derived from the properties of an ideal fluid with no support from experimental facts. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$, as so interpreted, are statistical in nature and readily visualizable in a mechanical and geometric sense. The electromagnetic-momentum theorem is shown to be a second-order effect stemming from nonlinear effects which were neglected in the derivation of the vacuum equations. The presence of voids (hollow vortex cores) in an otherwise incompressible fluid allows a mechanical interpretation of the Lorentz gauge. It is suggested as a matter of parsimony that Newton's laws may be replaceable by kinematics, thereby reducing electrodynamics to Euclidean geometry.

1. – Introduction.

A primary objection to most ether theories is esthetic; they simply do not meet the demands of parsimony. On the other hand, the richness of the « vacuum », the phenomenological identity of special relativity and at least one ether theory (1), and recent cosmological observations (2,3) suggest that re-

(1) H. ERLICHSON: *Amer. Journ. Phys.*, **41**, 1068 (1973).
(2) E. K. CONKLIN: *Nature*, **222**, 971 (1969).
(3) S. J. PROKHOVNIK: *Foun. Phys.*, **3**, No. 3, 351 (1973).

jection of the ether concept is not justified. The medium which is the subject of this paper satisfies some minimum requirements for an ether model by evincing electromagnetic properties with no more than an ideal fluid as the starting point. The reduction of ideal-fluid dynamics to kinematics is mentioned as a possibility for an increase in parsimony. Some of these ideas have already been published ([4]), but numerous weaknesses have required extensive reworking so that the present treatment is almost entirely self-contained. For brevity, historical background is left to ref. ([4]).

The postulates are those of an ideal fluid, structureless, homogeneous, incompressible, nonviscous and devoid of other intrinsic qualities such as gravitational and electrical ones. Newton's laws of motions are assumed, although in appendix C it is suggested that they may be simplified, or at least stated in terms of kinematics. To obtain nontrivial results we consider initial conditions which are somewhat complicated. The model can easily be visualized by imagining a box pierced through from all directions by pins. If the box is filled with a viscous fluid and all pins spun about their axes, vortex motion will arise around each pin. This motion persists if the viscosity is annihilated. Next, it can be imagined that all the pins are withdrawn so that the medium has hollow vortex tubes. The fluid is in cyclic irrotational motion about the tubes; these are flexible and, as it is impossible for them to remain straight in such an environment without internal support, they acquire a writhing motion, which persists as a property of the medium. In an undisturbed (neutral) state, the medium has no preferred directions. Now allow the number of tubes crossing the box to increase, while the tube diameters decrease so as to maintain a small ratio of the tube diameters to distances between tubes, until the individual tubes are unresolvable. The system now appears macroscopically as a continuum in which the extreme variations in velocity, acceleration and pressure over the microscopic distances from tube to tube are undiscernible. The detailed effects of the hydrodynamic laws are lost in the averaging effects of large numbers of tubes. Because of the voids (hollow vortex tubes) the bulk medium is compressible even though the fluid itself is not; however, if the volume fraction of void is small, as will be assumed, the compressibility is slight.

Although the magnitudes of the geometrical parameters (tube cross-section, length of tube per unit volume) are presently unknown, one can make rough estimates. For stability, the hollow tubes should never get too close to one another; furthermore, the effects to be considered involve only slight shifts of tubes from their average positions. The minimum distance between tubes might be 10^3 or more times the tube radius. If there is to be a chance of dealing with phenomena at the nuclear level, 10^6 or more tubes crossing an area of 10^{-28} square meter might be necessary to get good statistics. This corresponds to 10^{34} tubes per m^2, a distance between tubes of 10^{-17} m and a tube radius of

([4]) E. M. KELLY: *Amer. Journ. Phys.*, **31**, 785 (1963); **32**, 657 (1964).

10^{-20} m. If the tubes were arranged so that a third of them were parallel to each of three perpendicular axes, the tube length per unit volume (tube density, L) would be $3 \cdot 10^{34}$ meters per cubic meter. These quantities are not to be taken literally, but only to give a crude notion of the possible scale of the model.

Although at first glance the picture appears to be one of complete chaos, it will be seen by first considering a few properties of vortex tubes and then averaging these effects over many tubes that certain statistically defined quantities behaving like electrical quantities emerge.

When a gravitational field is present, the motion of a horizontal infinite cylinder in a liquid, where there is circulation about the cylinder, is readily found ([5]). The cylinder does not tend to sink even in the case where its weight exceeds the buoyant force; rather, the path of the center is a horizontal trochoid. A special case, occurring when the cylinder is released from rest, is an inverted cycloid, the cylinder accelerating downward at first, but then laterally also as the lift force becomes appreciable. Eventually, as the velocity increases and changes direction, the vertical component of the lift exceeds the weight and the cylinder accelerates upward, finally coming to rest at its original height but with a horizontal translation. The process then repeats itself indefinitely, since the initial conditions, except for the horizontal translation, are recovered periodically. The average effect over a long time is a sidewise translation of the cylinder at constant speed. This average speed will be denoted by $\dot{\xi}$ and the average lateral velocity (referred to as « drift ») by $\dot{\boldsymbol{\xi}}$; this vector is always perpendicular to the axis of the cylinder. It is customary to define the « strength of circulation » $\varkappa$ so that $2\pi\varkappa$ is the circulation about the cylinder. A cylinder moving about in the fluid carries its circulation with it; no change in the circulation is possible without friction or other extraneous forces which are, by hypothesis, not present in the fluid. It is also convenient to define a « spin vector » $\boldsymbol{\varkappa}$ directed along the cylinder so that, if one grasps the cylinder in the right hand with the thumb in the direction of $\boldsymbol{\varkappa}$, the fingers give the sense of circulation. A cylinder perpendicular to the paper with counterclockwise circulation will have $\boldsymbol{\varkappa}$ out of the paper. When such a cylinder starts to fall a lift force to the right develops; that is the drift $\dot{\boldsymbol{\xi}}$ is to the right. If the force $\boldsymbol{F}$ (including weight and buoyancy but not lift) is downward, the drift is in the direction of $\boldsymbol{\varkappa} \times \boldsymbol{F}$. Later, when the geometrical properties of the medium are emphasized, a unit vector $\boldsymbol{\lambda}$, parallel to $\boldsymbol{\varkappa}$, is introduced, so that $\boldsymbol{\varkappa} = \varkappa\boldsymbol{\lambda}$. The drift is parallel to $\boldsymbol{\lambda} \times \boldsymbol{F}$.

Now consider a rigid torus with circulation at rest in a liquid with no gravitational field. The liquid near the surface farthest from the center of the torus is moving more slowly than that near the nearest surface; thus there is a greater

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([5]) L. M. Milne-Thomson: *Theoretical Hydrodynamics*, 2nd edition (New York, N. Y., 1951), p. 179.

pressure at the outer surface. If the torus suddenly becomes a void, several changes occur; the tube is flexible and compressible, and has a smaller virtual mass (6) than a tube containing a ponderable substance. These changes result in an acceleration of the torus toward its center. The velocity thus acquired creates a lift force which is at first normal to the plane of the torus; that is, drift along the axis occurs just as in the case of the straight tube, except that here weight and buoyancy are absent and the net applied force is derived exclusively from pressure differences. The motion is like that of a smoke ring; if the magnitude of the vibratory motion is small, the torus appears to be advancing at constant speed. This case is treated in detail by HICKS (7).

The effect on the velocity of advance of a particular segment of a torus due to distant segments is negligible for a large thin ring, so that the motion is determined by local conditions. If the effects of variations in shape and size of cross-section are negligible, as may well be the case for exceedingly thin tubes, we may regard the tube as a line the motion of which is a function of curvature alone. This function has, presumably, a McLaurin's expansion (although see appendix A) so that for small curvatures we may write

$$\dot{\xi} = \eta C\,, \tag{1.1}$$

where C is the magnitude of the curvature and η a constant, the « drift coefficient ». The direction of $\dot{\boldsymbol{\xi}}$ is the same as that of the fluid on the inner (concave) side of the tube, so that when the vector curvature is $\boldsymbol{C}$, as in fig. 1,

$$\dot{\boldsymbol{\xi}} = \eta\boldsymbol{\lambda}\times\boldsymbol{C}\,.$$

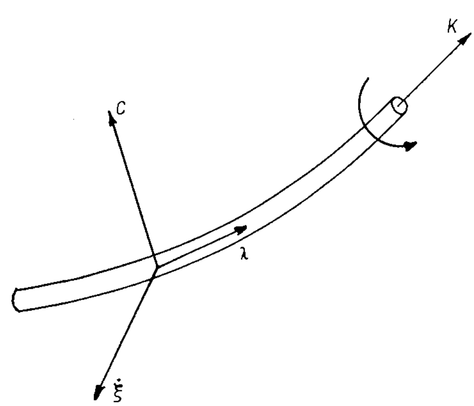

Fig. 1. – The drift $\dot{\xi}$ of a bent tube is in the direction of fluid flow on the side of the tube toward C.

(6) L. M. MILNE-THOMSON: *Theoretical Hydrodynamics*, 2nd edition (New York, N. Y., 1951), p. 228.
(7) W. M. HICKS: *On the steady motion and small vibrations of a hollow vortex*, in *Phil. Trans. Roy. Soc.*, p. 161-195 (1884).

When a cylinder with circulation moves laterally in a fluid of density ϱ the lift force on it is $2\pi\varkappa\varrho\dot{\xi}$ per unit length in the direction of $\boldsymbol{\lambda}\times\dot{\boldsymbol{\xi}}$; that is

(1.2) $$\text{lift per unit length} = 2\pi\varkappa\varrho\boldsymbol{\lambda}\times\dot{\boldsymbol{\xi}},$$

accompanied by a reaction thrust on the fluid

(1.3) $$\text{thrust per unit length} = 2\pi\varkappa\varrho\dot{\boldsymbol{\xi}}\times\boldsymbol{\lambda}.$$

For hollow tubes these forces are fictitious since a void can neither receive nor apply a force. They are, however, convenient fictions for expressing thrusts associated with momentum transfer rates arising from bent tubes. To see how momentum transfer is generated from bent tubes, consider fig. 2, showing first

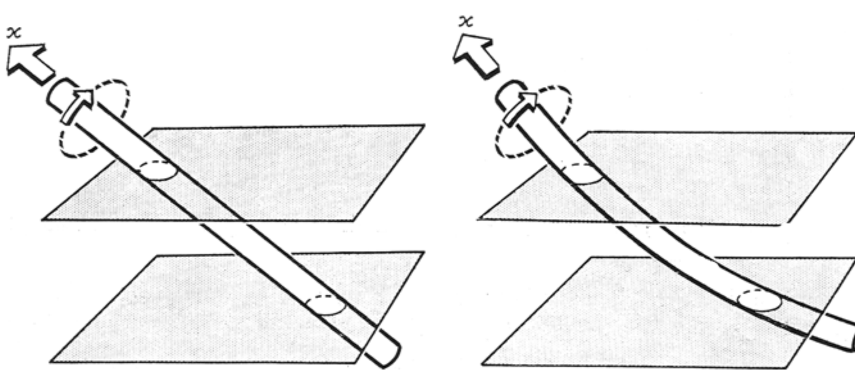

Fig. 2. – The rate of momentum transfer into a region between parallel planes is zero from straight tubes; when the tubes are bent the momentum transfer rate into the region is different for the two planes, resulting in thrust.

a straight tube and two parallel planes intersecting it. Fluid crosses each plane due to the circulation, but the rate of transfer of momentum into the space between planes is zero, since as much leaves across one plane as enters across the other. If the tube is curved this equality is upset, so that a net rate of transfer of momentum, a thrust, is created. When the thrusts from all tube sections in a volume V are added, the result divided by V is the average thrust per unit volume. In the limit of small V, this is defined as the thrust vector $\boldsymbol{T}$.

In the neutral state of the medium thrusts due to curved tubes are occurring everywhere, but, by definition of the neutral state, no appreciable macroscopic acceleration of fluid results; that is, on a sufficiently large scale the thrusts cancel with statistical accuracy. The medium, as with a quiescent gas, is in dynamic equilibrium. However, it is easy to imagine a state where the thrusts do not cancel. Consider a plastic in which many long straight threads are embedded in random directions; if the plastic is bent nearly all threads acquire curvature, although of different amounts and directions depending upon their original directions. The average vector curvature of the threads as a result of a displacement resulting in strain is in general nonzero. A quantity which has considerable prominence later is a bulk displacement $\boldsymbol{D}$. Vortex filaments, that is, fine vortex tubes containing fluid rather than void, move with the fluid [8]. Hollow tubes, except for motion due to curvature (and effects of large-scale pressure gradients, which will be ignored for the present), will do likewise. A bulk displacement as considered here is really a displacement of tubes rather than of fluid, since there is no attempt to identify and follow a particular blob of fluid. However, it is convenient for visualization to think of the medium being displaced as a whole, carrying the tubes with it, just like the plastic with embedded threads, while ignoring the microscopic cyclic motion around the individual tubes. The form of $\boldsymbol{D}$ determines the pattern of tube curvature and therefore the average drift and thrust on a region of fluid. Terms like $\partial D_y/\partial x$ are tangents of angles of rotation; when such terms are infinitesimal the angles are equal to the tangents, and, being infinitesimal, can be regarded as vectors. Thus, we may define

$$\boldsymbol{\theta} = \tfrac{1}{2}\nabla\times\boldsymbol{D}\,. \tag{1.4}$$

$\boldsymbol{D}$ and $\boldsymbol{\theta}$ appear later in close association with the vector potential and magnetic induction, respectively.

It is already evident that large-scale (bulk) effects are statistical; in calculating thrust per unit volume due to tube curvature, for example, we cannot actually allow the volume to vanish in the limit, but must keep it large enough to obtain a statistically valid average of thrusts from many tubes. In particular, it will become apparent that Maxwell's equations along with $\boldsymbol{E}$ and $\boldsymbol{B}$ for the medium have only statistical significance. The displacement $\boldsymbol{D}$ and angular bulk displacement $\boldsymbol{\theta}$, along with lateral tube displacement $\boldsymbol{\xi}$ and drift $\dot{\boldsymbol{\xi}}$, can all be considered as measured relative to a co-ordinate system which is at rest relative to the centroid of the fluid, assuming the latter to be of finite extent.

2. – Maxwell's vacuum equations.

Consider a region of the medium, initially neutral, to undergo a displacement which bends tubes. Tube bending depends partly on tube orientation; if, for instance, an initially flat slab is bent to form a cylindrical shell, fig. 3, tubes transverse to the axis have maximum bending, those parallel to the axis no bending, and tubes with intermediate orientations acquire intermediate cur-

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[8] L. M. Milne-Thomson: *Theoretical Hydrodynamics*, 2nd edition (New York, N. Y., 1951), p. 79.

vatures, the general effect being an average curvature along the curvature $\boldsymbol{K}$ of the cylinder. It is evident that tubes are drifting across the line of $\boldsymbol{K}$ in the sense of $\boldsymbol{\lambda}\times\boldsymbol{C}$ with speeds proportional to the magnitude of the latter vector. If we choose a direction, calling it « upward » for convenience, it is apparent that all tubes crossing $\boldsymbol{K}$ from right to left have upward spin components, while

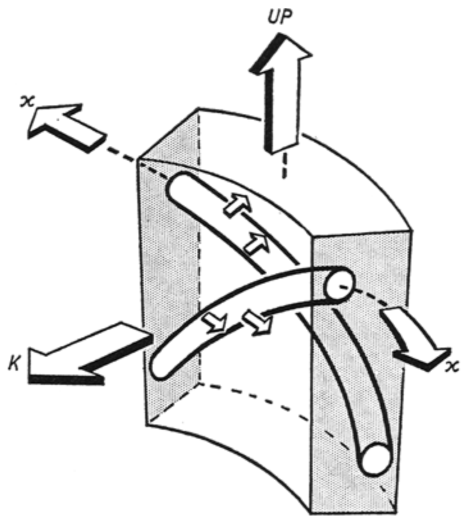

Fig. 3. – Preferential sorting of spin occurs as a result of medium bending. The short arrows normal to tubes show the direction of drift.

those crossing from left to right have downward spin components. Since each tube carries with it circulation of amount $2\pi\varkappa$, circulation is being sorted preferentially by medium bending. When drift varies in space, a net spin density may accumulate so that the medium acquires a rotation which appears macroscopically as vorticity, even though there is no vorticity at the microscopic level.

The expressions for thrust per unit length developed in the introduction permit us to write the thrust per unit volume $\boldsymbol{T}$ from drifting tubes in several ways:

$$(2.1)\qquad \boldsymbol{T}=V^{-1}\!\int 2\pi\varkappa\varrho\,\dot{\boldsymbol{\xi}}\times\boldsymbol{\lambda}\,\mathrm{d}l=V^{-1}\!\int 2\pi\varkappa\varrho\eta(\boldsymbol{\lambda}\times\boldsymbol{C})\times\boldsymbol{\lambda}\,\mathrm{d}l=V^{-1}\!\int 2\pi\varkappa\varrho\eta\boldsymbol{C}\,\mathrm{d}l\,,$$

where $\mathrm{d}l$ is the element of tube length and V the volume over which the integration extends. Although V cannot shrink to a point without vitiating the meaning of $\boldsymbol{T}$ as an average, we can take it small enough, while still retaining good statistics, to define a « quasi-point » or « statistical point » function.

Now consider a thin torus of radius r and small cross-sectional area s. The inertia of the fluid inside the torus is $\varrho(2\pi rs)$ and the tangential velocity is $r\omega$, ω being the angular speed. The angular momentum is then $2\varrho As\omega$, where

$A = \pi r^2$ is the area of the loop. The torque on the fluid inside the torus is $r(\boldsymbol{T}\cdot s\,\mathrm{d}\boldsymbol{a})$, where $\mathrm{d}\boldsymbol{a}$ is the element of arc, integrated around the loop. Equating torque to rate of change of angular momentum, we get

$$A^{-1}\int \boldsymbol{T}\cdot \mathrm{d}\boldsymbol{a} = 2\varrho\, \partial\boldsymbol{\omega}/\partial t\,. \tag{2.2}$$

For small loops the left-hand side approaches the component of $\nabla\times\boldsymbol{T}$ normal to the loop and since the integration loop is arbitrary we have

$$\nabla\times\boldsymbol{T} = 2\varrho\,\partial\boldsymbol{\omega}/\partial t\,. \tag{2.3}$$

This, like all similarly derived quantities, is only statistically valid, since the integration loop cannot shrink to a vanishing limit.

After tubes have drifted for a time from their neutral positions they have translated laterally by an amount $\boldsymbol{\xi}$. Since it turns out to have properties similar to those of the electric vector, we define

$$\boldsymbol{E} = k_1 V^{-1}\int 2\pi\varkappa\varrho\,\boldsymbol{\lambda}\times\boldsymbol{\xi}\,\mathrm{d}l\,. \tag{2.4}$$

Comparison with eq. (2.1) shows that this differs from $k_1\boldsymbol{T}$ by the presence of $\boldsymbol{\xi}$ instead of $\dot{\boldsymbol{\xi}}$, as well as in the order of cross-multiplication. The latter change is made so that later results will conform to analogues of electric and magnetic fields.

Inspection of (2.4) and (2.1) shows that

$$\boldsymbol{T} = -\,k_1^{-1}\,\partial\boldsymbol{E}/\partial t\,. \tag{2.5}$$

From eqs. (2.3) and (2.5) we get

$$k_1^{-1}\,\varrho^{-1}\,\nabla\times(\partial\boldsymbol{E}/\partial t) = -\,2\,\partial\boldsymbol{\omega}/\partial t\,.$$

Integration gives

$$k_1^{-1}\,\varrho^{-1}\,\nabla\times\boldsymbol{E} = -\,2\boldsymbol{\omega}\,, \tag{2.6}$$

where the function of integration vanishes if the initial state of the medium is neutral.

If the net spin density is spread uniformly across an area normal to $\boldsymbol{\omega}$ the area spins like a rigid body except for the microscopic turbulence. We consider only those cases where the medium experiences angular displacements which are so small that no appreciable error is incurred by treating them as vectors. Let the angular displacement be $\boldsymbol{\theta}$; it will be apparent shortly that this vector has similarities to the magnetic-induction vector, so we define a new vector

$$\boldsymbol{B} = 2k_2\boldsymbol{\theta} = k_2\nabla\times\boldsymbol{D}\,, \tag{2.7}$$

where $\boldsymbol{D}$ is the bulk medium displacement. Differentiation gives

$$\partial\boldsymbol{B}/\partial t = 2k_2\,\partial\boldsymbol{\theta}/\partial t = 2k_2\boldsymbol{\omega}\,, \tag{2.8}$$

which, combined with eq. (2.6), gives

$$k_1^{-1}k_2\,\varrho^{-1}\nabla\times\boldsymbol{E} = -\,\partial\boldsymbol{B}/\partial t\,. \tag{2.9}$$

Thus, the medium exhibits a property similar to that described by Faraday's law for stationary circuits. Here, it is simply the assertion that, when $\nabla\times\boldsymbol{E}$ does not vanish, more tubes of one sense of spin have entered a loop than have left it, the result being a net spin density proportional to $\nabla\times\boldsymbol{E}$, although oppositely directed, and a concomitant angular velocity proportional to $\partial\boldsymbol{B}/\partial t$. Consequently, there is bulk vorticity of amount $2\boldsymbol{\omega} = k_2^{-1}\partial\boldsymbol{B}/\partial t$.

In order to find further relationships between $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{T}$ in terms of $\boldsymbol{D}$ and its derivatives, we examine the effects of the medium displacement in more detail. Under a bulk displacement $\boldsymbol{D}$, an initially straight tube is, in general, rotated and bent by the strain. Consider a tube section between neighboring points P and Q, denoting the position vector of Q relative to P by $\mathrm{d}\boldsymbol{\lambda}$. After the displacement the points have moved to Q' and P', the new position vector being $\mathrm{d}\boldsymbol{\lambda}' = \mathrm{d}\boldsymbol{\lambda} + (\mathrm{d}\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}$. Only the component of $(\mathrm{d}\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}$ normal to $\mathrm{d}\boldsymbol{\lambda}$ produces rotation; for small angles (recall that $\boldsymbol{\lambda}$ is a unit vector tangent to the tube) the magnitude θ of the rotation is $|\boldsymbol{\lambda}\times(\mathrm{d}\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}|/\mathrm{d}\lambda = |\boldsymbol{\lambda}\times(\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}|$. Furthermore sufficiently small angles may be considered as vectors so that

$$\boldsymbol{\theta} = \boldsymbol{\lambda}\times(\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}\,. \tag{2.10}$$

The rotation of a tube segment at Q' relative to the rotation of a segment of the same tube at P' is

$$\mathrm{d}\boldsymbol{\theta} = (\mathrm{d}\boldsymbol{\lambda}\cdot\nabla)\,\boldsymbol{\theta}\,.$$

This differential rotation induces curvature in the (initially straight) tube. The part of $\mathrm{d}\boldsymbol{\theta}$ directed along the tube produces no curvature but only a twist which has no obvious significance for our present analysis. The component normal to the tube results in curvature of magnitude $\mathrm{d}\theta/\mathrm{d}\lambda$ concave in the direction of $\mathrm{d}\boldsymbol{\theta}\times\boldsymbol{\lambda}$. The curvature vector can therefore be written

$$\boldsymbol{C} = (\mathrm{d}\boldsymbol{\theta}\times\boldsymbol{\lambda})/\mathrm{d}\lambda = [(\mathrm{d}\boldsymbol{\lambda}\cdot\nabla)\,\boldsymbol{\theta}]\times\boldsymbol{\lambda}/\mathrm{d}\lambda = [(\boldsymbol{\lambda}\cdot\nabla)\,\boldsymbol{\theta}]\times\boldsymbol{\lambda} = -\boldsymbol{\lambda}\times[(\boldsymbol{\lambda}\cdot\nabla)\{\boldsymbol{\lambda}\times[(\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}]\}]\,.$$

Since $\boldsymbol{\lambda}$ is a constant vector this reduces to

$$\boldsymbol{C} = -\boldsymbol{\lambda}\times[\boldsymbol{\lambda}\times(\boldsymbol{\lambda}\cdot\nabla)\{(\boldsymbol{\lambda}\cdot\nabla)\boldsymbol{D}\}] = -\boldsymbol{\lambda}\times[\boldsymbol{\lambda}\times(\boldsymbol{\lambda}\cdot\nabla)^2\boldsymbol{D}],$$

which, upon expanding the triple vector product, becomes

$$\boldsymbol{C} = -[\boldsymbol{\lambda}\cdot(\boldsymbol{\lambda}\cdot\nabla)^2\boldsymbol{D}]\boldsymbol{\lambda} + (\boldsymbol{\lambda}\cdot\nabla)^2\boldsymbol{D}. \tag{2.11}$$

The curvature will vary with the tube direction; for the investigation of bulk properties the mean curvature is needed, since this determines the mean thrust from momentum transport arising from bent tubes. It is shown in appendix B that the mean curvature is

$$\overline{\boldsymbol{C}} = (4/15)\nabla^2\boldsymbol{D} - (2/15)\nabla(\nabla\cdot\boldsymbol{D}). \tag{2.12}$$

The thrust per unit volume is, from eq. (2.1), $V^{-1}\int 2\pi\varkappa\varrho\eta\boldsymbol{C}\,\mathrm{d}l$. Using the average value of $\boldsymbol{C}$, we get $\boldsymbol{T} = V^{-1}(2\pi\varkappa\varrho\eta\overline{\boldsymbol{C}})\int\mathrm{d}l = 2\pi\varkappa\varrho\eta L\overline{\boldsymbol{C}}$, where $L = V^{-1}\int\mathrm{d}l$, the tube length per unit volume. With the result of appendix B, and setting $G = (8/15)\pi\varkappa\varrho\eta L$, we get

$$\boldsymbol{T} = G\nabla^2\boldsymbol{D} - \tfrac{1}{2}G\nabla(\nabla\cdot\boldsymbol{D}). \tag{2.13}$$

When $\nabla\cdot\boldsymbol{D}$ vanishes this reduces to

$$\boldsymbol{T} = G\nabla^2\boldsymbol{D} = -G\nabla\times(\nabla\times\boldsymbol{D}), \tag{2.14}$$

a special case which will now be examined. With $\nabla\times\boldsymbol{D} = k_2^{-1}\boldsymbol{B}$, from eq. (2.7) and $\boldsymbol{T} = k_1^{-1}\,\partial\boldsymbol{E}/\partial t$ from eq. (2.5) we can write eq. (2.14) in the form

$$\nabla\times\boldsymbol{B} = k_1^{-1}k_2G^{-1}\,\partial\boldsymbol{E}/\partial t, \tag{2.15}$$

which is analogous to the second of Maxwell's curl equations for free space. Since values of k_1 and k_2 have not yet been chosen, we may set $k_1^{-1}k_2\varrho^{-1} = 1$ and $k_1^{-1}k_2G^{-1} = c^{-2}$, so that $G\varrho^{-1} = c^2$ and eqs. (2.9) and (2.15) take the familiar form of the « vacuum » equations

$$\nabla\times\boldsymbol{E} = -\,\partial\boldsymbol{B}/\partial t, \tag{2.16}$$

$$\nabla\times\boldsymbol{B} = c^{-2}\,\partial\boldsymbol{E}/\partial t. \tag{2.17}$$

Since, by definition, $\boldsymbol{B}$ is $k_2\nabla\times\boldsymbol{D}$ we get

$$\nabla\cdot\boldsymbol{B} = 0. \tag{2.18}$$

By taking the divergence of both sides of eq. (2.17) we see that $(\partial/\partial t)\nabla\cdot\boldsymbol{E}$ vanishes, so that $\nabla\cdot\boldsymbol{E}$ is a function of space only. Since this requires a field constant in time the only solution of interest for free space is

$$\nabla\cdot\boldsymbol{E}=0\,. \tag{2.19}$$

The dependence of eq. (2.19) on eq. (2.17) must be noted; eq. (2.19) is not necessarily valid when displacements have nonvanishing divergence. By contrast, eq. (2.18) is correct for all cases. That is, the theory prohibits magnetic monopoles but does not, at this point, prohibit electric monopoles. Without doing violence to the terms, we may refer to $\boldsymbol{E}$ and $\boldsymbol{B}$ as, respectively, electric and magnetic induction vectors. $\boldsymbol{E}$ is characterized by a lateral displacement of tubes in the sense of ξ described above. $\boldsymbol{B}$ is characterized by a bulk angular displacement small enough to be represented as a vector along $\boldsymbol{B}$. It is apparent that these are merely the most obvious characteristics; since $\boldsymbol{E}$ and $\boldsymbol{B}$ are complicated structures in the medium additional properties exist. Equation (2.16) states that the density of spin (bulk vorticity) resulting from $-\nabla\times\boldsymbol{E}$ is accompanied by bulk medium rotation, proportional to $\partial\boldsymbol{B}/\partial t$. $\boldsymbol{E}$ has been so defined that, if its curl is positive, a negative spin is induced, fig. 4. This equa-

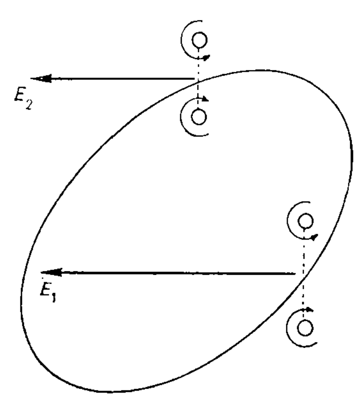

Fig. 4. – Vorticity results from net spin density; since E_1 exceeds E_2 more tubes of counterclockwise spin enter the loop so that the line integral around a loop corresponds to counterclockwise rotation.

tion is kinematic in character. Equation (2.17) is dynamic; $\nabla\times\boldsymbol{B}=k_2\nabla\times\times(\nabla\times\boldsymbol{D})$ is, except for a constant factor, the negative of the thrust per unit volume due to tube curvature, while $\partial\boldsymbol{E}/\partial t$ expresses the same thing in terms of the reaction to the lift force as in eq. (2.5).

It is sometimes convenient to deal with a simpler form of the medium in which the tubes are resolved along perpendicular axes. The electric field is then much simplified as it involves only tubes perpendicular to the field. The magnitude of $\boldsymbol{E}$, from eq. (2.4), reduces to $k_1 V^{-1}\int 2\pi\varkappa\varrho\xi\,\mathrm{d}l = k_1(2\pi\varkappa\varrho\xi L')$, where L' is the tube length per unit volume directed at right angles to $\boldsymbol{E}$. It is reasonable to suspect that the energy of the field derives from transfer of bulk kinetic energy of motion; that is, if initially E vanishes but a bulk velocity $\boldsymbol{q} = \partial\boldsymbol{D}/\partial t$ exists, the fluid does work on the tubes as they drift, at the expense of the bulk kinetic energy. The force on a tube in a current of speed q is $2\pi\varkappa\varrho q$ so that the work done in moving the tube length L' laterally by amount $\mathrm{d}\xi$ is

$$\mathrm{d}W = 2\pi\varkappa\varrho q L'\,\mathrm{d}\xi = -\,\mathrm{d}(\varrho q^2/2) = -\,\varrho q\,\mathrm{d}q\,,$$

the last term being the change in kinetic-energy density. The negative sign arises because $\mathrm{d}q$ is negative for positive $\mathrm{d}\xi$. Integration from $\xi = 0$ to $\xi = \xi_0$, with q varying from q_0 to 0, gives $2\pi\varkappa L'\xi_0 = q_0$. The original kinetic energy $\varrho q_0^2/2$ in terms of ξ_0 is $2\pi^2\varrho\varkappa^2 L'^2\xi_0^2$. This can be expressed in terms of $E_0 = k_1(2\pi\varkappa\varrho L'\xi_0)$, derived in the preceding paragraph, to give $W = E_0^2/2k_1^2\varrho$. If we identify the field with an actual electric field, we must identify k_1 as

$$k_1 = (\varrho\varepsilon_0)^{-\frac{1}{2}}\,. \tag{2.20}$$

It has been shown by somewhat different reasoning [9] that

$$k_2 = (G\mu_0)^{\frac{1}{2}}\,. \tag{2.21}$$

These results are useful in the next section.

3. – Second-order field effects.

In averaging the effects of tube bending (appendix B) the tubes were assumed to be at their neutral positions. Although this is a good approximation, it is not strictly correct, since the tubes share in the bulk displacement and, in addition, have the lateral displacements ξ. When these factors are taken into account, small second-order effects become evident. As an example of this (using a co-ordinate system where x is to the right, y toward the top of the page (upward) and z out of the paper) consider a plane wave propagating along x with $\boldsymbol{E}$ parallel to y and $\boldsymbol{B}$ parallel to z. The thrust $\boldsymbol{T}$ is also parallel (or antiparallel) to y. However, the medium has rotated during the displacement by an average angle θ. Tubes (in the resolved version of the medium) parallel to y do not change directions at all, while tubes in planes perpendicular to y have their directions of drift changed by 2θ; that is the vector $\boldsymbol{T}$ has changed from

(9) E. M. KELLY: *Amer. Journ. Phys.*, **32**, 658 (1964).

the y-direction by an angle 2θ which, from eq. (2.7), can be written as $k_2^{-1}B$. Thus $\boldsymbol{T}$ has a component $k_2^{-1}\boldsymbol{B}\times\boldsymbol{T}$ in the x-direction. With eqs. (2.5), (2.20) and (2.21) and interchange of order of multiplication, this can be written as $(k_1k_2)^{-1}(\partial\boldsymbol{E}/\partial t)\times\boldsymbol{B}=(\mu_0c^2)^{-1}(\partial\boldsymbol{E}/\partial t)\times\boldsymbol{B}$.

Another thrust arises because the existing electric field is rotating with average angular velocity ω (cf. eq. (2.8)). When a tube section $\mathrm{d}\boldsymbol{l}$ is displaced to form part of a field, one can think of this as equivalent to annihilating the original element by superimposing a tube element $-\mathrm{d}\boldsymbol{l}$ on it, then creating a new section $\mathrm{d}\boldsymbol{l}$ at a distance $\boldsymbol{\xi}$ from the original one. Finally a tube section $\boldsymbol{\xi}$ is added to one side and $-\boldsymbol{\xi}$ to the other to form a rectangular re-entrant vortex tube of area $\xi\,\mathrm{d}l$. Since this is done for all tube sections, the added tube section $\boldsymbol{\xi}$ for one $\mathrm{d}\boldsymbol{l}$ is annihilated by a $-\boldsymbol{\xi}$ for the next $\mathrm{d}\boldsymbol{l}$, so that nothing is added to the medium. If the rectangle rotates about edge $\mathrm{d}\boldsymbol{l}$ with angular velocity Ω, a thrust is developed by the opposite edge of $2\pi\varkappa\varrho$(length)(velocity)$\,=2\pi\varkappa\varrho\,\mathrm{d}l\,\xi\Omega$; it is apparent that the location of the axis of rotation is immaterial. For the same reason that 2θ was taken as angle of rotation of $\boldsymbol{T}$, the angular velocity of rotation of $\boldsymbol{E}$ is evidently $\boldsymbol{\Omega}=2\boldsymbol{\omega}$, so that the magnitude of the thrust is $2k_1^{-1}E\omega$. The direction of thrust is to the right when $\boldsymbol{E}$ is upward and $\boldsymbol{\omega}$ is out of the paper so that the thrust is $2k_1^{-1}\boldsymbol{E}\times\boldsymbol{\omega}$ which, with eqs. (2.8), (2.20) and (2.21), can be written $(k_1k_2)^{-1}\boldsymbol{E}\times(\partial\boldsymbol{B}/\partial t)=(\mu_0c^2)^{-1}\boldsymbol{E}\times(\partial\boldsymbol{B}/\partial t)$.

The total thrust from the two causes, change in direction of $\boldsymbol{T}$ and angular velocity of $\boldsymbol{E}$, is $(\mu_0c^2)^{-1}(\partial/\partial t)(\boldsymbol{E}\times\boldsymbol{B})$. Now although the devices of lift on a tube by the fluid and the reacting thrust on the fluid by the tube are extremely useful techniques because they are easy to use, it is obvious, as was mentioned previously, that they are fictitious. That is, the thrust on a volume of the medium must have its real origin in the rate of transfer of momentum across the surface of the volume, this arising from tube motions and differential changes in the angles at which tubes intersect the surface. In the plane wave, let $\boldsymbol{R}(x,t)$ be the rate of transfer of momentum across a unit area toward the right; for a slab of unit area and thickness Δx, the rate at which momentum enters from the left is $\boldsymbol{R}$ and the rate at which it leaves toward the right is $\boldsymbol{R}+(\partial\boldsymbol{R}/\partial x)\Delta x$, the rate of increase within the slab being $-(\partial\boldsymbol{R}/\partial x)\Delta x$. Thus, the thrust on the slab in the x-direction is

$$-(\partial\boldsymbol{R}/\partial x)\Delta x=(\mu_0c^2)^{-1}(\partial/\partial t)(\boldsymbol{E}\times\boldsymbol{B})\,.$$

For a wave the operator $-c^{-1}\partial/\partial t$ can replace $\partial/\partial x$, so that the preceding relations can be written in the equivalent form

$$\partial\boldsymbol{R}/\partial t=(\mu_0c)^{-1}(\partial/\partial t)(\boldsymbol{E}\times\boldsymbol{B})\,.$$

Since the space function of integration may be taken as zero in this case, integration gives

$$\boldsymbol{R} = (\mu_0 c)^{-1}(\boldsymbol{E}\times\boldsymbol{B}) = c^{-1}(\boldsymbol{E}\times\boldsymbol{H}), \tag{3.1}$$

where $\boldsymbol{H} = \boldsymbol{B}/\mu_0$, as the rate of transfer of momentum across a unit surface normal to the propagation velocity. This result is inherent in the model, but not in the analogues of Maxwell's vacuum equations, eqs. (2.16) and (2.17), since the latter were derived by neglecting the effects discussed above. That is, at least with respect to the model, the linear vacuum equations are incomplete. $\boldsymbol{R}$ is interpreted here as the momentum transfer rate per unit area normal to the wave front. That the interpretation applies only to wave motion is apparent from the substitution of $-c^{-1}\partial/\partial t$ for $\partial/\partial x$. As elsewhere, for example in ultra-sonics, it may not be profitable to rewrite the vacuum equations to include nonlinear terms, but rather to proceed from the solutions of the linear equations to second-order effects as was done to obtain eq. (3.1).

Classical derivations of radiation pressure and related phenomena usually start with the work done on charges by fields or, alternately, with the Maxwell stresses. In either case, the experimental fact of the existence of charge is needed. The above treatment has a clear advantage in economy, since the concept of charge is unnecessary.

As another example of second-order effects we consider an effect of compressibility which arises from the hollowness of the vortex cores. If a region of the medium expands by an increase in core volume, each volume element acts like a source of output $\nabla\cdot(\partial\boldsymbol{D}/\partial t)$ per unit volume. For irrotational bulk motion there is a bulk velocity potential Φ and a bulk velocity $-\nabla\Phi$. In addition there may be a divergenceless component of bulk velocity $\boldsymbol{Q}$, so the bulk velocity is

$$\partial\boldsymbol{D}/\partial t = -\nabla\Phi + \boldsymbol{Q},$$

and

$$\nabla\cdot(\partial\boldsymbol{D}/\partial t) = -\nabla^2\Phi. \tag{3.2}$$

Suppose that Φ satisfies the wave equation. Then

$$\nabla\cdot(\partial\boldsymbol{D}/\partial t) = -\nabla^2\Phi = -c^{-2}\partial^2\Phi/\partial t^2,$$

and an integration with respect to time gives

$$\nabla\cdot\boldsymbol{D} = -c^{-2}\partial\Phi/\partial t, \tag{3.3}$$

where the space function of integration may be taken as zero when no static fields exist. The pressure equation [10] is $P+\frac{1}{2}\varrho q^2-\varrho\,\partial\Phi/\partial t=\varrho C(t)$; if the medium at great distances is neutral, $C=0$, and if q is sufficiently small, we get

$$P=\varrho\,\partial\Phi/\partial t\,.$$

That is, P is the bulk pressure, superimposed upon the hydrodynamic pressure, which arises from bulk motions of the medium. Equation (3.3) asserts that the relative compression of the medium is proportional to the bulk (or excess) pressure $\varrho\,\partial\Phi/\partial t$.

The quantity $\boldsymbol{D}$ can be related to an electrical quantity by recalling that $\boldsymbol{B}$ was defined (eq. (2.7)) as $k_2\nabla\times\boldsymbol{D}$, so that $k_2\boldsymbol{D}$ is evidently the same as the ordinary electric vector potential $\boldsymbol{A}$. For Φ we can procede as follows: from eqs. (2.7) and (2.16) we have

$$\left\{\begin{aligned} &k_2\nabla\times\boldsymbol{D}=\boldsymbol{B}\,,\\ &\nabla\times k_2\partial\boldsymbol{D}/\partial t=\partial\boldsymbol{B}/\partial t=-\nabla\times\boldsymbol{E}\,,\\ &\nabla\times\partial\boldsymbol{A}/\partial t=-\nabla\times\boldsymbol{E}\,,\\ &\partial\boldsymbol{A}/\partial t=-\boldsymbol{E}-\nabla\varphi\,,\end{aligned}\right. \tag{3.4}$$

where φ is the electrical scalar potential. Taking the divergence of both sides of eq. (3.4), and confining our attention to cases where $\nabla\cdot\boldsymbol{E}$ vanishes, we get

$$\left\{\begin{aligned} &\nabla\cdot\partial\boldsymbol{A}/\partial t=-\nabla^2\varphi\,,\\ &\nabla\cdot\partial\boldsymbol{D}/\partial t=-\nabla^2\varphi/k_2\,.\end{aligned}\right. \tag{3.5}$$

Comparison of eqs. (3.2) and (3.5) shows that

$$\Phi=\varphi/k_2 \tag{3.6}$$

except possibly for an additive harmonic function which may be dismissed since such a function does not contribute to the electric or magnetic fields. Finally, eq. (3.3) can be written in terms of electrical, rather than bulk hydrodynamic, quantities:

$$\left\{\begin{aligned} &\nabla\cdot\boldsymbol{A}/k_2=-c^{-2}(\partial\varphi/\partial t)/k_2\,,\\ &\nabla\cdot\boldsymbol{A}=-c^{-2}\,\partial\varphi/\partial t\,.\end{aligned}\right. \tag{3.7}$$

Thus, within the stated limitations, we have obtained a physical interpretation for the Lorentz gauge, relating it to the compressibility of the medium.

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[10] L. M. MILNE-THOMSON: *Theoretical Hydrodynamics*, 2nd edition (New York, N. Y., 1951), p. 81.

4. – Discussion.

Several theorems identical in form to those of vacuum electrodynamics have been derived without using any facts of observation. Even Newton's second law and the concepts of force and inertia may, if appendix C proves cogent, be replaced by kinematic concepts of continuity and conservation of fluid volume. From the standpoint of postulational economy the medium appears to be promising as a candidate for an ether model.

The nature of matter as it relates to the medium has not been discussed here for the reason that virtually no progress has been made in this direction. Considerations of parsimony suggest that no new postulates are needed; that matter is a quasi-stable structure of the medium itself, an idea which is by no means novel. The attractiveness of this approach is enhanced by the reflection that through it the wave nature of matter and relativistic phenomena such as rod contraction and clock retardation become qualitatively intelligible. One quantitative property which may relate to matter is the circulation constant $\varkappa$. If the tubes are all assumed to have the same circulation (the simplest assumption) the medium has an intrinsic quantum; although one may suspect a connection between $\varkappa$ and Planck's constant, the connection has not yet been found.

Appendix A

Drift coefficient.

A fine hollow vortex ring advances approximately with speed

$$v = \tfrac{1}{2}\varkappa C[\ln(8/rC) - \tfrac{1}{2}]\,,$$

where C is the curvature of the vortex tube regarded as a circular line (that is, $C = 1/R$, where R is the radius of the aperture), $\varkappa$ is the strength of circulation and r is the radius of tube cross-section. When the radius of cross-section remains constant, the speed is a function of C only. Although v vanishes in the limit as C approaches zero, $\mathrm{d}v/\mathrm{d}C$ does not exist at $C = 0$; there is, consequently, no McLaurin's expansion, nor is there one for a more precise formula developed by Hicks ([11]). Two choices are available (other than solving a difficult problem): we can assume that the motion of the curved portions of a straight vortex tube slightly perturbed differs sufficiently from

([11]) W. M. Hicks: *On the steady motion and small vibrations of a hollow vortex*, in *Phil. Trans. Roy. Soc.*, p. 163 (1884).

that of a large axially symmetric vortex ring to have a McLaurin's expansion, or we can use the fact that the tubes in the medium are not really straight even in the neutral state. Additional tube curvature induced by a bulk displacement will then increase or decrease an already existing tube motion. For this case, a Taylor's expansion of v, which does exist for $C > 0$, can be used to calculate the change in motion due to the superimposed curvature. Since effects of tube curvature in the neutral state cancel for the phenomena considered here, we can specify, in principle, a drift coefficient η, such that $\dot{\xi} = \eta C$, where C is the superimposed curvature and $\dot{\xi}$ is an effective value which does not include the tube motion in the neutral state.

Appendix B

Calculation of the mean curvature.

The mean value $\bar{C}$ of C is obtained by averaging over the neutral medium where all tube directions are equally represented. In Cartesian co-ordinates,

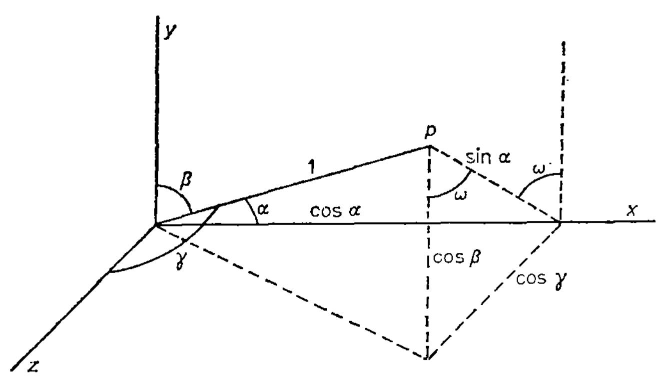

Fig. 5. – Co-ordinate system used for averaging C. The element of area on unit sphere is $\sin\alpha\, d\omega\, d\alpha$. Useful relations are $\cos\beta = \sin\alpha\cos\omega$ and $\cos\gamma = \sin\alpha\sin\omega$.

fig. 5, with $\boldsymbol{\lambda} = \boldsymbol{i}\cos\alpha + \boldsymbol{j}\cos\beta + \boldsymbol{k}\cos\gamma$ and $\boldsymbol{D} = \boldsymbol{i}u + \boldsymbol{j}v + \boldsymbol{k}w$, we get for x-components

$$[(\boldsymbol{\lambda}\cdot\nabla)^2\boldsymbol{D}]_x = [u_{xx}\cos^2\alpha + u_{yy}\cos^2\beta + u_{zz}\cos^2\gamma + \\ + 2u_{xy}\cos\alpha\cos\beta + 2u_{xz}\cos\alpha\cos\gamma + 2u_{yz}\cos\beta\cos\gamma]\,,$$

$$[\{\boldsymbol{\lambda}\cdot(\boldsymbol{\lambda}\cdot\nabla)^2\boldsymbol{D}\}\boldsymbol{\lambda}]_x = [\cos^2\alpha\{u_{xx}\cos^2\alpha + u_{yy}\cos^2\beta + \ldots + 2u_{yz}\cos\beta\cos\gamma\} + \\ + \cos\alpha\cos\beta\{v_{xx}\cos^2\alpha + \ldots + 2v_{yz}\cos\beta\cos\gamma\} + \\ + \cos\alpha\cos\gamma\{w_{xx}\cos^2\alpha + \ldots + 2w_{yz}\cos\beta\cos\gamma\}]\,.$$

The x-component of $\boldsymbol{C}$ is, from eq. (2.1),

$$C_x = \sin^2\alpha\{u_{xx}\cos^2\alpha + u_{yy}\cos^2\beta + u_{zz}\cos^2\gamma +$$
$$+ 2u_{xy}\cos\alpha\cos\beta + 2u_{xz}\cos\alpha\cos\gamma + 2u_{yz}\cos\beta\cos\gamma\} -$$
$$- \cos\alpha\cos\beta\{v_{xx}\cos^2\alpha + v_{yy}\cos^2\beta + \ldots + 2v_{yz}\cos\beta\cos\gamma\} -$$
$$- \cos\alpha\cos\gamma\{w_{xx}\cos^2\alpha + w_{yy}\cos^2\beta + \ldots + 2w_{yz}\cos\beta\cos\gamma\} .$$

The relative frequency of tubes having directions within the element of solid angle is $\sin\alpha\,\mathrm{d}\omega\,\mathrm{d}\alpha/4\pi$. We multiply this by C_x and integrate over the unit sphere, using the relations $\cos\beta = \sin\alpha\cos\omega$ and $\cos\gamma = \sin\alpha\sin\omega$ to express all variables in terms of α and ω. Many of the integrals vanish because of $\cos\omega$, $\sin\omega$ or $\cos^3\omega$ factors in the integrands; the nonvanishing ones for the mean value of the x-component of $\boldsymbol{C}$ are as follows (the ranges of integration being zero to 2π for ω and zero to π for α):

$$(4\pi)^{-1}u_{xx}\iint \sin^3\alpha\cos^2\alpha\,\mathrm{d}\omega\,\mathrm{d}\alpha = (2/15)\,u_{xx} ,$$
$$(4\pi)^{-1}u_{yy}\iint \sin^5\alpha\cos^2\omega\,\mathrm{d}\omega\,\mathrm{d}\alpha = (4/15)\,u_{yy} ,$$
$$(4\pi)^{-1}\,u_{zz}\iint \sin^5\alpha\sin^2\omega\,\mathrm{d}\omega\,\mathrm{d}\gamma = (4/15)\,u_{zz} ,$$
$$-2(4\pi)^{-1}\,v_{xy}\iint \sin^3\alpha\cos^2\alpha\cos^2\omega\,\mathrm{d}\omega\,\mathrm{d}\alpha = -(2/15)\,v_{xy} ,$$
$$-2(4\pi)^{-1}w_{xz}\iint \sin^3\alpha\cos^2\alpha\sin^2\omega\,\mathrm{d}\omega\,\mathrm{d}\alpha = -(2/15)\,w_{xy} .$$

If we add and subtract $(2/15)\,u_{xx}$, the sum of the integrals can be written

$$\bar{C}_x = (4/15)(u_{xx} + u_{yy} + u_{zz}) - (2/15)(u_{xx} + v_{xy} + w_{xz}) ,$$

which is recognizable as the x-component of $(4/15)\nabla^2\boldsymbol{D} - (2/15)\nabla(\nabla\cdot\boldsymbol{D})$. Since the choice of axes is arbitrary, we infer that

$$\bar{\boldsymbol{C}} = (4/15)\nabla^2\boldsymbol{D} - (2/15)\nabla(\nabla\cdot\boldsymbol{D}) .$$

Appendix C

Equivalence of kinematics and dynamics of the fluid.

Consider a fluid of finite extent where the complete boundary, including vortex cores, is specified at time zero, along with the normal component of fluid velocity at the boundary and the circulations associated with irreducible circuits which embrace vortex cores.

It is assumed that the given boundary velocities are compatible with the condition of incompressibility and with the continuity of the fluid. Details of the kinematic constraints may be found in standard treatises ([12]). For an incompressible fluid in irrotational motion this is sufficient information, in principle, to find the velocity potential $\varphi(x, y, z, t)$ at time zero by the solution of a «first» boundary-value problem. Among the kinematic constraints is the requirement that the velocity potential has continuous second derivatives with respect to time and any space co-ordinate so that the order of differentiation in terms like $\partial^2\varphi/\partial x\,\partial t$ is commutative. An «existence operator» ϱ may be defined such that ϱ vanishes throughout a region of void and has an unassigned but constant nonvanishing value throughout the fluid. We define a function

$$p = \varrho(\partial\varphi/\partial t) - \tfrac{1}{2}\varrho q^2 + \varrho C(t)\,,$$

where $\boldsymbol{q} = -\nabla\varphi$ and $C(t)$ is a continuous function of time only. The function p is continuous within the fluid because φ and its first derivatives are. However, let us require that it be continuous everywhere. It is zero in a void, since $\varrho = 0$ there, hence, for continuity at a boundary, p must approach zero as the boundary is approached along a path lying in the fluid. That is

$$\partial\varphi/\partial t - \tfrac{1}{2}q^2 + C(t) = 0 \qquad \text{on the boundary.}$$

At time zero this becomes

$$[\partial\varphi/\partial t + C(t)]_{B,0} = \tfrac{1}{2}q^2_{B,0}\,,$$

the subscripts indicating that the quantities are evaluated on the boundary at time zero. Since $\boldsymbol{q}_{B,0} = -(\nabla\varphi)_{B,0}$ is known from the solution of the first boundary-value problem, the quantity $\partial\varphi/\partial t + C(t)$ is therefore known on the boundary at time zero. It is harmonic, since φ and C are, and since its value is known on the boundary at time zero, the solution of a second boundary-value problem yields the value of $\partial\varphi/\partial t + C(t)$ at time zero, but throughout the fluid. That is, the function p is now known everywhere at time zero from the given boundary conditions and continuity assumptions.

The acceleration of a fluid particle is given by the convective derivative

$$\mathrm{d}\boldsymbol{q}/\mathrm{d}t = \partial\boldsymbol{q}/\partial t + (\boldsymbol{q}\cdot\nabla)\,\boldsymbol{q} = \partial\boldsymbol{q}/\partial t + \tfrac{1}{2}\nabla q^2$$

for irrotational motion. With $\boldsymbol{q} = -\nabla\varphi$ and $\nabla C(t) = 0$, and commutation of time and space differentiations, we get

$$\mathrm{d}\boldsymbol{q}/\mathrm{d}t = -\nabla[\partial\varphi/\partial t - \tfrac{1}{2}q^2 + C(t)]\,.$$

Multiplying throughout by ϱ and recalling the definition of p, we have

$$\mathrm{d}\boldsymbol{q}/\mathrm{d}t = -\nabla p/\varrho\,.$$

([12]) R. Avis: *Vectors, Tensors, and the Basic Equations of Fluid Mechanics* (Englewood Cliffs, N. J., 1962), p. 76.

The acceleration of every fluid particle is therefore known at time zero. The calculation of the state of the fluid at any other time is therefore reducible to the solution of a differential equation.

As an illustration of these definitions, consider a fluid region in the form of an infinitely long hollow cylinder ([13]) of inner and outer radii a and b, respectively. The circulation around the cylinder is $2\pi\varkappa$; for definiteness outward radial velocity is assumed. Polar co-ordinates r, θ in a plane normal to the cylinder axis may be used; incompressibility and continuity require that $2\pi r q_r = 2\pi m$, where m may be a function of time, but not of space, and q_r is the radial velocity. The normal components of velocity on the boundary can then be written as $q_r = m/a$ and m/b at the inner and outer boundaries, respectively. The first boundary-value problem can be solved here by inspection to get

$$\varphi = -m_0 \ln r - \varkappa\theta ,$$

where m_0 is the value of m at time zero. This is assumed to be given. Since

$$\partial\varphi/\partial t = -(\partial m/\partial t)\ln r = -\dot{m}\ln r$$

is harmonic, it constitutes the solution to the second boundary-value problem if we find the values of $\dot{m}$ and $C(t)$ at time zero. Note that although m_0 is given $\dot{m}_0$ is not.

The vanishing of p at $r = a$ and $r = b$, with $C(0) = C_0$, gives

$$-\dot{m}_0 \ln a - (m_0^2 + k^2)/2a^2 + C_0 = 0 ,$$

$$-\dot{m}_0 \ln b - (m_0^2 + k^2)/2b^2 + C_0 = 0 .$$

When these are solved for $\dot{m}_0$ and C_0, we get an expression for p throughout the fluid at time zero of the form

$$p = A_1 + A_2 \ln r + A_3 r^{-2},$$

where the A's contain only given quantities. The negative gradient of this divided by ϱ is then the acceleration.

Although the notation in the foregoing discussion was designed to be suggestive, it was not necessary to think of p as a pressure or ϱ as an inertia density. The equation $\varrho\,\mathrm{d}\boldsymbol{q}/\mathrm{d}t = -\nabla p$ is formally the same as Newton's second law for an inviscid liquid with no body forces; in the above treatment it is derived solely from kinematic concepts. It appears, therefore, that the continuity assumptions are equivalent to an assumption of inertia.

It is interesting that something like Mach's principle operates here; the acceleration of a fluid particle is determined by the *totality* of boundary conditions, so that the entire medium must, for complete accuracy, be taken into account. There is no velocity of propagation involved, for the acceleration of the particle is determined by the boundary conditions at the same instant.

([13]) An even simpler case is that of an expanding spherical shell.

● RIASSUNTO (*)

Se un ritorno ai concetti di etere diviene necessario o desiderabile, evento non tanto inverosimile come sembrava poco tempo fa, il metro trattato qui può acquistare interesse. Si deducono le equazioni di Maxwell dalle proprietà di un fluido ideale senza alcun sostegno derivante da fatti sperimentali. I campi $\boldsymbol{E}$ e $\boldsymbol{B}$, così interpretati, hanno una natura statistica e facilmente visualizzabile in un senso meccanico e geometrico. Si dimostra che il teorema del momento elettromagnetico è un effetto di secondo ordine derivante da effetti non lineari che sono stati trascurati nelle deduzioni delle equazioni nel vuoto. La presenza di vuoti (nuclei vorticali cavi) in un fluido d'altra parte incompressibile permette un'interpretazione meccanica della gauge di Lorentz. Si suggerisce per parsimonia che le leggi di Newton possono essere rimpiazzate dalla cinematica, riducendo così l'elettrodinamica alla geometria euclidea.

(*) *Traduzione a cura della Redazione.*

Электродинамика в вакууме, выведенная исключительно из свойств идеальной жидкости.

Резюме (*). — Если возвращение к концепциям эфира становится необходимым или желательным, то подход, развитый в этой работе, может оказаться интересным. Уравнения Максвелла в вакууме выводятся из свойств идеальной жидкости без использования экспериментальных фактов. В этом случае поля $\boldsymbol{E}$ и $\boldsymbol{B}$ являются по своей природе статистическими и могут быть наглядно представлены в механическом и геометрическом смысле. Показывается, что теорема об импульсе электромагнитного поля представляет эффект второго порядка, происходящий из нелинейных эффектов, которыми пренебрегают при выводе вакуумных уравнений. Наличие пустот (полые вихревые ядра) в несжимаемой жидкости допускает механическую интерпретацию калибровки Лорентца. Предполагается, что законы Ньютона могут быть заменены кинематикой, при этом электродинамика приводится к геометрии Эвклида.

(*) *Переведено редакцией.*