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Functional-Integral Approach to Chiral Anomalies in Supersymmetric Gauge Theories.

K. KONISHI

Istituto Nazionale di Fisica Nazionale - Sezione di Pisa
Dipartimento di Fisica dell'Università - 56100 *Pisa, Italia*

K. SHIZUYA

Department of Physics, Tohoku University - *Sendai* 980, *Japan*

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Summary. — We formulate anomalous chiral and related Ward-Takahashi identities in supersymmetric-gauge theories, by generalizing Fujikawa's functional-integral method to superspace. Our approach provides a manifestly supersymmetric and gauge-covariant treatment of the superspace Abelian anomalies, and is applicable to chiral- as well as to left-right symmetric theories. Non-Abelian anomalies are also discussed briefly. Superspace Abelian anomalies imply that particular composite operators, *i.e.* those containing the associated U_1 currents as a component, exhibit an anomalous supermultiplet structure. We discuss how this leads to various exact relations between scalar and gauge fermion condensates, thereby imposing strong constraints on possible chiral-symmetry realizations in supersymmetric-confining theories.

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1. – Introduction.

An interesing class of theories, which might find an application in the construction of a realistic model of particle physics at mass scale above 0(250 GeV), and whose dynamical properties are not yet fully explored, are supersymmetric gauge theories of strongly interacting particles. Recently a considerable improvement in our understanding of nonperturbative dynamics in these theories was made possible by combining the results of i) explicit calculation of instanton effects ([1-5]), ii) analyses of the U_1 and related anomalies ([6-8]), iii) studies on effective Lagrangians ([9-11]), and of iv) index analyses *à la* WITTEN ([12]). The emerging picture of chiral symmetry realizations in supersymmetric gauge theories (see, *e.g.*, ref. ([3]) and sect. **5** below) bears quite different features as compared to what one knows about the low-energy dynamics of conventional theories such as quantum chromodynamics (QCD).

It is the purpose of this paper to elaborate on one of the above-mentioned studies: chiral and related anomalies ([6-8]). In the first part (sect. **2-4**), we formulate the anomalous chiral Ward-Takahashi (WT) identities by use of a supersymmetric generalization of Fujikawa's functional-integral method ([13]). We present the derivation for sypersymmetric QCD (SQCD) first. The result will then be generalized to chiral theories. The case of the Abelian anomalies which generalize the well-known axial U_1 anomaly ([14]) will be studied in detail.

Importance of the particular regularization employed here will be emphasized, in connection with the absence of non-Abelian gauge anomalies in chiral theories.

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([5]) Y. MEURICE and G. VENEZIANO: *Phys. Lett. B*, **141**, 69 (1984); I. AFFLECK, M. DINE and N. SEIBERG: *Phys. Rev. Lett.*, **52**, 1677 (1984).
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([7]) T. E. CLARK, O. PIGUET and K. SIBOLD: *Nucl. Phys. B*, **159**, 1 (1979).
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([12]) E. WITTEN: *Nucl. Phys. B*, **202**, 253 (1982); S. CECOTTI and L. GIRARDELLO: *Phys. Lett. B*, **110**, 39 (1982); E. COHEN and C. GOMEZ: *Nucl. Phys. B*, **223**, 183 (1983).
([13]) K. FUJIKAWA: *Phys. Rev. Lett.*, **42**, 1195 (1979); *Phys. Rev. D*, **21**, 2848 (1980); **22**, 1499 (E) (1980); **29**, 285 (1984); A. P. BALACHANDRAN, G. MARMO, V. P. NAIR and C. G. TRAHERN: *Phys. Rev. D*, **25**, 2713 (1982); M. B. EINHORN and D. T. JONES: *Phys. Rev. D*, **29**, 331 (1984).
([14]) J. SCHWINGER: *Phys. Rev.*, **82**, 664 (1951); S. ADLER: *Phys. Rev.*, **177**, 2426 (1969); J. S. BELL and R. JACKIW: *Nuovo Cimento A*, **60**, 47 (1969).

The method of this paper can be used also in a more general study of non-Abelian anomalies [15-17]. In this paper we limit ourselves, however, to a simple case, namely the supersymmetric generalization of Bardeen anomaly in SQCD with external vector SU_{N_f} fields.

There has been a renewed interest [16] recently in chiral anomalies in gauge theories, partly triggered by the observation that anomalies, though seen as short-distance effects in perturbation theory, have a topological origin related to the global structure of gauge theories.

On the other hand, FUJIKAWA developed some time ago an elegant method [13] for formulating chiral anomalies within the functional-integral approach, that made the topological origin of anomalies manifest.

In view of the important role the functional-integral method plays in general, and in particular of the systematic manner in which it deals with anomalies, we feel it worthwhile to generalize this method to superspace, and formulate some of earlier results in a manifestly supersymmetric way.

The second part of the paper (sect. **5**) is dedicated to a discussion of those aspects of the superspace U_1 anomalies, that are specific to supersymmetric gauge theories. In fact, superspace U_1 anomalies imply [6] anomalous supersymmetry commutation relations among the components of particular composite vector supermultiplets, the ones containing the relevant U_1 current operators as a component.

It will be shown how these anomalous commutators can be used to obtain rigorous relations involving various scalar and gauge-fermion condensates. They are combined consequences of two symmetries: anomalus chiral U_1 symmetry and supersymmetry.

When used together with dynamical information such as about instanton effects, these relations allow us to *compute* the vacuum properties from the first principles, as in the examples of SQCD [2,3], a chiral SU_5 model [5] and a chiral SU_6 model [18]. In sect. **5** we review and discuss this important development, which is principally due to the authors of ref. [2-5].

Section **6** contains the summary and concluding remarks. Several technical details are grouped in appendices A-C.

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(17) T. E. CLARK and S. T. LOVE: *Phys. Lett. B*, **138**, 289 (1984); N. K. NIELSEN: *Nucl. Phys. B*, **244**, 499 (1984).

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2. – Preliminaries.

In this section we review briefly Fujikawa's functional-integral formulation of axial anomalies in conventional gauge theories ([13]).

Let us consider quantum electrodynamics. The matter part of the action is given by

$$S = \int \mathrm{d}^4x \, \{\tfrac{1}{2}\bar{\psi} i \overleftrightarrow{\check{D}} \psi - m\bar{\psi}\psi\}\,, \tag{2.1}$$

where $\check{D} = \gamma^\mu(\partial_\mu - ieA_\mu)$. The kinetic part of eq. (2.1) is invariant under global axial transformations

$$\psi(x) \to \psi'(x) = \exp\,[i\alpha\gamma_5]\,\psi(x)\,. \tag{2.2}$$

Fujikawa observed that the functional integral measure $\mathcal{D}\psi\,\mathcal{D}\bar{\psi}$, properly defined, is not invariant under the axial transformations and showed how the correctly treated Jacobian gives rise to the axial anomaly.

In Euclidean space-time, $\check{D}$ is a Hermitian operator, whose eigenfunctions $\varphi_n(x)$ form a complete orthonormal set

$$\check{D}(A)\varphi_n(x) = \lambda_n\varphi_n(x)\,, \qquad \int \mathrm{d}x^4 \varphi_m^\dagger(x)\varphi_n(x) = \delta_{m,n}\,. \tag{2.3}$$

By using the eigenmode expansion

$$\psi = \sum_n a_n\varphi_n(x)\,, \qquad \bar{\psi} = \sum_m \bar{b}_m\varphi_m^\dagger(x)\,, \tag{2.4}$$

the functional measure $\mathcal{D}\psi\,\mathcal{D}\bar{\psi}$ can be defined as

$$\mathcal{D}\psi \equiv \prod_n \mathrm{d}a_n\,, \qquad \mathcal{D}\bar{\psi} \equiv \prod_m \mathrm{d}\bar{b}_m\,. \tag{2.5}$$

In the formulation of the axial U_1 WT identities, one needs to consider *local* transformations, *i.e.* eq. (2.2) with a space-time dependent parameter $\alpha(x)$. The Jacobian of the infinitesimal transformation is found to be

$$\exp\left[-2i\int \mathrm{d}^4x\,\alpha(x)\,\mathfrak{A}(x)\right], \tag{2.6}$$

$$\mathfrak{A}(x) \equiv \sum_n \varphi_n^\dagger(x)\,\gamma_5\varphi_n(x)\,. \tag{2.7}$$

$\mathfrak{A}(x)$ is an ill-defined conditionally convergent quantity: the crucial step is to regularize it by suppressing the large eigenvalues of the operator $\check{D}$ (which does not commute with γ_5). Thus one finds

$$(2.8)\qquad \mathfrak{A}^{\text{reg}}(x) = \lim_{\tau\to 0}\sum_n \varphi_n^\dagger(x)\gamma_5\varphi(x)\exp[-\tau\lambda_n^2] = \lim_{\tau\to 0}\operatorname{tr}\langle x|\gamma_5\exp[-\tau\check{D}^2]|x\rangle = (e^2/16\pi^2)\,\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}F_{\mu\nu}F_{\varrho\sigma}\,.$$

The result can, furthermore, be shown to be independent of the particular (Gaussian in eq. (2.8)) cut-off employed. This stability of the axial anomaly is a reflection of its topological origin. Equation (2.7), integrated over x is formally equal to

$$(2.9)\qquad \int \mathrm{d}^4x\,\mathfrak{A}(x) = n_+ - n_-\,,$$

where $n_+(n_-)$ is the number of the zero eigenvalue modes of positive (negative) chirality. The index of the Dirac operator $\check{D}$, $n_+ - n_-$, is thus related via Atiyah-Singer theorem ([19]) to the topological charge density, $(e^2/32\pi^2)\cdot\varepsilon^{\mu\nu\varrho\sigma}F_{\mu\nu}F_{\varrho\sigma}$.

3. – Functional-integral formulation of anomalous chiral WT identities in supersymmetric gauge theories.

In this section we analyse the chiral anomalies in supersymmetric gauge theories by generalizing the functional-integral method to superspace.

For the clarity of presentation, we first treat the specific case of the U_1 anomalies in supersymmetric quantum chromodynamics (SQCD) in detail. Generalization to non-Abelian anomalies in SQCD and U_1 anomalies in chiral theories, and the question of the non-Abelian gauge anomaly cancellation in chiral theories, will be considered subsequently (see after eq. (3.18)).

SQCD is a supersymmetric version of QCD with SU_{N_c} gauge group. The matter fields are N_f pairs (flavours) of chiral superfields $\{\Phi_i, X^i\}$ $(i = 1, \dots, N_f)$ transforming as $\{N_c, N_c^*\}$ multiplets of the colour group. The gauge fields are contained in the vector superfields V^k $(k = 1, \dots, N_c^2 - 1)$.

The generating functional of SQCD is given by

$$(3.1)\qquad \exp Z[J] = \int \mathscr{D}\Phi\,\mathscr{D}\bar{\Phi} \dots \exp[i(S_{\text{matter}} + S_{\text{gauge}})]\,,$$

where the part of the action containing the matter fields is of the form (*)

([19]) M. Atiyah and I. Singer: *Ann. Math.*, **87**, 484 (1968).

(*) Our notation is that of ref. ([20]) except that we use the Bjorken-Drell metric. The superspace co-ordinate is denoted by $z = (x, \theta, \bar\theta)$ and integration measures by $\mathrm{d}^8z = \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta$, $\mathrm{d}^6z = \mathrm{d}^4x\,\mathrm{d}^2\theta$, and $\mathrm{d}^6\bar z = \mathrm{d}^4x\,\mathrm{d}^2\bar\theta$. Also, $D_\alpha = (\partial/\partial\theta^\alpha) + i(\sigma^\mu\bar\theta)_\alpha\partial_\mu$; $\bar D_{\dot\alpha} = -(\partial/\partial\bar\theta^{\dot\alpha}) - i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$. Here and whenever possible, the colour and flavour indices will be kept implicit.

([20]) J. Wess and J. Bagger: *Supersymmetry and Supergravity* (Princeton University Press, 1983).

(3.2) $$S_{\text{matter}} = \int d^8z\,(\bar{\Phi}\exp[V]\,\Phi + X\exp[-V]\bar{X}) + \int d^6z\,Xm\Phi + \text{h.c.} + S_{\text{sources}},$$

where

(3.3) $$V = V^k T^k$$

(T^k = the SU_{N_c} generators appropriate for Φ fields). For convenience, we have introduced in eq. (3.2) source terms for colour-singlet composite operators. For instance, they can be taken as

(3.4) $$S_{\text{sources}} = \int d^6z\,(J_T)^i_j X^j \Phi_i + \text{h.c.} + \int dz^8 (J_B)^i_j \bar{\Phi}^j \exp[V]\,\Phi_i + \dots .$$

The action eq. (3.2) is invariant under the local SU_{N_c} (generalized) gauge transformations,

(3.5) $$\begin{cases} (\Phi, \bar{\Phi}) \to (\exp[-i\Lambda]\,\Phi, \bar{\Phi}\exp[i\bar{\Lambda}]), \\ (X, \bar{X}) \to (X\exp[i\Lambda], \exp[-i\bar{\Lambda}]\bar{X}), \\ \exp[V] \to \exp[-i\bar{\Lambda}\exp[V]\exp[i\Lambda], \end{cases}$$

where $\Lambda = \Lambda^k T^k$ and Λ^k's are arbitrary chiral superfields.

The global axial U_1 transformation is defined as

(3.6) $$(\Phi, X) \to \exp[i\alpha]\,(\Phi, X), \qquad (\bar{\Phi}, \bar{X}) \to \exp[-i\alpha]\,(\bar{\Phi}, \bar{X}).$$

Now, in order to derive the WT identities, one must consider a local version of eq. (3.6),

(3.7) $$\begin{cases} (\Phi, X) \to \exp[iA(z)(\Phi, X)], \\ (\bar{\Phi}, \bar{X}) \to \exp[-i\bar{A}(z)(\bar{\Phi}, \bar{X})], \end{cases}$$

where $A(z)$ is an arbitrary chiral superfield proportional to a unit matrix in the colour and flavour spaces.

Instead of considering directly eq. (3.7), however, we shall proceed as follows. In the functional-integral formalism, Φ, $\bar{\Phi}$, X and $\bar{X}$ are all independent integration variables. Therefore, the effects of change of each variable can be and in fact will be studied separately, leading to four independent sets of WT identities. Of these four, two involve the U_1 associated with Φ, or $U_{1,\text{right}}$, the other two the $U_{1,\text{left}}$ related to the X fields. The axial U_1 identities can be found by taking an appropriate combination of the four (see sect. 4 below); the point, however, is that each set of the identities contain independent information about the theory and we wish to keep them all (see the applications discussed in sect. 5).

Also, the reality of the matter-field content plays no essential role in this approach, hence the generalization of the results to chiral models is straightforward.

Keeping these points in mind, let us consider the following change of variables (*):

$$\Phi \to \exp\,[iA(z)\,\Phi(z)]\,, \qquad \bar{\Phi},\ X,\ \bar{X}\ \text{invariant}, \tag{3.8}$$

in eq. (3.1). For an infinitesimal $A(z)$, the action transforms as

$$\delta S_{\text{matter}} = \int \mathrm{d}^8 z\,\bar{\Phi}\exp\,[V]\,iA\Phi + \int \mathrm{d}^6 z\, iAXm\Phi + \delta S_{\text{sources}}\,. \tag{3.9}$$

On the other hand, the Jacobian of the transformation is given by

$$\begin{aligned} J(\Phi'/\Phi) &= \det_c |\delta\Phi'_{z'}/\delta\Phi_z| = \\ &= \det_c \langle z'|\exp\,[iA]\,(-\bar{D}^2/4)|z\rangle = \exp\,[\mathrm{tr}_c\,\{iA(-\bar{D}^2/4)\}]\,. \end{aligned} \tag{3.10}$$

A co-ordinate representation in superspace [21] has been introduced above ($|z\rangle = |x, \theta, \bar{\theta}\rangle$ is the eigenstate of the co-ordinate operator; $\langle z'|z\rangle = \delta^8(z'-z)$, etc.), and we have used the fact that

$$\delta\Phi_{z'}/\delta\Phi_z = \langle z'|(-\bar{D}^2/4)|z\rangle\,, \tag{3.11}$$

which acts as a delta-function in combination with a chiral measure $\mathrm{d}^6 z$. The subscript c in eq. (3.10) means that the sum (integral) over z is taken with the chiral measure.

The exponent in eq. (3.10) is apparently vanishing, since $\langle z|\bar{D}^2|z\rangle = 0$. This is, however, analogous to the naive result, $\mathrm{tr}\,\gamma_5 = 0$, for the exponent of the Jacobian in the QED case, eq. (2.7). There, the correct Jacobian is found by regularizing the large eigenvalues of the operator $\check{D}$, which does not commute with γ_5 (eq. (2.8)).

We find that an appropriate supersymmetric generalization of Fujikawa's procedure is provided by the regularization

$$\mathrm{tr}_c^{(\text{reg})}\{iA(-\bar{D}^2/4)\} \equiv \lim_{M^2\to\infty} \mathrm{tr}_c\{iA\exp\,[L/M^2](-\bar{D}^2/4)\}\,, \tag{3.12}$$

(*) Of course, an analogous treatment is possible in the case of conventional theories, by considering the change of variables $\psi \to \exp\,[i\alpha(1+\gamma_5)]\,\psi$, $\bar{\psi} \to \bar{\psi}$. The Lagrangian must be taken in the Hermitian form, $(i/2)\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi$, not $i\bar{\psi}\gamma^\mu D_\mu \psi$. However, in QED, no new information will be obtained by doing this.

([21]) K. Shizuya and Y. Yasui: *Phys. Rev. D*, **29**, 1160 (1984).

where

$$L \equiv \bar{D}^2 \exp[-V]\, D^2 \exp[V]/16 . \tag{3.13}$$

Indeed, L is the simplest operator that respects i) manifest supersymmetry, ii) chirality and iii) gauge covariance, and that contains $\check{D}^2$ as a component. Note that L transforms as $L \to \exp[-i\Lambda]\, L \exp[i\Lambda]$ under the gauge transformation, eq. (3.5).

Computation of the matrix element appearing in eq. (3.12) is described in appendix A. The answer is

$$\mathrm{tr}_c^{(\mathrm{reg})}\{iA(-\bar{D}^2/4)\} = i\int \mathrm{d}^6 z\, (g^2/32\pi^2)\, \mathrm{Tr}\{A(W^\alpha W_\alpha)\} , \tag{3.14}$$

where $W^\alpha = -\frac{1}{4}\bar{D}^2 \exp[-V]\, D^\alpha \exp[V]$, and Tr stands for a trace in the colour space.

The last step in obtaining the WT identities is to collect all variations under eq. (3.8). Because a change of (functional) variables does not modify the integral itself, we find finally the desired relations (*)

$$\begin{aligned} 0 = \delta Z/i\,\delta A(z) = \\ = \left\langle -\frac{\bar{D}^2}{4}\bar{\Phi}\exp[V]\,\Phi + Xm\Phi + \frac{g^2}{32\pi^2}\mathrm{Tr}\, W^\alpha W_\alpha + \frac{\delta S_{\mathrm{sources}}}{i\,\delta A}\right\rangle^{\{J\}} , \end{aligned} \tag{3.15}$$

where $\delta S_{\mathrm{sources}}/i\,\delta A$ is inferred from eq. (3.4).

Up to now we studied the consequences of the invariance of the generating functional under the change of integration variable, eq. (3.8). In an analogous fashion, we obtain another set of WT identities

$$0 = \left\langle -\frac{\bar{D}^2}{4} X \exp[-V]\,\bar{X} + Xm\Phi + \frac{g^2}{32\pi^2}\mathrm{Tr}\, W^\alpha W_\alpha + \frac{\delta S_{\mathrm{sources}}}{i\,\delta A}\right\rangle^{\{J\}} \tag{3.16}$$

from the consideration of the change of variables, $X \to \exp[(iA)\, X]$, all other fields remaining invariant.

Furthermore, the study of change of variables, $\bar{\Phi} \to \bar{\Phi}\exp[-i\bar{A}]$, and $\bar{X} \to \bar{X}\exp[-i\bar{A}]$ lead (by use of the regularization kernel $L = (D^2 \exp[V]\cdot \cdot\bar{D}^2 \exp[-V])/16$ in the computation of the Jacobian) to the Hermitian conjugates of eqs. (3.15) and (3.16),

$$0 = \left\langle -\frac{D^2}{4}\bar{\Phi}\exp[V]\,\Phi + \bar{\Phi}\bar{m}\bar{X} + \frac{g^2}{32\pi^2}\mathrm{Tr}\, \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha} + \frac{\delta S_{\mathrm{sources}}}{\delta(-i\bar{A})}\right\rangle^{\{J\}} \tag{3.17}$$

(*) We remind that the coupling constant g has been reinstated by the replacement $V \to 2gV$.

and

$$(3.18)\qquad 0=\left\langle -\frac{D^2}{4}X\exp[-V]\bar{X}+\bar{\Phi}\bar{m}\bar{X}+\frac{g^2}{32\pi^2}\operatorname{Tr}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}+\frac{\delta S_{\text{sources}}}{\delta(-i\bar{A})}\right\rangle^{\{J\}}.$$

Equations (3.15)-(3.18), altogether, represent the anomalous $U_{A,1}$ and non-anomalous $U_{V,1}$ WT identities of SQCD, and their supersymmetric partners. $Xm\Phi$ term is the usual soft breaking term, $(g^2/32\pi^2)\operatorname{Tr}WW$ is the anomaly.

These identities have first been obtained for supersymmetric QED by CLARK, PIGUET and SIBOLD [7], who worked with BPHZ renormalization scheme. For SQCD, eqs. (3.15)-(3.18) have been found in ref. [8] and ref. [6] by using the point-splitting method and Pauli-Villars regularization (in ref. [6], these methods were employed in the component formalism). For completeness we briefly describe these methods in appendix B.

The WT identities of the nonanomalous $SU_{N_f}\times SU_{N_f}$ (namely, with no weak gauging of the flavour group), can be immediately found by taking $A(z)$ as a matrix in the flavour space,

$$(3.19)\qquad A(z)=\sum_{a=1}^{N_f^2-1}A^a(z)t^a,$$

where t^a's are the generators of SU_{N_f}, in the analysis made above. One finds

$$(3.20)\qquad 0=\delta Z/\delta A^a=\left\langle -\frac{\bar{D}^2}{4}\bar{\Phi}\exp[V]t^a\Phi+Xmt^a\Phi+\frac{\delta S_{\text{sources}}}{i\,\delta A^a}\right\rangle^{\{J\}}$$

and there other sets of identities analogous to eqs. (3.16)-(3.18). The identities used in the derivation of Dashen's formula and its supersymmetric generalization due to VENEZIANO [22], for instance, can be readily found by taking the first derivatives of these with respect to $J_T(z)$.

Because our regularization conserves the vector gauge invariance it can be used to find the supersymmetric generalization of Bardeen anomaly [15], in the presence of external SU_{V,N_f} fields (but with no external axial fields). One gets instead of eq. (3.20) an anomalous $SU_{N_f,\text{right}}$ identities,

$$(3.21)\qquad 0=\left\langle -\frac{\bar{D}^2}{4}\bar{\Phi}\exp[V]t^a\Phi+Xmt^a\Phi+\left(\frac{\hat{g}^2}{32\pi^2}\right)\operatorname{tr}(t^a\hat{W}\hat{W})+\frac{\delta S_{\text{sources}}}{\delta iA^a}\right\rangle^{\{J\}},$$

where $\hat{g}$ and $\hat{W}$ refer to the external SU_{V,N_f} vector superfields and tr stands for a trace in the flavour space. Combined with three other identities, analogous to eqs. (3.16)-(3.18), eq. (3.21) leads to nonanomalous $SU_{V,N_f}\times U_{V,1}$ and anomalous SU_{A,N_f} WT identities. Equation (3.21) agrees with ref. [17].

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[22] G. VENEZIANO: *Phys. Lett. B*, **128**, 199 (1983); see also G. SHORE: *Nucl. Phys. B*, **231**, 139 (1984).

As has been already emphasized, our method does not depend on the reality of the matter fields with respect to the gauge group, and as such, can be generalized to chiral theories in a straightforward manner. Consider a generic theory with a set of matter chiral superfields $\{\Phi\}$, and with a superpotential $\mathscr{P}(\{\Phi\})$. Following the example of SQCD, the effect of the change of functional variables can be studied separately for each matter field Φ_i (i denoting all the « flavour » indices, in particular specifying the representation of the gauge group according to which Φ_i transforms). The associated Jacobian can be computed as in eqs. (3.10)-(3.12), with the substitution

$$V \to V_i \equiv \sum_k V^k T_i^k \tag{3.22}$$

in L, where T_i^k are the gauge group generators appropriate for Φ_i.

With this regularization, we find the following U_1 WT identities (no sum over i) (*)

$$0 = \left\langle -\frac{\bar{D}^2}{4} \bar{\Phi}^i \exp[V_i]\, \Phi_i + \Phi_i \frac{\delta \mathscr{P}\{\Phi\}}{\delta \Phi_i} + \frac{g^2}{32\pi^2} \operatorname{Tr} W_i W_i + \frac{\delta S_{\text{sources}}}{\delta i A_i} \right\rangle^{\{J\}}, \tag{3.23}$$

where $W_i^\alpha = -\frac{1}{4}\bar{D}^2 \exp[-V_i]\, D^\alpha \exp[V_i]$. Equation (3.22) generalizes eq. (3.15) of SQCD. Simplest examples of eq. (3.23) have already been considered in ref. [5] for a chiral SU_5 model and subsequently for a chiral SU_6 model in ref. [18]. (See sect. **5**.) In ref. [18], the full WT identities eq. (3.23) have been taken into account in a study of low-energy effective actions, within a general class of supersymmetric confining theories, leading to the idea of the effective gauge symmetry.

That eqs. (3.12), (3.13) and (3.22) provide a correct regularization method in a chiral theory, is by no means a trivial statement. A justification of this method (hence of eq. (3.23)) comes from studies based on the point-splitting regularization [5,23,24]. (See also appendix B.)

A further and crucial support comes from the following observation: the regularization based on eqs. (3.12), (3.13) and (3.22) is the one that guarantees the gauge invariance of the theory. Indeed, by using the same regularization and repeating the analysis by considering a gauge transformation, *i.e.* with $A(z) = \sum_k A^k(z) T_i^k$, where T_i^k's are the generators of the gauge group, we get the gauge anomaly, ($(g^2/32\pi^2)$ times)

$$\sum_{\{\Phi\}} \operatorname{Tr}\,(T_i^k W_i^\alpha W_{i,\alpha}) = \tfrac{1}{2} \sum_{\{\Phi\}} \operatorname{Tr}\,(T_i^k \{T_i^l T_i^m\})\, W^{l,\alpha} W_\alpha^m . \tag{3.24}$$

(*) Summation over colour indices is implicit in the second term in eq. (3.23).

(23) Y. MEURICE: Ph. D. Thesis, in preparation.

(24) K. KONISHI: unpublished.

This, being proportional to the symmetric trace of three generators, can be cancelled among an appropriate set of the matter fields (such as between 5 and 10* in SU_5) in a usual manner (*).

We conclude this section by noting that the same method as presented here can be used in a more general study of non-Abelian anomalies; the precise form of the regulator kernel, eq. (3.13), however, depends upon the constraints on the anomalies one wishes to impose. These constraints, in turn, will depend on the physics one chooses to study. These issues, including the explicit computation of the «consistent» anomalies, will be discussed in a forthcoming paper ([26]).

4. – Axial U_{A1} anomaly in SQCD.

In this section we come back to the specific case of SQCD and study its axial $U_{A,1}$ identity. It is not difficult to extract from eqs. (3.15)-(3.18) the part corresponding to it. Applying D^2 on eqs. (3.15) and (3.16) and $\bar{D}^2$ on eqs. (3.17) and (3.18), and using the formula

$$[D^2, \bar{D}^2] = -i\partial^\mu(D\sigma_\mu\bar{D} - \bar{D}\bar{\sigma}_\mu D)\,, \tag{4.1}$$

one finds (summing over flavours)

$$0 = \langle \partial^\mu \Gamma^5_\mu - 2iM - a + s\rangle^{\{J\}}, \tag{4.2}$$

where

$$\Gamma^5_\mu = (D\sigma_\mu\bar{D} - \bar{D}\bar{\sigma}_\mu D)\,(\bar{\Phi}\exp[V]\Phi + X\exp[-V]\bar{X})/4\,, \tag{4.3}$$

$$M = -\{D^2(Xm\Phi) - \bar{D}^2(\bar{\Phi}\bar{m}\bar{X})\}/4\,, \tag{4.4}$$

$$a = -\frac{2N_f ig^2}{32\pi^2}\cdot \mathrm{Tr}(D^2W^2 - \bar{D}^2\bar{W}^2)/4\,, \tag{4.5}$$

and s is the contribution from the source terms.

(*) It is seen that our regularization automatically leads to the «covariant» rather than «consistent» anomaly ([25]). The same holds for the point-splitting method discussed in appendix B. Note that ([26]) the criterion for the gauge anomaly cancellation is the same in the consistent form of the anomaly, although the explicit form of the latter obtained so far (N. K. NIELSEN ([17]) and ref. ([26])) is rather complicated. We thank S. FERRARA, E. GUADAGNINI and M. MINTCHEV for discussions on the anomaly cancellation in supersymmetric theories.

([25]) W. A. BARDEEN and B. ZUMINO: *Nucl. Phys. B*, **244**, 421 (1984).
([26]) E. GUADAGNINI, K. KONISHI and M. MINTCHEV: *Phys. Lett. B*, **157**, 37 (1985); see also related works, O. PIGUET and K. SIBOLD: *Nucl. Phys. B*, **247**, 484 (1984); L. BONORA, P. PASTI and M. TONIN: *Phys. Lett. B*, **156**, 341 (1985); G. GIRARDI, R. GRIMM and R. STORA: *Phys. Lett. B*, **156**, 203 (1985).

Alternatively, eq. (4.2) can be obtained by considering the axial transformation, eq. (3.7), with ($\alpha(x)$ real)

$$A(z) = -\frac{\bar{D}^2}{4}\left(\bar{\theta}^2\alpha(x)\right), \qquad \bar{A}(z) = -\frac{D^2}{4}\left(\theta^2\alpha(x)\right) \tag{4.6}$$

and by setting $\delta Z/\delta\alpha(x) = 0$.

The lowest component of eq. (4.2) reduces to the standard axial $U_{A,1}$ identity of SQCD, with

$$a\big|_{\theta=\bar{\theta}=0} = 2N_f\cdot\frac{g^2}{32\pi^2}\,\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu} - \partial^\mu(\bar{\lambda}\gamma_\mu\gamma_5\lambda)\right) \tag{4.7}$$

(λ is the four-component Majorana gauge fermions, $\lambda = (\bar{\lambda}^{\dot{\alpha}}, \lambda_\alpha)^{\mathrm{tr}}$).

To make explicit the topological origin of the superspace axial anomaly, let us write the action eq. (3.2) in the form (for the use of the following notation, see ref. ([21])),

$$S = \Psi^*\circ\Omega[V]\circ\Psi, \tag{4.8}$$

$$\Omega[V] = \begin{bmatrix} m1_{++} & -(D^2/4)\exp[V]1_{--} \\ -\frac{1}{4}\bar{D}^2\exp[-V]1_{++} & m1_{--}\end{bmatrix}, \tag{4.9}$$

where $\Psi = (\bar{X}, \Phi)^{\mathrm{tr}}$ and $\Psi^* = (\bar{\Phi}, X)$ and $1_{--} = -\frac{1}{4}\bar{D}^2$, $1_{++} = -\frac{1}{4}D^2$. The dot $\circ$ implies a summation over superspace–co-ordinate labels using appropriate chiral or antichiral measures. In this notation, the chirality operator is a diagonal matrix

$$\Gamma_5 = \mathrm{diag}\,(1_{++}, -1_{--}), \tag{4.10}$$

which satisfies the relation

$$\Omega\circ\Gamma_5 + \Gamma_5\circ\Omega = 2m\Gamma_5\,. \tag{4.11}$$

There is clear one-to-one correspondence between the superfield action eq. (3.2) and the QED Lagrangian eq. (2.1) expressed in terms of two-component spinors

$$\mathscr{L} = \bar{\psi}\Gamma\psi, \tag{4.12}$$

$$\Gamma = i\check{D} - m = \begin{bmatrix} -m\delta^{\dot{\alpha}}_{\dot{i}} & i\bar{\check{D}}^{\dot{\alpha}\beta} \\ i\check{D}_{\alpha\dot{\beta}} & -m\delta^{\beta}_{\alpha}\end{bmatrix}, \tag{4.13}$$

where $\psi = (\bar{\zeta}^{\dot{\beta}}, \xi_\beta)^{\mathrm{tr}}$ and $\bar{\psi} = (\bar{\xi}_{\dot{\alpha}}, \zeta^\alpha)$. γ_5 is diagonal in this basis,

$$\gamma_5 = \mathrm{diag}\,(\delta^{\dot{\alpha}}_{\dot{\beta}}, -\delta^{\beta}_{\alpha}), \tag{4.14}$$

and obeys the relation

$$\{\gamma_5, \Gamma\} = -2\, m\gamma_5\,. \tag{4.15}$$

To $i\check{D}$, in particular, corresponds the superspace operator $\Omega(V;\, m = 0)$

$$i\check{D}[A] \Leftrightarrow \Omega[V;\, m = 0]\,. \tag{4.16}$$

Similarly, to the squared operator $\check{D}^2$ corresponds the superspace operator $-\,\Omega\circ\Omega$ (for $m = 0$):

$$\check{D}^2[A] \Leftrightarrow - \tag{4.17}$$
$$-\operatorname{diag}\left(\frac{1}{16}\, D^2 \exp[V]\bar{D}^2 \exp[-V]1_{++},\quad \frac{1}{16}\,\bar{D}^2 \exp[-V]D^2 \exp[V]1_{--}\right).$$

This provides a simple way, in the specific case of SQCD, to obtain the regulator kernel eq. (3.13) and its antichiral partners.

Noting the above correspondence, we use the operator Ω^2 to regularize the Jacobian associated with the axial rotation eq. (4.6). Then the (regularized) integrated axial anomaly, appearing in the exponent of the Jacobian, is cast in the form

$$\int \mathrm{d}^8 z\, \alpha(x)\, \mathfrak{A}(z) = \lim_{\tau\to 0} \mathrm{Tr}[\alpha\Gamma_5\circ\exp[\tau\Omega^2(m = 0)]] \tag{4.18}$$

in exact analogy with eqs. (2.6) and (2.8). Equation (4.18) may be formally interpreted as relating the superspace axial anomaly to the topological index of the operator $\Omega(V;\, m = 0)$, the superspace analogue of the Dirac operator $i\check{D}$.

5. – Anomalous supersymmetry commutators and properties of vacua in SQCD and in a chiral SU_5 model.

We turn in this section to the features more specific to supersymmetric theories, involved in the WT identities obtained in sect. **3**. The operator equation (corresponding to eq. (3.15) of SQCD)

$$(\bar{D}^2/4)\,\bar{\Phi}^i \exp[V]\,\Phi_i = X^i m_i \Phi_i + (g^2/32\pi^2)\,\mathrm{Tr}\, WW \tag{5.1}$$

(for each i: $i = 1, \dots, N_f$ is the flavour index) can be easily written in components, in the Wess-Zumino gauge (see appendix C). From these equations (eqs. (C.1)-(C.3)) it follows that the higher components (θ^2, $\bar{\theta}^2$ $\theta^2\bar{\theta}$, $\bar{\theta}^2\theta$

and $\theta^2\bar\theta^2$) of the composite operator $\bar\Phi \exp[V]\Phi$ contain anomalous terms involving the gauge multiplet (see eq. (C.4)) (*).

On the other hand, the lower components ($\theta = \bar\theta = 0$, θ, $\bar\theta$ and $\theta\bar\theta$) cannot, and in fact do not, contain anomalous terms because there exist no gauge-invariant operators formed out of gauge and gauge-fermion fields that have the right dimension and chirality.

As a consequence, the lowest component of eq. (5.1) can be cast into the form of an anomalous anticommutation relation ([6]), (**)

$$\{\bar Q_{\dot\alpha}, \bar\psi^{\dot\alpha,i}\phi_i\}/2\sqrt{2} = -m_i\eta^i\phi_i + (g^2/32\pi^2)\lambda\lambda \tag{5.2}$$

valid for each i.

This relation was obtained in ref. ([6]) by working directly in the component formalism.

Taking the vacuum expectation value of the both sides of eq. (5.2), we get a rigorous result ([6])

$$m_i\langle\eta^i\phi_i\rangle = (g^2/32\pi^2)\langle\lambda\lambda\rangle\,, \tag{5.3}$$

since supersymmetry is not spontaneously broken for $m \neq 0$ ([12]), (***).

An analogous reasoning, starting from the nonanomalous $SU_{N_f}\times SU_{N_f}$ identities, eq. (3.20), leads to

$$\langle\eta^j\phi_i\rangle = 0 \qquad (i \neq j) \tag{5.4}$$

in the basis in which the mass matrix is diagonal.

(*) A detailed study in the Pauli-Villars regularization method in the component formalism ([6,24]) confirms these statements. More precisely, gauge-noninvariant « anomalous » terms in the lower components arise from regulator loops that are divergent, and are taken care by the ordinary subtraction procedure of divergent contributions. On the contrary, the anomalous pieces in the higher component (which are gauge-invariant) come from finite graphs involving regulator loops. It is this situation that makes the separation between the nonanomalous and anomalous parts of $\bar\Phi\exp[V]\Phi$ look apparently nonsupersymmetric. Actually this anomaly is perfectly compatible with supersymmetry as is evident in the present approach.

(**) We use the following notation for the component fields of SQCD:

$$\Phi_i^\alpha = (\phi + \sqrt{2}\,\theta\psi + \theta^2 F_\phi + \ldots)_i^\alpha\,,$$
$$X_\alpha^i = (\eta + \sqrt{2}\,\theta\chi + \theta^2 F_\eta + \ldots)_\alpha^i \qquad (i = 1, \ldots, N_f;\ \alpha = 1, \ldots, N_c)\,,$$
$$W^\alpha = -i\lambda^\alpha + \frac{i}{2}(\sigma^\mu\bar\sigma^\nu)^{\alpha\beta}F_{\mu\nu}\theta_\beta + \ldots\,.$$

Also, the mass matrix was taken, without losing the generality, to be flavour-diagonal.

(***) Note that if it were not for the anomalous right-hand side of eq. (5.3) one would have (incorrectly) concluded that $\langle\eta^i\phi_i\rangle = 0$ for all massive flavours.

Furthermore, in the massless case (or if at least one of the flavours is massless), one gets (from eq. (5.2))

$$\langle\lambda\lambda\rangle = 0 \tag{5.5}$$

if one assumes supersymmetry not to be broken dynamically.

Equations (5.3)-(5.5) strongly constrain the possible low-energy realization of the chiral $SU_{N_f}\times SU_{N_f}\times U_{V,1}$ symmetry of SQCD.

Important as it may be, eq. (5.3) gives only relations among the condensates and tells nothing about their absolute values. Crucial information in this sense came from the recent study of instanton effects [2-5] in supersymmetric gauge theories. For SQCD, they imply [2,3]

$$\langle\lambda\lambda\rangle^{N_c-N_f}\prod_{i=1}^{N_f}\langle\eta^i\phi_i\rangle = \text{const}\,\Lambda^{3N_c-N_f}, \tag{5.6}$$

where Λ is the renormalization invariant mass scale of the theory.

When eq. (5.6) is combined with eqs. (5.3)-(5.5), one arrives at the following picture of SQCD vacua (*).

i) *Massive theory.* The symmetry of the theory is reduced to $\prod_{i=1}^{N_f} U_{i,1}$ (for unequal masses). The scalar and gauge-fermion condensates are *determined* to be

$$m_i\langle\eta^i\varphi_i\rangle = (g^2/32\pi^2)\langle\lambda\lambda\rangle = \text{const}\prod_{j=1}^{N_f} m^{1/N_c}\cdot\Lambda^{3-N_f/N_c}\exp[i2\pi k/N_c]\,. \tag{5.7}$$

There is a N_c-ple degeneracy of the vacua ($k=1,\dots,N_c$) corresponding to Witten's index. The remarkable formula eq. (5.7) was originally obtained in an effective Lagrangian approach [10].

ii) *Massless theory.* In the massless theory, the net result can be summarized by eq. (5.5) and

$$\begin{cases}\varepsilon_{i_1\dots i_{N_c}}\langle\phi_{i_1}\dots\phi_{i_{N_c}}\rangle = \varepsilon_{i_1\dots i_{N_c}}\langle\eta^{i_1}\dots\eta^{i_{N_c}}\rangle = \text{const}\,\Lambda^N & (\text{for } N_f=N_c=N)\,,\\ \qquad = 0 & (\text{otherwise})\,,\end{cases} \tag{5.8}$$

and by

$$\prod_{j=1}^{N_f}\langle\eta^j\phi_j\rangle = \begin{cases}\infty & \text{for } N_f<N_c\,,\\ \text{const}\,\Lambda^{2N_f} & \text{for } N_f=N_c\,,\\ 0 & \text{for } N_f>N_c\,.\end{cases} \tag{5.9}$$

(*) In this discussion we shall restrict ourselves to continuous global symmetries commuting with supersymmetry.

Each of $\langle\eta^i\phi_i\rangle$, however, remains undetermined. There exist thus a large degeneracy of possible vacua, which can be interpreted as being related by $SL_{N_f,C}\times SL_{N_f,C}\times U_{V,1}$ (*) (**), a complex extension of $SU_{N_f}\times SU_{N_f}\times U_{V,1}$. Note that these possible vacua are not all equivalent under $SU_{N_f}\times SU_{N_f}\times U_{V,1}$; this is a reflection of the flat directions in the scalar potential in the perturbation theory.

Nonetheless, not all the perturbative vacua survive the nonperturbative effects, according to eq. (5.9). For $N_f\leqslant N_c$, eq. (5.9) shows that some of the perturbative vacua (*e.g.* $\phi_i=\eta^i=0$ for all i) are not found in the true vacua: the nonrenormalization theorem ([26]) is thus invalidated ([2,3]).

For $N_f>N_c$, eq. (5.9) alone might suggest ([4]) the full perturbative degeneracy of vacua to survive. However, the result of the massive case, eq. (5.7) (which was shown to hold for all N_f and N_c ([3,27])), indicates that the nonperturbative effects invalidate the nonrenormalization theorem in this case as well (***).

On the other hand, eqs. (5.7)-(5.9) do suggest the dynamical possibility of chirally symmetric vacua when $N_f>N_c$. In this respect, it is interesting that, for the particular case of $N_f=N_c+1$, there exists a simple plausible set of massless composite chiral superfields. Indeed, the following set ($N_c=N_f$, $N_f=N+1$)

$$\begin{cases} T_i^j\equiv X^j\Phi_i\,, \\ B^i\equiv\varepsilon^{ii_1\dots i_N}\varepsilon_{\alpha_1\dots\alpha_N}\Phi_{i_1}^{\alpha_1}\dots\Phi_{i_N}^{\alpha_N}\,, \\ C_j\equiv\varepsilon_{jj_1\dots j_N}\varepsilon^{\beta_1\dots\beta_N}X_{\beta_1}^{j_1}\dots X_{\beta_N}^{j_N}\,, \end{cases}$$

satisfies the 't Hooft anomaly matching equations with respect to the full global symmetry group, $SU_{N+1}\times SU_{N+1}\times U_{V,1}\times U_{X,1}$, for any N.

It is also interesting to compare eq. (5.5) with the result for pure Yang-Mills theory ([9,28])

$$\langle\lambda\lambda\rangle=O(\Lambda^3)\neq0 \qquad \text{(no matter fields).} \tag{5.10}$$

How can massless matter loops, which are suppressed by powers of $1/N_c$, modify the large-N_c result eqs. (5.10) to (5.5)? The formula eq. (5.7) suggests the answer: the $1/N_c$ expansion is invalidated by infra-red divergences in the massless limit.

(*) The condensates of eq. (5.7) suggest a larger symmetry of the vacuum: a $GL_{N_f,C}$. This is, however, not the case, because of the presence of other types of condensates such as $\langle\phi^*\phi\rangle$ and $\langle\eta^*\eta\rangle$, which break this $GL_{N_f,C}$ completely ([18]).

([27]) M. T. GRISARU, W. SIEGEL and M. ROCEK: *Nucl. Phys. B*, **159**, 429 (1979).

(**) $U_{V,1}$ factor is absent for $N_f=N_c$.

(***) In this respect we disagree with the conclusions of ref. ([4]) for the cases $N_f>N_c$.

([28]) E. COHEN and C. GOMEZ: *Phys. Rev. Lett.*, **52**, 237 (1984).

Importance of the mass singularities was also pointed out, in explaining how the result such as eqs. (5.3) and (5.4) can be smoothly connected to the massless $SU_{N_f} \times SU_{N_f}$ symmetric limit (3).

This concludes our discussion on the properties of the ground states of SQCD.

As an example of application of the WT identities eq. (3.22) to a chiral theory, let us consider a SU_5 model studied in ref. (5). The matter chiral multiplet of the model are

$$\Phi_i^\alpha \quad \text{and} \quad X^a_{\alpha\beta} = -X^a_{\beta\alpha} \qquad (\alpha, \beta = 1, \ldots, 5;\ i, a = 1, 2), \tag{5.11}$$

namely, two sets of $(5 + 10^*)$'s. The SU_5-invariant superpotential

$$\mathscr{P}(\Phi, X) = h_c^{ij} \Phi_i \Phi_j X^c \equiv h_c \Phi\Phi X \tag{5.12}$$

is assumed to be present ($h_c^{ij} = -h_c^{ji} = \varepsilon^{ij} h_c$).

The simplest of the WT identities eq. (3.22) for this model can be cast into the form (following the same reasoning as in the SQCD case) (*)

$$\langle \{\bar{Q}_{\dot\alpha}, \bar{\psi}^{\dot\alpha}\phi\}/2\sqrt{2}\rangle = -2h_c\langle\phi\phi\eta^c\rangle + \frac{g^2}{32\pi^2}\langle\lambda\lambda\rangle\,, \tag{5.13}$$

$$\langle \{\bar{Q}_{\dot\alpha}, \bar{\chi}^{\dot\alpha}\eta\}/2\sqrt{2}\rangle = -h_c\langle\phi\phi\eta^c\rangle + 3\,\frac{g^2}{32\pi^2}\langle\lambda\lambda\rangle\,. \tag{5.14}$$

Furthermore there exists a (nonanomalous) anticommutation relation (for each i, a, and b) (5)

$$\langle\{\bar{Q}_{\dot\alpha}, \bar{\psi}^{\dot\alpha, i}_{\beta_1}\eta^c_{\beta_2\beta_3}\eta^b_{\beta_4\beta_5}\varepsilon^{\beta_1\ldots\beta_5}\}\rangle/2\sqrt{2} = -2h_c^{ij}\langle\phi_j^\alpha\eta^c_{\beta_1\alpha}\eta^a_{\beta_2\beta_3}\eta^b_{\beta_4\beta_5}\rangle\varepsilon^{\beta_1\ldots\beta_5}\,. \tag{5.15}$$

Meurice and Veneziano (5) have, furthermore, computed the instanton effects in this model, which imply

$$\varepsilon_{ab}\varepsilon_{ij}\langle\lambda\lambda\rangle\cdot\langle A_i^a\rangle\langle A_j^b\rangle = \text{const}\,\Lambda^{11} \neq 0\,, \tag{5.16}$$

where

$$A_i^a \equiv \varepsilon_{cd}\,\varepsilon^{\beta_1\ldots\beta_5}\eta^c_{\alpha\beta_1}\eta^d_{\beta_2\beta_3}\eta^a_{\beta_4\beta_5}\phi_i^\alpha\,. \tag{5.17}$$

(*) We use the following notation below for the component fields:

$$\Phi_i^\alpha = (\phi + \sqrt{2}\,\theta\psi + \ldots)_i^\alpha\,, \qquad X^a_{\alpha\beta} = (\eta + \sqrt{2}\,\theta\chi + \ldots)^a_{\alpha\beta}\,.$$

Gauge fermions are denoted by λ as in SQCD.

As noted by the authors of ref. ([5]), there are no « solutions » of eqs. (5.13)-(5.15) for the condensates, if supersymmetry is to remain unbroken. The only way out seems to be to conclude that supersymmetry is, actually, dynamically broken in this model ([5]).

If supersymmetry is indeed spontaneously broken, one finds from eqs. (5.13)-(5.15) current-algebra–type formulae (*)

$$f_s\langle\dot\alpha|\bar\psi^{\dot\alpha}\phi|0\rangle/2\sqrt{2} = 2h_c\langle\phi\phi\eta^c\rangle + (g^2/32\pi^2)\langle\lambda\lambda\rangle\,, \tag{5.18}$$

$$f_s\langle\dot\alpha|\bar\chi^{\dot\alpha}\eta|0\rangle/2\sqrt{2} = h_c\langle\phi\phi\eta^c\rangle + 3(g^2/32\pi^2)\langle\lambda\lambda\rangle \tag{5.19}$$

and

$$f_s\langle\dot\alpha|\bar\psi^{\dot\alpha}\eta^a\eta^b\,|0\rangle/2\sqrt{2} = -\,2h_c\langle\phi\eta^c\eta^a\eta^b\rangle\,, \tag{5.20}$$

where $|\dot\alpha\rangle$ is the massless Goldstone fermion and f_s is the strength of the supersymmetry breaking

$$\langle 0|S^\mu_\alpha|\dot\alpha\rangle = f_s\sigma^\mu_{\alpha\dot\alpha} \tag{5.21}$$

(S^μ_α = the supersymmetry current). The derivation of eqs. (5.18)-(5.20) employs the standard current algebra techniques.

Furthermore, according to eqs. (5.18)-(5.20), the low-energy effective Lagrangian of the present model will contain, as effective degrees of freedom, composite supermultiplets of general types, $\bar\Phi\exp[V]\Phi$, $\bar X\exp[\tilde V]X$ and $\bar\Phi\exp[V]XX$ as well as a few composite chiral supermultiplets (*) (**).

A few other simple models have been studied along the line of analysis sketched here; they suggest that a rich variety of patterns of realization of chiral symmetries and supersymmetry are possible, depending upon the details of the model considered.

6. – Summary and concluding remarks.

In the first part of the paper, we discussed a (superfield-) functional-integral formulation of anomalous chiral WT identities in supersymmetric gauge theories. The gauge covariance and supersymmetry have been kept manifest throughout. Our U_1 identities have thus gauge-invariant form.

Supersymmetric generalization of axial SU_{A,N_f} anomalies have been obtained, in the simple case in which only the external vector SU_{V,N_f} fields are

(*) Equation (5.18) was first derived in AFFLECK *et al.* ([5]).

(**) In fact, simple effective Lagrangians which contain composite *chiral* superfields only lead to incorrect results in this model ([24]).

present. Our discussion also included the gauge anomaly cancellation in chiral theories.

In the second part (sect. **5**), we have shown how the supersymmetric partners of the U_1 identities lead to exact relations among scalar and gauge-fermion condensates. When combined with dynamical information based on explicit instanton calculus, they make it possible to compute the vacuum properties starting from the first principles. We discussed two examples, SQCD and a chiral SU_5 model, following the works of ref. [2-5].

We conclude with a few general remarks.

We are aware of the formal and somewhat heuristic nature of our discussions; in particular, the effects of renormalization have not been properly discussed. Also, our considerations based on the functional integration do not tell whether or not the anomaly term receives corrections of higher orders in g. The only result concerning this point we know of comes from the work on SQED by CLARCK, PIGUET and SIBOLD [7], done within the BPHZ renormalization scheme. Their result suggests that the form of the superspace WT identities is not modified by renormalization [*].

It is, of course, easy to check that each term of eq. (5.2), for instance, is invariant under renormalization, to one loop. To higher orders, the first term on the right-hand side of eq. (5.2) (also in eqs. (5.13) and (5.14)) remains invariant, due to the nonrenormalization theorem. The other two terms will mix under renormalization, but we have assumed that the form of the equation remains unchanged after renormalization, just as in the case of usual axial-current divergence equation [**].

Finally, we wish to point out that the superspace U_1 WT identities (eqs. (3.15)-(3.18), eq. (3.20), eq. (3.23)) contain actually more information than has been taken into account in the discussion of sect. **5**. Recently, the full information contained in such superspace WT identities was exploited in a study of the structure of the low-energy effective action, in the context of general supersymmetric confining theories [18]. The outcome, emergence of the «effective gauge symmetry» (an exact local symmetry structure at the tree level of the low-energy effective action) and of a generationlike structure of composite matter multiplets [18], seems to be an encouraging sign that super-

(*) According to SIBOLD (private communication) this can be proven for SQCD as well.

(**) In fact, a plausibility argument can be given for the form invariance of the WT identities under renormalization. We first consider the component containing the current divergence equations, and generalize the usual argument for the softly broken current divergences [29] (by using the nonrenormalization theorem) and for the anomalous $U_{A,1}$ current divergence [30] (by using the form of the anomaly, eq. (4.7)); we then invoke supersymmetry.

(29) See, *e.g.*, D. GROSS: *Les Houches Lectures, 1975*, edited by R. BALIAN and J. ZINN-JUSTIN (North Holland Publishing Company, 1976).

(30) See, *e.g.*, R. CREWTHER: *Status of the U_1 problem*, CERN preprint, TH-2546 (1978).

symmetric gauge theories might one day find their place within a realistic composite theory of particle physics.

* * *

This work started when both of us were visiting CERN for a short period. We thank its theory division for hospitality.

APPENDIX A

In this appendix we describe the calculation of the matrix element

$$\langle z|iA\exp[L/M^2](-\bar{D}^2/4)|z\rangle \tag{A.1}$$

appeared in eq. (3.12). (For SQED, this matrix element has been known [21].) The first step is to observe that the operator $L = \bar{D}^2\exp[-V]D^2\exp[V]/16$ always acts on $\bar{D}^2(\ldots)$ so that L can be rewritten as $(1_{--} = -\bar{D}^2/4)$

$$L1_{--} = (P^2 - \tfrac{1}{2}W^\alpha D_\alpha + C^\mu P_\mu + F)1_{--}\,, \tag{A.2}$$

where

$$\begin{cases} W^\alpha \equiv -\frac{1}{4}\,(\bar{D}^2\exp[-V]D^\alpha\exp[V])\,, \\ C^\mu \;\equiv -\frac{1}{2}\,\sigma^\mu_{\alpha\dot{\alpha}}(\bar{D}^{\dot{\alpha}}\exp[-V]D^\alpha\exp[V])\,, \\ F \;\equiv \frac{1}{16}\,(\bar{D}^2\exp[-V]D^2\exp[V])\,, \end{cases} \tag{A.3}$$

and P^μ is the momentum operator which acts as $\langle x|P^\mu\ldots = \int\mathrm{d}y\, i\cdot\partial^\mu\delta(x-y)\cdot\langle y|\ldots$. The brackets in eqs. (A.3) mean that the covariant derivatives $\bar{D}^\alpha$ and D^α do not act outside them.

To evaluate the finite (as $M^2\to\infty$) part of eq. (A.1), we observe the following two facts. First, in order to get a nonvanishing diagonal element $\langle\theta\bar{\theta}|\ldots|\theta\bar{\theta}\rangle$ we need precisely two D's and two $\bar{D}$'s operating on $|\theta\bar{\theta}\rangle$, *e.g.*

$$\langle\theta,\bar{\theta}|D_\alpha D_\beta\bar{D}^2|\theta,\bar{\theta}\rangle = 8\,\varepsilon_{\alpha\beta}\,. \tag{A.4}$$

It means that the $W^\alpha D_\alpha$ term in the exponent should be expanded at least to second order.

On the other hand, the matrix element $\langle x|\ldots|x\rangle$ is at most of order $O(M^4)$, since

$$\langle x|\exp[P^2 + \text{lower order in } P]/M^2|x\rangle = \int\frac{\mathrm{d}^4k}{(2\pi)^4}\exp[-k^2/M^2]\,(1+O(1/M)) = iM^4/16\pi^2 + O(M^3)\,. \tag{A.5}$$

Since the expansion in $-\frac{1}{2}W^\alpha D_\alpha + C^\mu P_\mu + F$ brings down a factor of $1/M^2$ per each power, it follows in the light of the above observation that the finite part of eq. (A.1) comes exclusively from the second-order expansion of $-\frac{1}{2}W^\alpha D_\alpha$ (and zeroth order in $C^\mu P_\mu + F$). A simple calculation, by using eq. (A.4) and eq. (A.5), leads to the answer (after rescaling, $V \to 2gV$),

$$\text{(A.6)} \qquad \lim_{M^2\to\infty} \langle z|iA \exp[L/M^2](-\bar{D}^2/4)|z\rangle = iA(z)(g^2/32\pi^2)\,W^\alpha(z)\,W_\alpha(z)\,.$$

It is easy to check the stability of the answer, when the Gaussian cut-off is replaced by an arbitrary function $f(L/M^2)$ that satisfies

$$\text{(A.7)} \qquad \begin{cases} f(\infty) = f'(\infty) = \ldots = 0 \qquad \text{and} \\ f(0) = 1\,. \end{cases}$$

Indeed the only change would be in eq. (A.5) which, however, leads to the same result as before

$$\text{(A.8)} \qquad \int \frac{d^4k}{(2\pi)^4} f''(\{k^2 + O(k)\}/M^2) =$$
$$= \frac{i}{16\pi^2}\int_0^\infty dk^2 k^2 f''(k^2/M^2) + \ldots = \frac{iM^4}{16\pi^2} + O(M^3)\,.$$

Appendix B

In this appendix we describe the derivations of eq. (3.15) by the point-splitting and the Pauli-Villars regularization method, both of which were employed in ref. (6) in the component formalism. These derivation are close to those given in ref. (8).

In the point-splitting method, we start with the definition

$$\text{(B.1)} \qquad \bar{\Phi}\exp[V]\Phi \equiv \lim_{\varepsilon\to 0} \bar{\Phi}_{x+\varepsilon}\bar{U}(x+\varepsilon,\ x-\varepsilon)\,(\exp[V]\Phi)_{x-\varepsilon}\,,$$

where $\bar{U}$ is the string operator

$$\text{(B.2)} \qquad \bar{U}(x+\varepsilon,\ x-\varepsilon) = \mathscr{P}\exp\left[\int_{x-\varepsilon}^{x+\varepsilon} dy^\mu \bar{F}_\mu(y)\right]$$

with $\bar{F}_\mu = i\frac{1}{4}D\sigma_\mu \exp[V]\bar{D}\exp[-V]$. Here only the space-time co-ordinate has been point-split. $\bar{U}$ is constructed in such a way that the nonlocal operator $\bar{\Phi}_{x+\varepsilon}\bar{U}(\exp[V]\Phi)_{x-\varepsilon}$ is invariant under the gauge transformation, eq. (3.5) (6).

Applying $\frac{1}{4}D^2$ to eq. (B.1), one finds that

$$\text{(B.3)}\qquad \tfrac{1}{4}D^2\{\bar{\Phi}_{x+\varepsilon}\bar{U}(\exp[V]\Phi)_{x-\varepsilon}\} = \\ = \tfrac{1}{4}\bar{\Phi}_{x+\varepsilon}(\bar{D}^2\bar{U})\exp[V]\Phi + \tfrac{1}{2}\bar{\Phi}_{x+\varepsilon}(D^\alpha\bar{U})D_\alpha(\exp[V]\Phi) + m\bar{\Phi}_{x+\varepsilon}\bar{X}_{x-\varepsilon}\,.$$

It is easy to show, using eq. (B.2), that

$$\text{(B.4)}\qquad \begin{cases} D^\alpha\bar{U} = 2\varepsilon^\mu D^\alpha\bar{F}_\mu + O(\varepsilon^2)\,, \\ D^2\bar{U} = 4\varepsilon^\mu\varepsilon^\nu(D^\alpha\bar{F}_\mu)(D_\alpha\bar{F}_\nu) + O(\varepsilon^3)\,. \end{cases}$$

Hence the problem is now reduced to calculating the singular parts of the propagator $\langle T(\exp[V]\Phi)_{x-\varepsilon}\bar{\Phi}_{x+\varepsilon}\rangle$ in the presence of the external gauge superfield V. For an actual calculation, one may use the explicit representation of the propagator ([21])

$$\text{(B.5)}\qquad \langle T(\exp[V]\Phi)_{x-\varepsilon}\bar{\Phi}_{x+\varepsilon}\rangle = \frac{i}{16}\langle x-\varepsilon, \theta, \bar{\theta}|\exp[V]\cdot \\ \cdot\left(m^2 - \frac{\bar{D}^2\exp[-V]D^2\exp[V]}{16}\right)^{-1}\bar{D}^2\exp[V]D^2|x+\varepsilon, \theta, \bar{\theta}\rangle\,,$$

or use the superfield Feynman rules. We quote only the result

$$\text{(B.6)}\qquad \langle T(\exp[V]\Phi)_{x-\varepsilon}\bar{\Phi}_{x+\varepsilon}\rangle = (16\pi^2\varepsilon^2)^{-1}\{1 - 2\varepsilon^\mu\bar{F}_\mu + O(\varepsilon^2)\}\,,$$

which is combined with eq. (B.3) to give eq. (5.1).

We emphasize the importance of the point-splitting method in deriving the anomalous U_1 identities for chiral theories, hence justifying the regularization, eqs. (3.12), (3.13) and (3.22), for such theories. Note that the Pauli-Villars regularization cannot be used in these cases. (For simplicity of calculation, we have used the expression eq. (B.5) valid only in SQCD, but this can be avoided. Evaluation of the matrix element eq. (B.5) has been done, without using the right-hand side of it, by MEURICE ([23])).

In the Pauli-Villars regularization method (useful only in left-right symmetric theories) all operators are local but the regulator chiral superfield X_r and Φ_r also contribute to the regularized equation

$$\text{(B.7)}\qquad \tfrac{1}{4}\bar{D}^2(\bar{\Phi}\exp[V]\Phi) = mX\Phi - MX_r\Phi_r\,.$$

Accordingly, it is necessary to extract from the Green's function $\langle T(X_r\Phi_r)...\rangle$ the part which behaves as $1/M$ as the regulator mass goes to infinity. The functional representation of $\langle X_r\Phi_r\rangle$ is given by

$$\text{(B.8)}\qquad iM\langle X_r\Phi_r\rangle = M^2\langle x,\theta,\bar{\theta}|\left(M^2 - \frac{1}{16}\bar{D}^2\exp[-V]D^2\exp[V]\right)^{-1}1_{--}|x,\theta,\bar{\theta}\rangle\,,$$

which turns into the anomaly term of eq. (5.1) in the $M\to\infty$ limit, as readily verified by following the reasoning similar to the one in appendix A.

APPENDIX C

Equation (5.1) can be written in components in the Wess-Zumino gauge as $(C \equiv \bar{\Phi} \exp[V] \Phi)$,

$$-C^{(\bar{\theta}^2)} = m\eta\phi - (g^2/32\pi^2)\lambda\lambda\,, \tag{C.1}$$

$$-C_\alpha^{(\bar{\theta}^2\theta)} - \frac{i}{2}(\sigma^\mu \partial_\mu C^{\bar{\theta}})_\alpha = \tag{C.2}$$

$$= \sqrt{2}m(\eta\psi_\alpha + \chi_\alpha\phi) + \frac{g^2}{32\pi^2}\{-2i\lambda_\alpha D + (\sigma^\mu\bar{\sigma}^\nu\lambda)_\alpha F_{\mu\nu}\}\,,$$

$$-C^{(\theta^2\bar{\theta}^2)} + \frac{i}{2}\partial^\mu C_\mu^{(\theta\sigma^\mu\bar{\theta})} + \frac{\square}{4}C^{(0)} = m(\eta F_\phi + F_\eta\phi - \chi\psi) + \tag{C.3}$$

$$+\frac{g^2}{32\pi^2}\left\{D^2 - 2i\lambda\sigma^\nu D_\nu\bar{\lambda} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{4}\varepsilon_{\mu\nu\varrho\sigma}F_{\mu\nu}F_{\varrho\sigma}\right\}.$$

Other components of eq. (5.1) prove to be either vanishing, equivalent to one of eqs. (C.1)-(C.3), or Hermitian conjugates of them.

Equations (C.1)-(C.3) agree with the result found in ref. (6),

$$\bar{\Phi}\exp[V]\Phi = \tag{C.4}$$

$$= (\bar{\Phi}\exp[V]\Phi)_{\text{naive}} - \frac{g^2}{32\pi^2}\left[\bar{\theta}^2 W^\alpha W_\alpha + \text{h.c.} - \frac{\theta^2\bar{\theta}^2}{2}(WW|_{\theta^2} + \bar{W}\bar{W}|_{\bar{\theta}^2})\right],$$

where $(\bar{\Phi}\exp[V]\Phi)_{\text{naive}}$ means that the components of the first term are the operators obtained by the usual expansion of $\bar{\Phi}$, Φ and $\exp[V]$ in terms of the component fields.

● RIASSUNTO (*)

Si formulano le identità anomale chirali e correlate di Ward e Takahashi nelle teorie di gauge supersimmetrica, per mezzo della generalizzazione del metodo dell'integrale funzionale di Fujikawa al superspazio. Il nostro approccio fornisce un trattamento manifestamente supersimmetrico e di gauge covariante delle anomalie abeliane del superspazio, ed è applicabile a teorie chirali come pure a teorie simmetriche destre-sinistre. Si discutono anche brevemente le anomalie non abeliane. Le anomalie abeliane del superspazio implicano che particolari operatori composti, cioè quelli contenenti le correnti associate di U_1 come componente, esibiscano una struttura di supermultipletto anomalo. Si discute come ciò porti a varie relazioni esatte tra scalari e condensati di fermioni di gauge, imponendo cosí forti vincoli sulle possibili realizzazioni di simmetria chirale nelle teorie a confinamento supersimmetrico.

(*) *Traduzione a cura della Redazione.*

Подход с использованием функционального интегрирования к киральным аномалиям в суперсимметричных калибровочных теориях.

Резюме (*). — Обобщая метод функционального интегрирования Фуджикавы на суперпространство, получаются киральные тождества и тождества, родственные тождествам Уорда-Такахаши, в суперсимметричных калибровочных теориях. Предложенный в этой работе подход для абелевых аномалий является в явном виде суперсимметричным и применим к теориям с киральной симметрией и с лево-правой симметрией. Неабелевы аномалии могут быть рассмотрены аналогичным образом, но в этой работе обсуждаются только вкратце. Абелевы аномалии в суперпространстве подразумевают аномальную супермультиплетную структуру для некоторых составных операторов. Обсуждается, как этот подход приводит к различным точным соотношениям, включающим скалярный и калибровочно-фермионный конденсаты и, следовательно, накладывающим сильные ограничения на реализации киральний симметрии (и суперсимметрии) в суперсимметричных удерживающих теориях.

(*) *Переведено редакцией.*