New Light on the Neveu-Schwarz Model.

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Summary. — The Neveu-Schwarz model is revealed as possessing the same Koba-Nielsen structure as the original dual-resonance model in a formalism where the Koba-Nielsen variables are extended by the incorporation of anticommuting variables.

1. – Introduction.

The first consistent ghost-free dual model to be discovered after the initial Veneziano model with intercept 1 was the Shapiro-Virasoro model with intercept 2 (1,2), subsequently generalized to show operator factorization by YOSHIMURA (3) and DI VECCHIA and DEL GIUDICE (4). The relationship between this model and the original model can best be seen in the Koba-Nielsen representation where the modification consists of the replacement of the factors $(z_i - z_j)^{-2k_i k_j}$ in the integrand with $|z_i - z_j|^{-2k_i k_j}$ and the extension of the integration region from the real axis or the unit circle to the entire complex plane. The purpose of this paper is to present a formalism which reveals the Neveu-Schwarz (5) model as an extension of the original model of a similar nature; the factor $z_i - z_j$ is replaced by a different distance function and the integration region is appropriately modified. The completely symmetric

(1) M. A. VIRASORO: *Phys. Rev.*, **177**, 2309 (1969).
(2) J. SHAPIRO: *Phys. Lett.*, **33** B, 361 (1970).
(3) M. YOSHIMURA: *Phys. Lett.*, **34** B, 79 (1971).
(4) E. DEL GIUDICE and P. DI VECCHIA: *Nuovo Cimento*, **5** A, 90 (1971).
(5) A. NEVEU and J. H. SCHWARZ: *Nucl. Phys.*, **31** B, 86 (1971).

pion model of SCHWARZ [6] which after the work of OLIVE and SCHERK [7] is likely to be the odd-G-pomeron sector of the Neveu-Schwarz model is obtained from the Shapiro-Virasoro model by an analogous construction. Briefly the construction involves the introduction of anticommuting variables $\varphi_i (i = 1, ..., N)$ associated with the N Koba-Nielsen variables, and forming a Grassmann al gebra [*] [8]:

$$\varphi_i \varphi_j + \varphi_j \varphi_i = 0 \qquad \text{(all } i, j) . \tag{1.1}$$

With the formal definition of integration as

$$\begin{cases} \int \mathrm{d}\varphi_i = 0 , \\ \int \varphi_i \, \mathrm{d}\varphi_i = 1 , \end{cases} \tag{1.2}$$

where the differentials $\mathrm{d}\varphi_i$ also anticommute with each other and with φ_j $(j \neq i)$ we show that the N-point pion Born term in the Neveu-Schwarz model may be written in complete analogy with the original model as

$$\iint \prod_{i<j} (z_i - z_j - \varphi_i \varphi_j)^{-2k_i \cdot k_j} \prod_{i=1}^{N} \theta(z_{i+1} - z_i) \cdot \\ \cdot \prod_{i=1}^{N} \mathrm{d}\varphi_i \prod_{i=1}^{N-3} \mathrm{d}z_i (z_N - z_{N-1})(z_{N-1} - z_{N-2})(z_N - z_{N-2}) . \tag{1.3}$$

The somewhat unusual appearance of the formalized integration (1.2), which has already been applied to the evaluation of multiloop amplitudes in the Neveu-Schwarz model by MONTONEN [8], may be rendered more palatable when it is realized that the integration process is equivalent to the evaluation of (1.3) with the anticommutation relations

$$[\varphi_i, \varphi_j]_+ = 0 \qquad (i \neq j) , \tag{1.4}$$

integration over φ_i being interpreted as a contour integral round the origin

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([6]) J. SCHWARZ: Princeton preprint PURC 3072-06 (1972).
([7]) D. OLIVE and J. SCHERK: CERN preprint 1635 (1973).
([8]) C. MONTONEN: Cambridge preprint (March 1973).
([*]) Historical note. It is amusing to find in W. BLASCHKE (*Vorlesungen über Differentialgeometrie*, Vol. 1 (Berlin, 1924), p. 192) numbers of the form $z + \varphi$ described with astonishing prescience as dual numbers. I am indebted to T. J. WILLMORE for this reference.

with measure $\varphi_i^{-2}\,\mathrm{d}\varphi_i$ giving

$$(1.5)\qquad \int\prod_{i=1}^{N}\theta(z_{i+1}-z_i)\prod_{i=1}^{N-3}\mathrm{d}z_i(z_N-z_{N-1})(z_{N-1}-z_{N-2})(z_N-z_{N-2})\cdot \cdot\oint\ldots\oint\prod_{i<j}(z_i-z_j-\varphi_i\varphi_j)^{-2k_ik_j}\prod_{i=1}^{N}\frac{\mathrm{d}\varphi_i}{\varphi_i^2}\,.$$

After establishing these results, we go on to show that, if we try to use this device on the Neveu-Schwarz model itself, we do not generate a further model, but the resulting amplitude vanishes. We then present a simple algorithm for the systematic replacement of the factors in the one-loop integrals for the original model (9) by factors incorporating the anticommuting elements φ_i which when integrated formally reproduces the results of GODDARD and WALTZ (10) and CLAVELLI and SHAPIRO (11) on the one-loop calculations in the Neveu-Schwarz model. From this we infer and prove the algorithm which, when applied to the multiloop amplitudes in the original model by LOVELACE (12) and ALESSANDRINI and AMATI (13,14), reproduces the multiloop calculations by MONTONEN (8) in the Neveu-Schwarz model. The Born term is discussed in Sect. **2**, and the details of the contributions of single-loop orientable graphs in Sect. **3**. The reader who is interested only in general principles can ignore this Section and proceed to Sect. **4** where the essence of the general construction is given.

2. – Tree graphs.

Consider the formal expansion of the expression (1.3) by expanding each factor in the integrand as

$$(2.1)\qquad (z_i-z_j-\varphi_i\varphi_j)^{-2k_ik_j}=(z_i-z_j)^{-2k_ik_j}\left(1+\frac{2k_ik_j\varphi_i\varphi_j}{z_i-z_j}\right)$$

on account of the anticommutation relations (1.1), and then integrating using the formal device (1.2). Then it is easy to see that the only contribution will come from the coefficient of $\varphi_1\varphi_2\varphi_3\ldots\varphi_N$ in the expansion of the integrand.

(9) See, for example, V. ALESSANDRINI, D. AMATI, M. LE BELLAC and D. OLIVE: *Phys. Rep.*, **1** C, 269 (1971).

(10) P. GODDARD and R. E. WALTZ: *Nucl. Phys.*, **34** B, 99 (1971).

(11) L. CLAVELLI and J. A. SHAPIRO: *Nucl. Phys.*, **57** B, 490 (1973).

(12) C. LOVELACE: *Phys. Lett.*, **34** B, 500 (1971).

(13) V. ALESSANDRINI: *Nuovo Cimento*, **2** A, 321 (1971).

(14) V. ALESSANDRINI and D. AMATI: *Nuovo Cimento*, **4** A, 793 (1971).

This coefficient will be

$$(2.2)\qquad 2^{N/2}\sum(-1)^P\frac{(k_{i_1}\cdot k_{i_2})(k_{i_3}\cdot k_{i_4})\dots(k_{i_{N-1}}\cdot k_{i_N})}{(z_{i_1}-z_{i_2})(z_{i_3}-z_{i_4})\dots(z_{i_{N-1}}-z_{i_N})}\prod_{i<j}(z_i-z_j)^{-2k_ik_j},$$

where $\sum$ denotes the sum over all permutations $i_1i_2\dots i_N$ of the indices $1\dots N$ subject to the restrictions

$$(2.3)\qquad i_1<i_2;\; i_3<i_4,\dots,i_{N-1}<i_N,$$

$$(2.4)\qquad i_1<i_3<i_5\dots<i_{N-1},$$

and P denotes the parity of the permutation $i_1\dots i_N$. The sign simply arises from the fact that every elementary transposition in bringing $\varphi_{i_1}\varphi_{i_2}\dots\varphi_{i_N}$ into standard numerical order introduces a negative sign on account of (1.1), and the term will be positive or negative according to the number of elementary transpositions.

This expression (2.2) when integrated over the variable z_i with a measure to ensure Möbius invariance may be recognized as the Neveu-Schwarz model in the Koba-Nielsen formulation [15]. There are two points which are noteworthy in this formulation. Firstly $z_i-z_j-\varphi_i\varphi_j$ plays the role of a distance function, and is antisymmetric under the interchange of i with j. Secondly we can see how Möbius invariance of the expression (1.3) is guaranteed directly if $k_i^2=-\frac{1}{2}$, if we postulate that under a Möbius transformation

$$(2.5)\qquad z_i\to\frac{az_i+b}{cz_i+d},\qquad ad-cb=1,$$

φ_i simultaneously transforms as

$$(2.6)\qquad \varphi_i\to\pm\frac{\varphi_i}{cz_i+d}.$$

There is a sign ambiguity in this transformation, which is important when we come to study loop graphs. Of course the sign chosen must be the same for all φ_i.

Then

$$(2.7)\qquad z_i-z_j-\varphi_i\varphi_j\to\frac{z_i-z_j-\varphi_i\varphi_j}{(cz_i+d)(cz_j+d)},$$

and as usual

$$(2.8)\qquad \mathrm{d}z_i\to\frac{\mathrm{d}z_i}{(cz_i+d)^2}.$$

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[15] D. B. Fairlie: *Nucl. Phys.*, **42** B, 253 (1972).

However we also have $d\varphi_i \to (cz_i + d)\,d\varphi_i$ and this relation calls for further comment. Essentially it arises from the fact that we have defined the integration operation by (1.2). This relationship is preserved if

$$\varphi_i \to \lambda\varphi_i\,, \qquad d\varphi_i \to \frac{1}{\lambda}\,d\varphi_i\,. \tag{2.9}$$

Alternatively giving this formal integration the significance of contour integration under $\varphi \to \lambda\varphi$ the integration measure

$$\oint \frac{d\varphi}{\varphi^2} \to \oint \frac{1}{\lambda}\,\frac{d\varphi}{\varphi^2}\,. \tag{2.10}$$

Putting (2.7), (2.8) and (2.9) together we see that we have a Möbius-invariant expression (1.3) provided that $k_i^2 = -\frac{1}{2}$, the mass shell condition of the Neveu-Schwarz amplitude.

What happens when we iterate this operation by applying it to the amplitude (2.2)? We might expect to obtain thereby an amplitude which is fully factorizable and describes the scattering of zero-mass particles. However the outcome for the four-point function is identically zero, a result both satisfying from the point of view of self-consistency and disappointing as it closes this avenue to further extensions of dual models.

The method may be readily adapted however to construct the symmetric dual-pion model [6] which is the analogue of the Virasoro-Shapiro model [1,2] starting from the latter. All that is necessary is to introduce elements of the algebra of involutions of φ_i, *i.e.* φ_i^* in addition to φ_i, $\varphi_i^* \neq \varphi_i$, and replace $|z_i - z_j|^{-2k_ik_j}$ by the expression

$$\left((z_i - z_j)(z_i^* - z_j^*) - \varphi_i\varphi_i^*\varphi_j\varphi_j^*\right)^{-k_i\cdot k_j} \tag{2.11}$$

in the Koba-Nielsen integrand and integrate formally over all φ_i, φ_i^* as well as over all complex z_i. It is clear that this model is Möbius invariant and correponds to the scattering of particles with $k^2 = -1$.

3. – One-loop graphs.

We first of all discuss the representation of the planar loop graph in the Neveu-Schwarz model in terms of our formalism. GODDARD and WALTZ [10] first evaluated these contributions: their result for the planar loop in $d = 10$ dimensions is

$$\int \prod_{i=1}^{N} dx_i \frac{\omega^{-\frac{3}{2}}}{\ln^2 \omega}\,[f(\omega)]^{-8} \prod_{1\leqslant i\leqslant j\leqslant N} \psi(x_{ij})^{-k_i\cdot k_j} \sum (-1)^P \varphi_+^8(\omega) \prod_{\text{pairs}} k_{i_l}\cdot k_{i_m}\chi^+(x_{i_li_m})\,, \tag{3.1}$$

where the sum in the second factor is over all sets of pairs $i_1 i_2 \ldots i_{N-1} i_N$ with the sign just as in eq. (2.2) for the three graphs: the first factor is just the planar loop for the original model ([9]) with standard conventions

$$(3.2)\qquad \begin{cases} x_{ij} = x_i x_{i+1} \ldots x_j & (i<j)\,, \\ \prod_{i=1}^{N} x_i = \omega\,, \\ f(\omega) = \prod_{n=1}^{\infty}(1-\omega^n)\,, \\ \psi(x) = -2\pi i \exp\left[\dfrac{\ln x}{\ln \omega}\right]^2 \dfrac{\theta_1(\ln x/2\pi i|\ln \omega/2\pi i)}{\theta_1'(0|\ln\omega/2\pi i)}\,. \end{cases}$$

The power of the partition function is chosen so that only physical intermediate states contribute to the loop according to the recent results of BRINK and OLIVE ([16,17]). The other functions appearing in the correction factor are defined by

$$(3.3)\qquad \chi^{\pm}(x) = \sum_{n=1}^{\infty} \frac{x^{n-\frac{1}{2}} \pm (\omega/x)^{n-\frac{1}{2}}}{1 \pm \omega^{n-\frac{1}{2}}}\,,$$

$$(3.4)\qquad \varphi^{\pm}(\omega) = \prod_{n=1}^{\infty}(1 \pm \omega^{n-\frac{1}{2}})\,.$$

It is expedient to transform by a Jacobi transformation to see the connection between χ and ψ. If we define

$$(3.5)\qquad \nu = \frac{\ln x}{2\pi i}\,, \qquad \tau = \frac{\ln \omega}{2\pi i}\,,$$

$$(3.6)\qquad -i \ln \frac{z_i}{z_j} = \theta_i - \theta_j = 2\pi \frac{\ln x_{ij}}{\ln \omega}\,, \qquad \ln q \ln \omega = 2\pi^2\,,$$

it is shown in ref. ([10]) that χ^+ may be expressed in terms of theta-functions as

$$(3.7)\qquad \chi^+ = \frac{1}{2}\, i\, \theta_2(0|\tau)\theta_4(0|\tau)\frac{\theta_3(\nu|\tau)}{\theta_1(\nu|\tau)}\,.$$

CLAVELLI and SHAPIRO ([11])(*) use this relation to express χ^+ as a derivative

$$(3.8)\qquad \chi^+ = \frac{i}{\pi}\frac{\mathrm{d}}{\mathrm{d}\nu} \ln \frac{\theta_1(\nu/2|\tau)\theta_3(\nu/2|\tau)}{\theta_2(\nu/2|\tau)\theta_4(\nu/2|\tau)}\,.$$

([16]) L. BRINK and D. OLIVE: *Nucl. Phys.*, **56** B, 253 (1973).
([17]) L. BRINK and D. OLIVE: *Nucl. Phys.*, **58** B, 237 (1973).
(*) After correcting a small omission in ([11]).

When we recall the expression for $\theta_1(\nu|\tau)$ in terms of half-angles ([18])

$$\theta_1(\nu|\tau)\theta_1'(0|\tau) = 2\theta_1\left(\frac{\nu}{2}\middle|\tau\right)\theta_2\left(\frac{\nu}{2}\middle|\tau\right)\theta_3\left(\frac{\nu}{2}\middle|\tau\right)\theta_4\left(\frac{\nu}{2}\middle|\tau\right), \tag{3.9}$$

we see that there is a close relationship between χ^+ and the logarithmic derivative of ψ. The precise connection is made explicit by the following device. By using the Jacobi transformation on the function $\psi(x_{ij},\tau)$ it may be expressed ([18]) in terms of the function $\theta_1(\nu/\tau|1/\tau)/\theta_1'(0|1/\tau)$ giving

$$\psi(x_{ij}) = \frac{z_i - z_j}{\sqrt{z_i z_j}} \prod_{n=1}^{\infty} \frac{(z_i - q^{2n} z_j)(z_j - q^{2n} z_i)}{(z_i - q^{2n} z_i)(z_j - q^{2n} z_j)}. \tag{3.10}$$

From (3.8) we observe that the function $\chi^+(x_{ij})$ may be expressed as an infinite series of the form

$$\chi^+(x_{ij}) = -\frac{\ln q}{\pi i}\sqrt{z_i z_j}\sum_{n=-\infty}^{\infty}\frac{(-q)^n}{z_i - q^{2n} z_j}. \tag{3.11}$$

From (3.10) and (3.11) we can see directly that if $\psi'(x_{ij},\tau)$ represents the function obtained by term by term replacement of each factor $z_i - q^{2n} z_j$ in (3.10) by $z_i - q^{2n} z_j + (-q)^n \varphi_i \varphi_j$, then the coefficient c_{ij} of $\varphi_i\varphi_j$ in the expansion of $\psi'(x_{ij},\tau)^{-k_i k_j}$ will be given by

$$c_{ij} = k_i \cdot k_j \frac{\pi i}{\ln q \sqrt{z_i z_j}} \psi(x_{ij})^{-k_i k_j} \chi^+(x_{ij}). \tag{3.12}$$

The rationale behind this substitution is the following: $z_i - q^{2n} z_j$ is the difference between z_i and the n times iterated Möbius transformation of z_j by q^2. The standard form corresponding to this iterated transform has

$$a = d^{-1} = \pm q^n. \tag{3.13}$$

Thus we expect that φ_i should appear with the n times iterated action of this transformation on φ_j, *i.e.* with $(-1)^n q^n \varphi_j$. Here we have made a specific choice of sign in (3.13). The motivation for this choice will be clarified when we come to consider nonplanar graphs. The extra factors $(\log q\sqrt{z_i z_j})^{-1}$ are accounted for by the slight difference in the exponents $k_1 \cdot k_j$ in the original model where the momenta satisfy the mass shell condition $k_i^2 = -1$ and (3.12)

([18]) E. T. WHITTAKER and G. N. WATSON: *Modern Analysis* (London, 1927) p. 488.

where the mass shell condition is $k_i^2=-\frac{1}{2}$. The overall dependence on $\ln q\sqrt{z_i z_j}$ in the complete integrand is therefore the same in both cases. The integration measure must also change in accordance with the introduction of the variables φ_i. We do not have any real understanding of this effect at present, but would like to draw attention to certain numerical coincidences. In the original model, after correcting the exponent of the partition function to $d-2$ in accordance with the elimination of the contribution of spurious states to the trace operations in accordance with the results of BRINK and OLIVE [16,17], for the critical dimension $d=26$ there is a factor [11]

$$f(q^2)^{-24}=\prod_{n=0}^{\infty}(1-q^{2n})^{-24}=2^8q^2\theta_1'(0|\tau)^{-8}\,, \tag{3.14}$$

whereas the corresponding factor in the Neveu-Schwarz model for the planar loop is [10,11]

$$\left(\frac{\varphi^+(q^2)}{f(q^2)}\right)^8=2^4q\left(\frac{\theta_3(0|\tau)}{\theta_1'(0|\tau)}\right)^4=2^4q(\theta_2(0|\tau)\theta_4(0|\tau))^{-4}\,. \tag{3.15}$$

It may be significant that the coefficients of the theta-functions appearing in (3.2) and (3.7) are $\theta_1'(0|\tau)^{-1}$ and $\theta_2(0|\tau)\theta_4(0|\tau)$ respectively.

In order to treat the case of nonplanar orientable loops, it proves more convenient to work in the representation where the Koba-Nielsen variables lie on the real line. To make contact with the notation of ref. [11] we call these variables ϱ_i and define $x_{ij}=\varrho_j/\varrho_i$, where the range of integration of the variables ϱ_i is given by

$$-1=\varrho_1<\varrho_2\ldots<\varrho_N<-\omega<0<w\varrho_{N+1}<\varrho_{N+M}<\ldots<\varrho_{N+1}\text{ and }w<\varrho_{N+M}<1\,, \tag{3.16}$$

where there are N particles on one boundary, $N+M$ on the other.

We must now distinguish between the case where N is even and N is odd. The integral is given by

$$\int_0^1\frac{d\omega}{\omega^2}\prod_{i=2}^{N+M}\frac{d\varrho_i}{\varrho_i}\omega^{\frac{1}{2}}\left(\frac{\varphi^{\pm}(\omega)}{f(\omega)}\right)^8\prod_{1\leqslant i<j\leqslant N+M}\psi_{(T)}(x_{ij})^{-k_i\cdot k_j}\sum(-1)^P\prod k_i\cdot k_j\chi^{\pm}_{(T)}(x_{ij})\,, \tag{3.17}$$

where $\psi_{(T)}(x_{ij})$ is read as ψ if i and j are on the same boundary and as $\psi=\psi(x_{ij}\exp[i\pi],\omega)$ if i and j are on different boundaries. Similarly

$$\chi^{\pm}_T(x_{ij},\omega)=\chi^{\pm}(x_{ij}\exp[i\pi],\omega)\,. \tag{3.18}$$

By formal expansion of the denominator in $\chi^{\pm}(x_{ij})$ and interchange of orders of summation we find

$$\chi^{\pm}(x) = \sum_{n=-\infty}^{\infty} \frac{(\mp 1)^n \omega^{n/2}}{1-\omega^n x}. \tag{3.19}$$

Since $\psi(x_{ij}|\tau)^{-k_ik_j}$ has an expansion in terms of ϱ_i, ϱ_j of the same form as (3.10) apart from an exponential factor, we again find that, if $\varrho_i - \omega^n \varrho_j$ is replaced by $\varrho_i - \varrho^n \varrho_j - (\mp 1)^n \omega^{n/2} \varphi_i \varphi_j$ in the product expansion of $\psi(x_{ij}|\tau)^{-k_ik_j}$, then the coefficient of $\varphi_i \varphi_j$ in the expansion will be $-2k_ik_j\psi(x_{ij}|\tau)^{-k_ik_j}\sqrt{\varrho_i\varrho_j}\,\chi^{\pm}(x_{ij})$. By definition of ψ and $\chi^{\pm}$ (3.18) it is clear that the similar expansion of the function ψ of the extended variables will give analogous factors $\chi^{\pm}$. Thus we see that the resolution of the ambiguity in sign of the action of a Möbius transform on φ_i is related to whether we have an even or odd G-parity pomeron. If N is even we choose the sign $(-1)^n$, if N is odd we choose the positive sign. To reproduce the amplitude starting from the original model we must replace the partition function by $(\varphi^{\pm}(\omega)/f(\omega))^8$, replace $\varrho_i - \omega^n \varrho_j$ as indicated above and perform the formal integral over the $N+M$ variables φ_i.

4. – Multiloop amplitudes.

From the work of LOVELACE [12] and ALESSANDRINI [13] we know that the structure of the integrand for the multiloop graphs involves principally factors of the form $\exp[k_ik_j\Omega(\varrho_i, \varrho_j)]$, where $\Omega(\varrho_i\varrho_j)$ is an automorphic function. If T_α is a member of this group, then $\Omega(\varrho_i\varrho_j)$ will have essentially the form given by

$$\exp[\Omega(\varrho_i, \varrho_j)] = \prod_\alpha \frac{(\varrho_i - T_\alpha(\varrho_j))(\varrho_j - T_\alpha(\varrho_i))(\varrho_i - \varrho_j)}{(\varrho_i - T_\alpha(\varrho_i))(\varrho_j - T_\alpha(\varrho_j))\sqrt{\varrho_i\varrho_j}}. \tag{4.1}$$

From the similarity of this expression to eq. (3.10) we infer the rule for deriving a Neveu-Schwarz amplitude: replace all factors of the form $\varrho_i - T_\alpha(\varrho_j)$ in (4.1) by

$$\varrho_i - T_\alpha(\varrho_j) \to \varrho_i - T_\alpha(\varrho_j) - \frac{(\mp)^{n_\alpha}}{c_\alpha \varrho_j + d}\varphi_i\varphi_j, \tag{4.2}$$

and formally integrate over all φ_i, where n_α is the order of the transformation T_α, *i.e.* the sum of the powers of the fundamental generators in a representation of T_α in terms of these transformations:

$$T_\alpha(z) = \frac{a_\alpha z + b_\alpha}{c_\alpha z + d_\alpha},$$

and the negative or positive sign is chosen according to the G-parity in the appropriate channel. The timely appearance of ref. (8) enables us to prove this algorithm very simply: it follows directly from eqs. (6.3) and (6.4) of this paper, which have the joint consequence (remembering that a Pfaffian is the square root of a skew-symmetric determinant (15,19)) that the modification factor may be expressed as

$$\prod_{\text{pairs}} (-1)^P k_{i_1} k_{i_2} F(\varrho_i \varrho_j) \sqrt{\varrho_i \varrho_j} \tag{4.3}$$

with

$$F(\varrho_i, \varrho_j) = \sum_\alpha \frac{(\mp 1)^{n_\alpha}}{(c_\alpha \varrho_j + d_\alpha)(\varrho_i - T_\alpha(\varrho_j))}\,. \tag{4.4}$$

Comparison of (4.1) after the replacement (4.2) shows that the coefficient of $\varphi_i \varphi_2 \dots \varphi_N$ in the N-particle amplitude is just the original amplitude modified by (4.3), leaving aside the partition function.

5. – Conclusion.

The essence of this paper is the demonstration that the Neveu-Schwarz model is an extension of the original model of essentially the same nature as the Virasoro-Shapiro model. There is one important difference however, and that is in the different power of the partition function which is present in the Neveu-Schwarz model for the critical dimension (20) $d = 10$ and that for the Virasoro-Shapiro model which shares the same critical dimension $d = 26$ with the original model. We suspect that further analysis of this question may cast some light on the dimension problem. Further reference to Grassmann algebras may be found in ref. (21).

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We are indebted to C. MONTONEN, whose work on functional integrals over anticommuting variables led us to the formalism presented here.

D. MARTIN thanks the S.R.C. for a Research Award.

(19) A. C. AITKEN: *Determinants and Matrices* (London, 1948).

(20) P. GODDARD and C. B. THORN: *Phys. Lett.*, **40** B, 235 (1972).

(21) F. A. BEREZIN: *The Method of Second Quantization* (New York, 1966).

● RIASSUNTO (*)

Si trova che il modello di Neveu-Schwarz possiede la stessa struttura di Koba-Nielsen del modello originale delle risonanze duali in un formalismo in cui le variabili di Koba-Nielsen sono ampliate con l'inclusione di variabili che anticommutano.

(*) *Traduzione a cura della Redazione.*

Новая информация о модели Невэ-Шварца.

Резюме (*). — Показывается, что модель Невэ-Шварца обладает той же структурой Коба-Нильсена, как исходная дуальная резонансная модель, в том случае, когда переменные Коба-Нильсена расширяются благодаря включению антикоммутируюших переменных.

(*) *Переведено редакцией.*