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A Special Law of Variation for Hubble's Parameter (*) (**).

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Summary. – A law of variation for Hubble's parameter in evolutionary models, that yields a constant value for the deceleration parameter, is presented. It leads naturally to the exclusion of open universes.

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1. – Introduction.

Let us consider a universe satisfying Robertson-Walker's metric, whose line element is given by

$$\mathrm{d}s^2 = -\frac{R^2(t)}{(1+kr^2/4)^2}\,[\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2] + \mathrm{d}t^2\,, \tag{1}$$

Hubble's parameter is defined by

$$H = \frac{\dot{R}}{R}\,. \tag{2}$$

The law to be examined in this paper is

$$H = DR^{-m}\,, \tag{3}$$

where D and m are constants.

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

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2. – Obtention of q.

The parameter of deceleration (q) is defined by

$$q = -\frac{\ddot{R}R}{\dot{R}^2}. \tag{4}$$

The experimental values for it are (¹)

$$q_0 \simeq 1.0 \pm 0.5 , \tag{5}$$

where the index 0 means its present value.

From (2) and (3) we have

$$\dot{R} = DR^{-m+1} , \tag{6}$$

$$\ddot{R} = -D^2(m-1)R^{-2m+1} , \tag{7}$$

$$\therefore q = m - 1 ; \tag{8}$$

we see that q is also a constant, in our case.

3. – Einstein's equations and the values of ϱ and p.

For a RW metric, Einstein's equations reduce to (¹)

$$3\dot{R}^2 = \varkappa\varrho R^2 - 3k , \tag{9}$$

$$6\ddot{R} = -\varkappa(\varrho + 3p)R , \tag{10}$$

where ϱ and p are density and pressure, respectively. We have neglected the cosmological constant.

From (9), (10), (6) and (7) we obtain

$$\varrho = \frac{3D^2}{\varkappa} R^{-2m} + \frac{3k}{\varkappa} R^{-2} , \tag{11}$$

$$p = \frac{(2m-3)D^2}{\varkappa} R^{-2m} - \frac{k}{\varkappa} R^{-2} . \tag{12}$$

(¹) R. ADLER, M. BAZIN and M. SCHIFFER: *Introduction to General Relativity*, 2nd edition (New York, N.Y., 1975), p. 421, 426 and 434.

4. – Variation of R with time.

From (6) we obtain the law for R

$$R = (Dt)^{1/m}\,, \tag{13}$$

where we assume for $t = 0$ the value $R = 0$.

If $m > 0$, we have

$$\lim_{t\to\infty} R = \infty\,, \qquad \lim_{R\to\infty} p = 0\,, \qquad \lim_{R\to\infty} \varrho = 0\,,$$

as expected.

5. – Choice of k and m.

If we consider that the gravitational-mass density (²)

$$\sigma = \varrho + 3p$$

should be always positive, from (10) we see that

$$\ddot{R} < 0\,,$$

or that, from (7),

$$m > 1\,, \qquad i.e. \qquad q > 0\,.$$

If we impose the condition

$$\varrho \geqslant 0\,, \tag{14}$$

it is necessary to adopt, for k, either

$$k = +1\,, \tag{15}$$

or

$$k = 0\,. \tag{16}$$

For $k = 0$, if we impose

$$p \geqslant 0\,, \tag{17}$$

(²) W. H. McCrea: *Proc. R. Soc. London Ser. A*, **206**, 562 (1951).

we have

$$m \geqslant 1.5\,, \tag{18}$$

or, equivalently,

$$q \geqslant 0.5\,, \tag{19}$$

which agrees with the experimental values given in (5).

For $k = +1$, we see that negative pressures appear. Negative pressures are acceptable in general relativity, as was pointed out by Mc Crea [2]. If we impose that, for low values of R, we should have positive pressures, we have again to consider that

$$m \geqslant 1.5\,,$$

or

$$q \geqslant 0.5 \qquad \text{equivalently.}$$

In this case, as R increases, p assumes positive values, then we have $p = 0$ for

$$R_0 = [D\sqrt{2m-3}]^{1/(m-1)}\,,$$

then it assumes negative values with a minimum at

$$R_1 = [D^2 m(2m-3)]^{1/2(m-1)}\,,$$

and then it tends to zero asymptotically, as $R \to \infty$.

6. – Conclusions.

Law (3) for Hubble's parameter is a possible law of nature. It leads to the exclusion of open universes. Closed universes and flat ones are possible and both obey law (13). To decide between them, experimental values must be obtained. The requirements that law (3) makes on q should be compared with the Friedman models [1], for which

$$k = 0 \Rightarrow q = \tfrac{1}{2} \quad \text{and} \quad k = 1 \Rightarrow q > \tfrac{1}{2}\,.$$

For those who like simplicity, the case $k = 0$ is especially challenging, because we have

$$\varrho = \frac{3D^2}{\varkappa} R^{-2m}\,,$$

$$p = \frac{(2m-3)D^2 R^{-2m}}{\varkappa} = \frac{2m-3}{3}\varrho\,.$$

● RIASSUNTO (*)

Si presenta una legge di variazione per il parametro di Hubble nei modelli evolutivi, che dà un valore costante per il parametro di decelerazione. Essa porta naturalmente all'esclusione di universi aperti.

(*) *Traduzione a cura della Redazione.*

Резюме не получено.