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Quantum Theory of Measurement without Wave Packet Collapse.

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Summary. — A schematization of the measurement process in quantum mechanics is presented leading to a unified treatment both of measurements performed by means of polarized counters and of measurements made with Stern-Gerlach-like set-ups. In this way it is shown that the so-called wave packet collapse is not an absolute postulate which should be added from outside to the laws of quantum mechanics, but rather a consequence —though not an exact one but valid to a very high degree of accuracy— of these laws. The limits of the deviations from exact collapse are expressed explicitly in terms of quantities related to the macroscopic character of the experimental device. The relationship between irreversibility and this (pseudo) collapse is discussed and shown to arise from their common origin represented by the large numbers implied in this macroscopic character. However, one can have collapse without irreversibility, although not vice-versa. It is shown that all the so-called paradoxical features of the measurement problem stem from the confusion between the level of small quantum numbers and the level of very large ones. It is only at this latter level that the equivalence between the pure state vector of the total system « object+apparatus » and the statistical matrix representing the possible outcomes of their interaction ensures that the « observer » does not have any power of « creating » reality, but merely obtains from an objective, although probabilistic, representation of reality all the statistical information available.

1. – Introduction.

The wave packet collapse (or projection) postulate is an extra assumption one has to add to the closed system of rules which form the theory of quantum mechanics in order to give a physical interpretation to its mathematical formalism.

It is well known to all undergraduates in physics that, if the state vector ψ of a system S is expanded in terms of the eigenvectors $|n\rangle$ of an operator G representing an observable $\mathcal{G}$, the probability of finding any eigenvalue g_n, as a result of a measurement of $\mathcal{G}$ by means of a suitable apparatus M_G, is given by $|c_n|^2$, where c_n is the coefficient of the eigenvector $|n\rangle$ in this expansion.

However, in order to justify this interpretation, one has to assume that, once a well-determined result g_r has been actually obtained, the state vector is no longer the same ψ, but has collapsed into $|r\rangle$, because the result of a measurement repeated immediately after the first one must be, with certainty, again g_r.

This postulate introduces, therefore, in addition to the causal and reversible Schrödinger time evolution given by the theory, an acausal and irreversible source of sudden change of the state vector arising from the act of measurement.

Attempts to dispense with the projection postulate go back as far as 1935. MARGENAU (1) pointed out at that time that by so doing not only had one the advantage of eliminating an element of incoherence within the theory's body, but also the understanding of some of its apparently paradoxical features might be facilitated. His proposal, however, was not accepted, partly because it did not explain satisfactorily how the certainty of the result in a repeated measurement could be accounted for, but mostly, in my opinion, because the majority of physicists was not interested in engaging themselves into debates on the foundations of the theory, and accepted acritically the prevailing Göttingen-Copenhagen interpretation according to which it is the observer who « creates » reality in the act of « looking » at it. The idea of doing away with the projection postulate has been proposed again in 1957 by EVERETT and further developed by others (2). This important step, however, failed to lead to a satisfactory solution of the measurement problem, because it circulated under the queer form of a many-world theory.

Attempts at building a realistic quantum theory of measurement based on a detailed analysis of the interaction between a quantum microsystem and a suitable macroscopic-measurement apparatus have been pursued during these fortyfive years (3,4) with important results. On the whole, however, one cannot say that this effort has led to the construction of a « paradigm » shared by the majority of the physicist community; and not only for lack of a definitely satisfactory solution of the problem, but also for the persistence of ideological prejudices biased against a realistic epistemological stand.

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(1) H. MARGENAU: *Phys. Rev.*, **49**, 240 (1936).

(2) H. EVERETT: *Rev. Mod. Phys.*, **29**, 454 (1957); L. N. COOPER and D. VAN VECHTEN: *Am. J. Phys.*, **37**, 1212 (1960); P. A. MOLDAUER: *Phys. Rev. D*, **5**, 1028 (1972).

(3) G. LUDWIG: *Z. Phys.*, **135**, 483 (1953); A. DANERI, A. LOINGER, G. M. PROSPERI: *Nucl. Phys.*, **33**, 297 (1962); *Nuovo Cimento B*, **44**, 119 (1966).

(4) K. HEPP: *Helv. Phys. Acta*, **45**, 237 (1972).

The vast majority of this community still takes for granted that the wave function collapse following the act of measurement is a basic postulate of the accepted theory and any attempt to « understand » it would be regarded as meaningless. The remaining minority is split. Most of those who are prepared to admit the existence of a problem believe that one should « recognize the fundamental paradox of quantum mechanics namely that the total system (object + apparatus) is always a superposition and that our feeling that things must come out one way or the other is an illusion » ([5]). On the other hand, a few people seem to assume the question has already been positively solved ([6]).

In a sense it is true that the right answer to the question about the origin of the so-called wave function collapse has been outlined already in the manual on quantum mechanics by GOTTFRIED ([7]). This answer, however, has remained to such an extent unnoticed for more than fifteen years that not only most of the books on this subject continue to adopt the wave packet projection postulate, but even the most comprehensive text dedicated to the « conceptual foundations of quantum mechanics » ([8]) makes no mention at all of it, and presents the problem as still essentially open.

Gottfried's argument consists in the discussion of a Stern-Gerlach experiment in which counting devices are inserted on the paths of the distinct beams corresponding to the different eigenvalues of the spin component. Its main result is that, for this experimental arrangement, the density matrix ϱ describing the actual pure state of the total system « particle + apparatus » is shown to be indistinguishable from the density matrix $\hat{\varrho}$ describing the mixture formed by the different states representing the possible outcomes of the measurement, each one consisting of the particle in a given spin eigenstate + the apparatus in the corresponding macroscopically defined state. It is precisely the macroscopic distance between the different spatial locations of the counting devices which leads to this equivalence, based on the negligibly small magnitude of the interference terms in ϱ, for all the observables which discriminate between the counters' locations.

Quite independently, although many years later, a very similar approach to the problem of the wave function collapse has been proposed by DE MARIA,

([5]) Anonymous referee of *Foundations of Physics*. A second referee of the same journal has, however, expressed the opposite view that the problem has already been solved.

([6]) A very good review of the point of view of this minority can be found in the excellent article by J. M. LÉVY-LEBLOND: *Towards a proper quantum theory* in *Quantum Mechanics, a Half Century Later*, edited by J. L. LOPES and M. PATY (Dordrecht, 1977). I am indebted to LÉVY-LEBLOND for having called my attention to many contributions in the literature which I had overlooked.

([7]) K. GOTTFRIED: *Quantum Mechanics* (New York, N. Y., 1966), sect. 20.

([8]) B. D'ESPAGNAT: *Conceptual Foundations of Quantum Mechanics*, 2nd ed. (New York, N. Y., 1976).

MATTIOLI, NICOLÒ and myself [9]. In this case the macroscopic system involved in the measuring process was assumed to be a « polarized counter » consisting of a large number N of particles, each one having the property of switching, as a consequence of a suitable interaction with the observed microsystem (a fixed spin) from one stable state (indicated as nonionized for analogy with real counters) to another one (called ionized). It was then shown that, as soon as a sufficiently large time has elapsed, the counter, initially assumed to be in the neutral state (all N particles nonionized) could be found either to have remained undisturbed in this state, or to have been thrown into a state characterized by the « ionization » of a large fraction n of its N particles, exhibiting a perfect correlation between each of the two counter states and the two spin states of the observed particle.

The important result is that, once again, also for this « experimental » arrangement, the density matrix of the pure state « particle + apparatus » is practically equivalent to the reduced density matrix $\hat{\varrho}$ of the mixture formed with the states corresponding to the two possible outcomes of the measurement. In this case it is the macroscopic difference between the number of ionized particles ($n \approx N$) in the discharged counter's state and the number of nonionized particles (N) in the neutral state which makes the interference terms negligibly small for all the counter's observables.

Clearly these two results reinforce each other and strongly support the thesis that the so-called wave packet reduction is not an absolute postulate which should be added from the outside and incompatible with the laws of quantum mechanics, but rather is a consequence—though not an exact one, still valid to a very high degree of approximation—of these laws. In other words there is a clear indication that, in spite of the impossibility for the wave function to undergo an exact collapse as a consequence of the Schrödinger time evolution, everything happens as if the collapse had indeed occurred during the interaction with the measuring apparatus.

It should be noted that this situation implies a close analogy with the second law of thermodynamics, which is valid to a very high degree of approximation in spite of its being incompatible with the time reversibility of the equations of motion. In fact, the probability of possible deviations from the wave packet collapse in a measurement process is so low that their detection is as difficult as the detection of deviations from irreversibility implied by the second law. This does not exclude the possibility that, in the interaction of a microscopic system with a macroscopic body, prepared in very exceptional and cleverly planned conditions, interference effects might be detected. This behaviour would be, however, incompatible with the performance required from a measurement apparatus, because, as we shall see, it implies the observation

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(9) M. CINI, M. DE MARIA, G. MATTIOLI, F. NICOLÒ: *Found. Phys.*, **9**, 479 (1979).

of effects which do not discriminate between the states of the macroscopic body which ought to be correlated with those of the microscopic system.

It seems, therefore, important to make an effort for ensuring a wide recognition to these contentions, raising them to the status of an accepted theory.

The purpose of the present paper is to contribute to this effort. Its main result consists in showing that a schematization of measurement processes is possible leading to a unified treatment both of counter and of Stern-Gerlach devices, by means of which the explicit mechanism of the approximate wave function collapse is exhibited and expressed in terms of quantities related to the macroscopic character of the experimental set-up.

The limit of the deviations from exact collapse found in this way is clearly an upper limit, because all the neglected effects, such as the irreversible character of the secondary ionization cascade, tend to wash out the coherence of the contributions from the different quantum states. This means that, if these results are sufficient in order to give firm foundations to an objective theory of measurement in quantum mechanics, the same conclusions will hold with greater strength for the real measuring processes.

Summing up this work shows clearly that all the discussions on the measurement problem based on a description of the measuring apparatus which does not explicitly take into account its macroscopic character are meaningless, because a microscopic quantum system cannot be used as a measuring apparatus. In other words all the so-called paradoxical features of this problem stem from the confusion between the level of small quantum numbers and the level of very large quantum numbers. It is only at this latter level that the equivalence between the pure state vector of the total system « object + apparatus » and the statistical matrix representing the possible outcomes of their interaction ensures that the « observer » does not have any power of « creating » reality, but merely obtains from an objective, although probabilistic, representation of reality all the statistical information available.

2. – Measurement by means of a polarized counter.

a) The interaction. – In this section we wish to describe the interaction between a quantum microsystem S, to which the variable submitted to measurement belongs, and a counter M, made of N particles, devised to perform this measurement. A schematization of this interaction will be introduced which, while closely resembling the one introduced in ref. [9], will prove to be, at the same time, simpler and more general. It should be viewed, therefore, already at this stage, not so much as the proposal of another model of measuring *apparatus*, but rather as an attempt to represent schematically some general features of a measuring *process*, shared by different experimental set-ups. The justification of this change in perspective will, however, become fully clear only later on (sect. **3**).

Without loss of generality we can still assume S to be a two-level quantum system whose state is described, before interacting with the apparatus, by the state vector

$$\chi = c_+ u_+ + c_- u_- , \tag{1}$$

where $u_{\pm}$ are the usual σ_3 eigenstates up and down.

Each one of the N particles of the counter is also assumed to have only two possible states, say ω_0 and ω_1. Since all their other degrees of freedom are neglected (notably the space ones), they should be, therefore, treated as indistinguishable. This may seem, at first sight, as an oversimplification. The real atoms which form the active element of a detecting instrument are certainly more complicated than that.

However, in the last instance, what matters is their being left after the interaction with S, either in their ground state or in their ionized state (*). It is the presence of a large number n (of the order of N) of ionized particles which characterizes the discharged state of the counter, making it (macroscopically) different from its initially neutral state in which all the N particles are in their ground state. Our schematization, therefore, consisting in suppressing the various co-ordinates which do not have direct bearing on this dichotomy, does not alter substantially the main property of the real atoms of which the real instruments are made. The only thing it does is to largely overestimate their quantum-mechanical coherence, by neglecting the phase randomization actually introduced by the neglected degrees of freedom. This approximation, however, goes just in the right direction, since we want to evaluate an upper limit for the quantum effects shown by the apparatus. The neglected complexities will only make the real effects much lower than the calculated ones.

Our schematization proceeds with the choice of a suitable interaction between S and the counter's particles. The mechanism involved is direct ionization of the latter by the former. Here also important complexities are neglected, such as all secondary ionization effects which arise from the mutual interactions of the counter's particles, thus again greatly enhancing the coherence of their dynamical evolution. The final step of our schematization consists in assuming that only one of the two independent states of S (say u_+) is capable of interacting with the counter's particles, the other one (u_-) being isolated and, therefore, stationary. This is what we mean by polarized counter, namely a counter which selects between the different values of the measured variable. The term « polarized » stresses the distinction with a counter as a counting device, which merely counts the number of particles of a given kind, by simply

(*) Sometimes it is an excited state. The difference is, however, unimportant in the light of the following discussion.

detecting their presence. Furthermore, since we want to treat the ideal case of a measurement of first kind ([10]), the interaction of S should not change its state ([*]).

The Hamiltonian will, therefore, be given by the expression

$$H(a) = g' \tfrac{1}{2}(1 + \sigma_3)(a_0^* a_1 + a_0 a_1^*)\,, \tag{2}$$

where, as usual, a_0^*, a_1^* are the creation operators in the states ω_0, ω_1 obeying boson commutation relations

$$[a_0, a_0^*] = [a_1, a_1^*] = 1\,, \qquad [a_0, a_1] = [a_0^*, a_1^*] = 0\,. \tag{3}$$

A given state of the counter will be defined by giving the number n of particles in the neutral ω_0-state. Clearly $N - n$ will be the number of particles in the ionized ω_1 state. This state will be, therefore, defined by

$$|n, N - n\rangle = \frac{1}{\sqrt{n!}}\,\frac{1}{\sqrt{N-n!}}\,(a_0^*)^n (a_1^*)^{N-n}|0\rangle\,. \tag{4}$$

The only matrix elements of H different from zero will be the following:

$$\begin{cases} \langle n, N-n|(u_+^\dagger, Hu_+)|n+1, N-n-1\rangle = g'\sqrt{n+1}\sqrt{N-n}\,, \\ \langle n, N-n|(u_+^\dagger, Hu_+)|n-1, n-n+1\rangle = g'\sqrt{n}\sqrt{N-n+1}\,. \end{cases} \tag{5}$$

All the others, notably those containing u_-, will vanish.

We are ready now to set up the machinery for the solution of the equation of motion. Before doing this, however, it is useful to briefly discuss the meaning of the coupling constant g'. From eq. (5) it follows immediately that, for $n = N$,

$$\langle N, 0|(u_+^\dagger, Hu_+)|N-1, 1\rangle = g'\sqrt{N}\,. \tag{6}$$

This means that, when the initial counter's state is chosen to be the neutral state ($n = N$), the time τ_0 required to ionize the first particle is of the order

$$\tau_0 \cong \frac{\hbar}{g'\sqrt{N}}\,. \tag{7}$$

([10]) W. Pauli: *Handb. Phys.* (*Enciclopedia of Physics*), Vol. 5, edited by S. Flügge (Berlin, 1958), p. 73.

([*]) This is a definite difference with the model in (9) in which each interaction of S produced a flip from u_+ to u_- and viceversa.

Physically, however, the time τ_0 should be, at least approximately, independent of the number of particles. It is, therefore, reasonable to redefine the coupling constant g of our model Hamiltonian in such a way that

$$g' = \frac{g}{\sqrt{N}} . \tag{8}$$

This will make all considerations about the dependence of the counter's discharge time on the number of particles more reliable.

b) The dynamical evolution of the total system. – Starting with an initial state of the total system $S + M$ at $t = 0$ (*)

$$\Psi(0) = \chi \otimes |N, 0\rangle , \tag{9}$$

the Schrödinger time evolution

$$\Psi(t) = \exp\left[-\frac{i}{\hbar} Ht\right] \Psi(0) \tag{10}$$

is explicitly evaluated by means of the standard transformation

$$\left\{\begin{aligned} a_0 &= \frac{1}{\sqrt{2}}(b_0 - b_1) , \qquad b_0 = \frac{1}{\sqrt{2}}(a_0 + a_1) , \\ a_1 &= \frac{1}{\sqrt{2}}(b_0 + b_1) , \qquad b_1 = \frac{1}{\sqrt{2}}(a_1 - a_0) , \end{aligned}\right. \tag{11}$$

leading, after substitution in eq. (2), to a diagonalized Hamiltonian

$$\bar{H}(b) = \frac{1}{2} \frac{g}{\sqrt{N}} (1 + \sigma_3)[b_0^* b_0 - b_1^* b_1] . \tag{12}$$

If we define the eigenstates of (12) in terms of the operators b_0, b_1 as follows:

$$|\lambda, N - \lambda\} u_+ = \frac{1}{\sqrt{\lambda!}} \frac{1}{\sqrt{N - \lambda!}} (b_0^*)^\lambda (b_1^*)^{N-\lambda} |0\rangle u_+ , \tag{13}$$

it is clear that

$$H|\lambda, N - \lambda\} u_+ = \frac{g}{\sqrt{N}} (2\lambda - N) |\lambda, N - \lambda\} u_+ . \tag{14}$$

(*) We will drop for simplicity the symbol $\otimes$ for the Krönocker product of the Hilbert spaces of S and M in all the subsequent formulae.

The initial state $\Psi(0)$ is now written as

$$\Psi(0) = \left(\frac{1}{\sqrt{2}}\right)^N \sum_{\lambda=0}^{N} \frac{\sqrt{N!}\,(-1)^{N-\lambda}}{\sqrt{\lambda!}\sqrt{N-\lambda!}}\,|\lambda, N-\lambda\}\,\chi\,. \tag{15}$$

Inserting (14), (15) into (10), one gets immediately

$$\Psi(t) = c_+\left[\left(\frac{1}{\sqrt{2}}\right)^N \sum_{\lambda=0}^{N} \frac{\sqrt{N!}\,(-1)^{N-\lambda}}{\sqrt{\lambda!}\sqrt{N-\lambda!}} \exp\left[\frac{i}{\hbar}\frac{g}{\sqrt{N}}(2\lambda-N)\,t\right]|\lambda, N-\lambda\}\right]u_+ + c_-|N, 0\rangle u_-\,. \tag{16}$$

We have now to go back to the physical states ω_0, ω_1 defined by the operators a_0, a_1. The transformation is standard, but requires some relabelling of summation indices. The result is

$$\Psi(t) = c_+u_+\left[\sum_{n=0}^{N} a_n(t)|n, N-n\rangle\right] + c_-u_-|N, 0\rangle \tag{17}$$

with

$$a_n(t) = \frac{\sqrt{N!}\,i^{N-n}}{\sqrt{n!}\sqrt{N-n!}}\left(\cos\frac{gt}{\hbar\sqrt{N}}\right)^n\left(\sin\frac{gt}{\hbar\sqrt{N}}\right)^{N-n}. \tag{18}$$

The probability P_n of finding n neutral particles at time t is, therefore, the probability of n independent trials with probability $p(t)$ given by

$$p(t) = \cos^2\alpha(t)\,, \qquad q(t) = 1 - p(t) = \sin^2\alpha(t)\,, \qquad \alpha(t) = \frac{gt}{\hbar\sqrt{N}}\,, \tag{19}$$

namely

$$P_n(t) = \binom{N}{n} p(t)^n q(t)^{N-n}\,. \tag{20}$$

c) The correlation between states of the counter and states of S. As is well known, for large values of N the distribution of P_n is very strongly peaked around its maximum. In fact, at a given time t, when n is equal to

$$\bar{n}(t) = Np(t)\,, \tag{21}$$

one has

$$P_{\bar{n}} = \frac{N!}{\bar{n}!\,N-\bar{n}!}\left(\frac{\bar{n}}{N}\right)^{\bar{n}}\left(\frac{N-\bar{n}}{N}\right)^{N-\bar{n}}, \tag{22}$$

which, within the limits of validity of Stirling's formula, becomes

$$P_{\bar{n}} \simeq 1\,. \tag{23}$$

Since, however,

$$\sum_{n=0}^{N} P_n = 1 \,, \tag{24}$$

eq. (23) shows that, for any given time t, the probability of finding $n \neq \bar{n}(t)$ is negligible. More precisely P_n is, near $\bar{n}$, a Gaussian (De Moivre-Laplace theorem)

$$P_{\bar{n}+\Delta n} = \frac{1}{\sqrt{2\pi pqN}} \exp\left[-\frac{(\Delta n)^2}{2Npq}\right]. \tag{25}$$

The ratio between the width and the total number of particles N tends to zero as $N^{-\frac{1}{2}}$. At $t = 0$, one clearly finds

$$P_n(0) = \delta_{nN} \tag{26}$$

and, for $t_0 = (\pi/2)(\hbar\sqrt{N}/g)$,

$$P_n(t_0) = \delta_{n0} \,, \tag{27}$$

namely all particles are ionized. The time for complete discharge of the counter is, therefore, proportional to $N^{\frac{1}{2}}$.

In the limit of very large values of N one can approximately write eq. (17) in the form

$$\Psi(t) = c_+|\bar{n}(t),\, N - \bar{n}(t)\rangle\, u_+ + c_-|N, 0\rangle\, u_- \,. \tag{28}$$

This is just what one would expect for an ideal quantum measuring instrument, having a one-to-one correlation between its states and those ones of the microsystem whose variable is measured. It should be stressed that this only occurs when N is very large. For N small there would be a considerable overlap between states with different values of n and eq. (20) would give a nonnegligible probability, even for $t \neq 0$, of finding the neutral state of the counter ($n = N$) associated with the up state of S. This shows that, apart from the considerations about the wave function collapse to be developed later, a good counter must be made of a very large number of particles if an unambiguous correlation between the values of the counter's variables and those of the microsystem's variables should be maintained for a sufficiently large interval of time.

There are, however, cases in which the measurement of a particle spin variable is performed by means of a Stern-Gerlach–type set-up in which a correlation is established between the two spin states and the space variables of the same single particle, whose presence in one or the other different spacial regions is successively detected by means of a simple counting device (*not* a polarized counter). It would seem, therefore, that in this case the space variables of the microsystems itself replace the N-particle counter in the role of the measuring instrument. This would contradict the above conclusion about the essential role of the number of particles for a proper performance of a measuring process. The following section is dedicated to the clarification of this question.

3. – Measurement by means of a macroscopic variable.

a) Angular momentum as a macroscopic variable. We will show now that the time evolution of the initial state (9) determined by the Hamiltonian (2) can be interpreted from a completely different point of view in terms of the motion of a single particle. This will lead to a description of a measurement process based on the correlation between the microsystem spin values and the values of a single-particle variable characterized by very large quantum numbers. This is, therefore, a sort of idealized Stern-Gerlach set-up. The argument goes as follows.

If one compares the two forms of the Hamiltonian (2) and (12) which can be obtained from each other by means of the canonical transformation (11), one is immediately led to investigate the form of its generator. It is easy to check that the transformation can be written as

$$\begin{cases} b_0 = \exp\left[i\frac{\pi}{2}L_2\right]a_0\exp\left[-i\frac{\pi}{2}L_2\right], \\ b_1 = \exp\left[i\frac{\pi}{2}L_2\right]a_1\exp\left[-i\frac{\pi}{2}L_2\right] \end{cases} \tag{29}$$

with

$$L_2 = \frac{i}{2}\Big((a_0^* a_1 - a_1^* a_0)\Big) \tag{30}$$

Furthermore, from eq. (12) we obtain

$$H(a) = \bar{H}(b) = \exp\left[i\frac{\pi}{2}L_2\right]\bar{H}(a)\exp\left[-i\frac{\pi}{2}L_2\right]. \tag{31}$$

If one now defines L_1, L_3 by means of

$$H(a) = \frac{1}{2}\frac{g}{\sqrt{N}}(1+\sigma_3)(a_0^* a_1 + a_1^* a_0) = \frac{g}{\sqrt{N}}(1+\sigma_3)L_1, \tag{32}$$

$$\bar{H}(a) = \frac{1}{2}\frac{g}{\sqrt{N}}(1+\sigma_3)(a_0^* a_0 - a_1^* a_1) = \frac{g}{\sqrt{N}}(1+\sigma_3)L_3, \tag{33}$$

it follows immediately that L_1, L_2, L_3 obey the angular-momentum commutation relations

$$[L_1, L_2] = iL_3 \,. \tag{34}$$

This result exhibits the formal identity existing between the measurement of the spin of a particle by means of a polarized counter and the measurement of the same spin by means of a Stern-Gerlach–type set-up. In fact, if we interpret L_1, L_2, L_3 as the three components of the particle orbital angular momentum, the Hamiltonian (32) describes a spin-orbit interaction of a peculiar type between the up state of the spin component in the 3-direction and the angular-momentum component in the 1-direction.

Let us discuss the time evolution of the system. Both the initial state (9) and the states $|n, N-n\rangle$ appearing in the state vector (17) are eigenstates of L_3:

$$L_3|N, 0\rangle = \tfrac{1}{2}N|N, 0\rangle \,, \tag{35}$$

$$L_3|u, N-n\rangle = \left(n - \frac{N}{2}\right)|n, N-n\rangle \,. \tag{36}$$

It is, therefore, convenient to introduce the usual notation for the eigenfunctions and eigenvalues of the angular momentum by setting

$$n - \frac{N}{2} = m \,, \qquad N = 2l \,, \tag{37}$$

$$|n, N-n\rangle = Y^{(3)}_{lm} \,. \tag{38}$$

It follows that the state $\Psi(t)$ given by eq. (17), as a consequence of the time evolution induced by the Hamiltonian (32), becomes, when $l \gg 1$, of the form (28), namely

$$\Psi(t) = c_+ Y^{(3)}_{l\overline{m}(t)} u_+ + c_- Y^{(3)}_{ll} u_- \tag{39}$$

with

$$\overline{m}(t) = \overline{n}(t) - \frac{N}{2} = l \cos 2\alpha(t) \,. \tag{40}$$

In other words, the 3rd component of the angular momentum of the particle can be identified with the pointer of a spin-measuring instrument provided its two eigenvalues, correlated with the two spin states, are macroscopically distinguishable.

This result shows that the general condition for the reliability of a measurement process is that the quantum numbers labelling the states of the « apparatus », which are in a one-to-one correspondence with the eigenvalues of the measured microsystem variable, should differ from each other by a very large number.

This condition is reasonable, almost trivial. Most of the papers on the quantum theory of measurement, however, do not take it explicitly into account.

b) The time evolution of the angular-momentum components. – In the previously discussed schematization, the orbital angular momentum of the particle interacts selectively with the spin in such a way that the up state induces a time variation of its component along the 3-axis. We want to discuss in more details the time variation of all its components.

Initially L_3 has a well-defined value l and the mean values of both L_1, L_2 vanish. At the same time the mean square deviations of the latter are

$$\frac{\Delta L_1}{l} = \frac{\Delta L_2}{l} = \frac{1}{\sqrt{2l}}. \tag{41}$$

This means that, for $l \to \infty$, the angular momentum becomes a classical variable with well-defined simultaneous values of all its components. This property is maintained at any later time t. In fact, the angular-momentum state of the particle (in its up spin state) is easily obtained from eqs. (17), (18), (37), (38) as

$$Z(t) = \sum_{m=-l}^{l} \sqrt{\frac{(2l)!}{(l+m)!\,(l-m)!}}\; i^{l-m}\, p^{(l+m)/2}\, q^{(l-m)/2}\, Y^{(3)}_{lm}, \tag{42}$$

which is readily found to be the eigenstate $Y^{(\alpha)}_{ll}$ with eigenvalue l of the component $L_{\alpha(t)}$ defined by

$$L_{\alpha(t)} = L_3 \cos 2\alpha(t) - L_2 \sin 2\alpha(t) = L_3 \frac{\overline{m}(t)}{l} - L_2 \frac{\sqrt{l^2 - \overline{m}^2}}{l} \tag{43}$$

with $\alpha(t)$ given by eq. (19).

Instead of the approximate equation (39) we can, therefore, write the exact equation

$$\Psi(t) = c_+ Y^{(\alpha)}_{ll} u_+ + c_- Y^{(3)}_{ll} u_- . \tag{44}$$

The correlation is now between the up spin state with the value l of the angular-momentum component L_α, on the one hand, and the down spin state with the value l of the angular-momentum component L_3, on the other. The uncertainties in both L_2 and L_3 when L_α has the well-defined value l are of course always given by the width of the Gaussian (25), namely

$$\frac{\Delta L_i}{l} < \frac{1}{\sqrt{2l}} . \tag{45}$$

In other words the angular-momentum components L_1, L_2, L_3 which obey the quantum-mechanical equations of motion

$$\begin{cases} \dot{L}_1 = 0 , \\ \dot{L}_2 = \frac{i}{\hbar}[H, L_2] = -\frac{2g}{\hbar\sqrt{2l}} L_3 , \\ \dot{L}_3 = \frac{i}{\hbar}[H, L_3] = \frac{2g}{\hbar\sqrt{2l}} L_2 \end{cases} \tag{46}$$

behave as classical variables given by their mean values

$$\bar{L}_1 = 0 , \tag{47}$$

$$\bar{L}_2 = -l \sin 2\alpha(t) = -\sqrt{l^2 - \overline{m}^2(t)} , \tag{48}$$

$$\bar{L}_3 = l \cos 2\alpha(t) = \overline{m}(t) \tag{49}$$

with uncertainties given by (41) and (45). Equations (39) and (44) are, therefore, equivalent as long as the uncertainties are negligible.

It may be useful to point out, in this connection, that a slightly modified form of the Hamiltonian (32) may be chosen in order to treat in a perhaps more conventional fashion the selection between the two spin states. In fact, if one writes

$$H = \frac{g}{\sqrt{2l}} \sigma_3 2L_3 , \tag{50}$$

the equations of motion for the components of the angular momentum become

$$\begin{cases} \dot{L}_1 = -\frac{2g}{\hbar\sqrt{2l}} \sigma_3 L_2 , \\ \dot{L}_2 = \frac{2g}{\hbar\sqrt{2l}} \sigma_3 L_1 , \\ \dot{L}_3 = 0 . \end{cases} \tag{51}$$

Now the 3rd component is fixed (equal to zero in the classical limit) and the correlation between the two eigenvalues of σ_3 is established with two different eigenstates of the component of $\boldsymbol{L}$ which is chosen to have mean value zero at $t = 0$. In fact, if L_1 has initially the eigenvalue l, one has

$$\bar{L}_1 = l \cos 2\alpha(t) = \bar{m}(t)\,, \tag{52}$$

$$\bar{L}_2 = \sigma_3 l \sin 2\alpha(t) = \sigma_3 \sqrt{l^2 - \bar{m}^2}\,. \tag{53}$$

$$\bar{L}_3 = 0\,. \tag{54}$$

In the Schrödinger picture, this corresponds to a wave function at a time t of the form

$$\Psi(t) \simeq c_+ u_+ Y^{(2)}_{l,\sqrt{l^2-\bar{m}^2}} + c_- u_- Y^{(2)}_{l,-\sqrt{l^2-\bar{m}^2}}\,. \tag{55}$$

This means that the spin orbit interaction (50) produces a clockwise rotation of the angular momentum in the (L_1, L_2)-plane when the spin is up and a counter-clockwise rotation with spin down. Equation (55) becomes, at $t = t_0/2 = (\pi/4)(\hbar/g)\sqrt{2l}$,

$$\Psi\left(\frac{t_0}{2}\right) = c_+ u_+ Y^{(2)}_{l,l} + c_- u_- Y^{(2)}_{l,-l} \tag{56}$$

because of eq. (43).

We can state, therefore, that the greater is l, the better can we regard eqs. (46) and (51) as classical equations of motion for the particle angular momentum. The measurement consists in this case in recording the value of the appropriate angular-momentum component as the instrument « pointer » and deducing by inference the corresponding eigenvalue of the spin component. This is conceptually the same procedure followed in a Stern-Gerlach experiment.

A few words are in order at this point to make some comments on the features which make a system, whose states are *not* characterized by macroscopic values of its variables, unsuitable to be used as a measuring apparatus. To this purpose let us consider our solution (42) for $l = 1$.

In this case the total wave function will be given, at time t, by eq. (44) with

$$Z(t) = \frac{1}{2}(\cos 2\alpha - 1)\, Y^{(3)}_{1,-1} + \frac{i}{\sqrt{2}} \sin 2\alpha\, Y^{(3)}_{10} + \frac{1}{2}(\cos 2\alpha + 1)\, Y^{(3)}_{11}\,. \tag{57}$$

Clearly only for $\alpha = \pi/2$ is there a strict correlation in the wave function between the spin state up with the value $m = -1$ and the spin state down with the value $m = 1$ of L_3. For all other times the values $m = 0, 1$ can also be found together with the spin up with probabilities comparable with that of $m = -1$. This means that the angular-momentum component for small values of l is not a good pointer for the measurement of spin. This is, for instance, the case of a model recently proposed ([11]) in order to evaluate explicitly the magnitude of the deviations from ideal measurement due to the existence of additive conserved quantities ([12]), based on the Hamiltonian (50), with $l = 1$. For the reasons stated above, however, to assume such a system as a good example of a reliable measuring apparatus is clearly too optimistic. On the other hand, the expected magnitude of these deviations, which according to the Yanase lower bound should be of order l^{-2}, becomes anyway negligible in the $l \gg 1$ case, corresponding to a good ideal instrument. This shows, in our opinion, that the condition of macroscopicity for the instrument variable should always explicitly be taken into account in order to avoid misleading discussions about expected deviations from the behaviour of a « normal » measuring instrument.

4. – Deviations from wave function collapse and objectivity of measurement.

The main argument advanced by those who consider inadequate the description of a measuring process in quantum mechanics, in terms of the interaction between microsystem and apparatus, consists in noticing that the total system state vector, resulting from the standard Schrödinger time evolution, is always a superposition, namely it predicts interference effects between terms belonging to different eigenstates of the apparatus. This is what the physicist quoted in ([5]) meant when he was saying that the « belief that things must come out one way or another is an illusion ». For the case of the counter this amounts to pointing out that the density matrix corresponding to wave function collapse (a mixture of spin up with discharged counter and spin down with neutral counter)

$$\hat{\varrho} = |c_+|^2|\bar{n}, N - \bar{n}\rangle\langle\bar{n}, N - \bar{n}|u_+\bar{n}_+^\dagger + |c_-|^2|N, 0\rangle\langle N, 0|u_-\bar{n}_-^\dagger \tag{58}$$

is different from the actual density matrix ϱ of the pure state (28):

$$\varrho = \hat{\varrho} + c_+ c_-^* |\bar{n}, N - \bar{n}\rangle \langle N, 0|u_+ u_-^\dagger + \text{c.c.}\,. \tag{59}$$

In order to assess whether the departure of ϱ from wave function collapse may give rise to observable consequences of some kind, we must, therefore, examine the properties of the interference terms. Clearly no difference between ϱ and $\hat{\varrho}$ can be detected by actually counting, or recording by means of any

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([11]) G. C. Ghirardi, F. Miglietta, A. Rimini, T. Weber: *Limitations on Quantum Measurements I, II*, ICTP preprint IC/81/5 and IC/81/14.
([12]) E. P. Wigner: *Z. Phys.*, **133**, 101 (1952); M. M. Yanase: *Phys. Rev.*, **123**, 666 (1961).

other device, the number N_1 (or N_0) of ionized (or neutral) particles (*) in the counter, because the interference terms in (59) give vanishing contributions in performing the partial trace on the counter's variables of ϱ with $N_1 = a_1^* a_1$.

To detect the difference, it is necessary (but not sufficient) to record the values of a variable having nonvanishing matrix elements between states of the counter widely differing in the number of ionized particles (**). This is not enough, however, to obtain evidence of the difference between ϱ and $\hat{\varrho}$. In fact, if only the values of the counter's variables are recorded, *without performing a second measurement of the microsystem's variables*, ϱ and $\hat{\varrho}$ will give the same results. This follows from the observation that these results are obtained by performing the partial trace on the spin variables, if their values are not independently specified by means of a second measurement. The trace, however, eliminates the interference terms in ϱ. In other words, in order to detect the effects of a possible departure from wave function collapse, it is necessary to compare the results of two independent measurements performed on S by two independent macroscopic instruments M_1, M_2 [13].

To see how the procedure works, it is useful to discuss in detail the case in which the first measurement is performed by recording the value of the particle angular-momentum component (Stern-Gerlach set-up), and the second one by means of a polarized counter.

Consider the state vector (44) of the system $S + M_1$ after the first measurement. If θ, φ are the polar angles of the particle orbital motion referred to L_3 taken as a polar axis and $\bar{\theta}$, $\bar{\varphi}$ are the polar angles referred to L_α, one can write explicitly

$$\Psi(t) = A[c_+ u_+ \exp[il\bar{\varphi}] (\sin\bar{\theta})^l + c_- u_- \exp[il\varphi] (\sin\theta)^l] , \tag{60}$$

where A is the appropriate normalization constant. If we choose for simplicity of reasoning to localize the particle at $\varphi = \bar{\varphi} = \pi/2$ (in the (L_2, L_3)-plane), then, because of (43), the relation between θ and $\bar{\theta}$ is simply $\bar{\theta} = \theta - 2\alpha$.

One sees immediately that the probability distribution for the particle's space location is given by

$$\Psi^*\Psi = A^2[|c_+|^2[\sin(\theta - 2\alpha)]^{2l} + |c_-|^2 (\sin\theta)^{2l}] , \tag{61}$$

namely by the sum of the two distributions corresponding to spin up and spin

(*) Since $N_0 + N_1 = N$, to count the number of ionized particles or the number of neutral particles is obviously the same thing.

(**) A suitable variable might be the counter's rate of discharge $\dot{N}_1$, which does not commute with N_1.

([13]) See on this point D. GUTKOSWSKI, M. V. VALDES FRANCO: *On the quantum-mechanical superposition of macroscopically distinguishable states.* Preprint Catania, February 1982.

down, respectively. The first one practically vanishes out of the plane $\theta = \pi/2 + 2\alpha$ and the second one vanishes out of the plane $\theta = \pi/2$. This, of course, is what one expects for the planes of the orbits of a classical particle having, respectively, either an angular momentum with components given by eqs. (47)-(49) or by $L_3 = l \gg 1$, $L_2 = L_1 = 0$.

The density matrices $\hat{\varrho}$ and ϱ are now

$$\hat{\varrho} = A^2[|c_+|^2[\sin(\theta - 2\alpha)]^{2l} u_+ u_+^\dagger + |c_-|^2(\sin\theta)^{2l} u_- u_-^\dagger] , \tag{62}$$

$$\varrho = \hat{\varrho} + A^2[\sin(\theta - 2\alpha)\sin\theta]^l [c_+ c_-^* u_+ u_-^\dagger + c_- c_+^* u_- u_+^\dagger] . \tag{63}$$

Here comes the necessity of a second measurement of the particle spin. Of course we should measure the spin component in a different direction, because a repeated measurement of σ_3 would lead to perform a trace of $(\varrho\sigma_3)$ from which the off-diagonal terms again disappear. We need, therefore, another instrument M_2. Let us assume it to be a counter polarized along the 2-direction, capable of selecting between the two eigenstates $v_\pm$ of the spin component σ_2. If we assume that v_+ discharges the counter and v_- does not, then the state vector Φ of the total system $S + M_1 + M_2$ will be

$$\begin{aligned}\Phi = \frac{A}{\sqrt{2}}\{&[c_+[\sin(\theta - 2\alpha)]^l + c_-(\sin\theta)^l] v_+ |\bar{n}, N - \bar{n}\rangle + \\ &+ [c_+[\sin(\theta - 2\alpha)]^l - c_-(\sin\theta)^l] v_- |N, 0\rangle\} ,\end{aligned} \tag{64}$$

because the eigenstates $u_\pm$ of σ_3 have been expressed in terms of $v_\pm$ by means of the relations

$$u_\pm = \frac{1}{\sqrt{2}}(v_+ \pm v_-) , \tag{65}$$

in order to obtain the expression equivalent to eq. (28) for a counter polarized in the 2-direction. We are now able to evaluate the difference between the predictions of (62) and (63) in terms of correlations between M_1 and M_2.

In fact, if we now eliminate the spin variables of S, because we do not make any further observation on them, we obtain the density matrix W of the system $M_1 + M_2$ as follows:

$$\begin{aligned}W &= \mathrm{Tr}_s(\Phi\Phi^\dagger) = \\ &= A^2\{\tfrac{1}{2}|c_+[\sin(\theta - 2\alpha)]^l + c_-(\sin\theta)^l|^2 |\bar{n}, N - \bar{n}\rangle\langle\bar{n}, N - \bar{n}| + \\ &\quad + \tfrac{1}{2}|c_+[\sin(\theta - 2\alpha)]^l - c_-(\sin\theta)^l|^2 |N, 0\rangle\langle N, 0|\} .\end{aligned} \tag{66}$$

This expression is the main result of our discussion. It shows a departure from the expression

$$(67)\qquad \hat{W} = A^2\{|c_+|^2[\sin(\theta - 2\alpha)]^{2l} + |c_-|^2(\sin\theta)^{2l}\}\cdot \cdot\{\tfrac{1}{2}|\bar{n}, N-\bar{n}\rangle\langle\bar{n}, N-\bar{n}| + \tfrac{1}{2}|N, 0\rangle\langle N, 0|\}$$

that one would have obtained if the measurement of σ_3 performed by detecting the orbital plane of the particle had given exact wave function collapse. Had this been the case, the two subensembles in the states u_+, u_- (with populations proportional to $|c_+|^2$ and $|c_-|^2$, respectively, the first one with orbits lying in the plane perpendicular to L_α and the second one in the plane perpendicular to L_3) would be separated, at their turn, into equally populated subensembles corresponding to the eigenstates v_+, v_- of σ_2. The difference between (66) and (67) stems from the interference effects between the macroscopic states with $L_\alpha = l$ and $L_3 = l$ which lead to different populations of the subensembles with opposite eigenvalues of σ_2 (discharged counter and neutral counter, respectively). These effects, we shall see, are exceedingly small for large values of l.

Before discussing their magnitude, however, we wish to point out that their supposed detectability is anyway in contradiction with the necessary discriminating properties of a measurement apparatus. In fact, eq. (66) shows that the interference effects have to be looked for in the region (near $\theta = \pi/2 + \alpha$) between the two classical orbital planes ($\theta = \pi/2 + 2\alpha$ and $\theta = \pi/2$) because on these planes one of the two factors practically vanishes. This means, however, that the events which might prove the presence of interference effects do not allow one to choose between the two spin values. In other words, this shows that a macrosystem can perform the function of a measuring apparatus only when it does not show interference effects.

Quantitatively, for $\theta = \pi/2 + \alpha$, one has

$$(68)\qquad 2\,\mathrm{Re}\,(c_+^* c_-)[\sin(\theta - 2\alpha)\sin\theta]^l = 2\,\mathrm{Re}\,(c_+^* c_-)(\cos\alpha)^{2l} < \exp[-l\alpha^2]\,.$$

This should be compared with $|c_+|^2$ and $|c_-|^2$, representing the intensities of the events lying on one or the other orbital plane. The effect is clearly by far unobservable, for values of l corresponding to macroscopic orbits. One should notice, however, that, in the classically forbidden region $\theta = \pi/2 + \alpha$, the interference terms are not small compared to the diagonal terms. It is simply that there are practically no events resulting from the first measurement yielding values of the macroscopic variable θ such as to allow their observation.

Once more one should compare this result with the situation in which l is small. For $l = 1$, the ratio of the intensity in the middle plane $\theta = \pi/2 + \alpha$ to the intensity in either of the orbital planes, for the two σ_2 eigenstates, is

$$(69)\qquad R_\pm = \frac{\cos^2\alpha\,|c_+ \pm c_-|^2}{|c_\pm|^2\cos^2\alpha + |c_\mp|^2}\,.$$

This expression gives, for the optimal case $c_+ = c_-$ and $\alpha = \pi/6$,

$$R_\pm = \begin{cases} 0 \\ \frac{3}{5}\,. \end{cases} \tag{70}$$

The effect is now quite substantial. Again it is clear that a microsystem cannot perform the function of a measuring apparatus.

The impossibility of detecting any physical difference between ϱ and $\hat{\varrho}$ is, therefore, a consequence of the interaction of S with a suitable macroscopic variable of M. The existence of a one-to-one correlation between the eigenvalues of the quantum variable to be measured and different macroscopic values of an appropriate variable of the measuring apparatus is, therefore, sufficient to ensure that the system $S + M$ always behaves as predicted by the projection postulate, even if the wave function collapse actually never occurs. Measurements in quantum mechanics are, therefore, objective, they are not the result of the observer's consciousness.

5. – Reversible and irreversible measuring apparatus.

It is useful at this point to discuss briefly the possibility that, after a sufficiently long interval of time, the Schrödinger evolution leads back to the initial state of the total $S + M$ system. If the interaction between microsystem and apparatus is reversible (Hermitian Hamiltonian), this will always be the case, for a time $t = t_r$ large enough. In our schematization of the measuring process, $t_r = \pi\hbar\sqrt{N}/g$. In real counters, however, this time is practically infinite, simply because of the second law of thermodynamics. No counter will ever recharge itself because all the electrons and ions will spontaneously recombine to form the initial neutral state.

For Stern-Gerlach–type set-ups, however, reversibility may be obtained provided the magnets are conveniently designed to recombine the beams split by the interaction between the microscopic variable (spin) and the macroscopic space variable.

In this case, the actual density matrix ϱ (see eq. (63)) is again given, at $t = t_r$, by its initial form

$$\varrho(t_r) = \varrho(0) = Y^{(3)}_{ll}\, Y^{(3)*}_{ll}(c_+ u_+ + c_- u_-)(c^*_+ u^\dagger_+ + c^*_- u^\dagger_-)\,. \tag{71}$$

This is clearly different from the value of $\hat{\varrho}$ at the same time,

$$\hat{\varrho}(t_r) = Y^{(3)}_{ll}\, Y^{(3)*}_{ll}\big(|c_+|^2 u_+ u^\dagger_+ + |c_-|^2 u_- u^\dagger_-\big)\,. \tag{72}$$

It is sometimes argued that, if the observer had decided to *look* at the apparatus at a time $t < t_r$, then (72) would be the correct statistical matrix, if not, then (71) would hold. This is again a surreptitious way of introducing the observer's subjectivity into a matter which can be settled with purely physical arguments. In fact, the real alternative is whether an instrument is brought in to interact with the variable which acts as a « pointer » for the measurement of the microscopic variable. If it is not, then at $t = t_r$ the correlation between the two spin eigenvalues and *two different macroscopic values* of the « pointer » variable is lost, because the latter has acquired back again its *unique initial value*. The interference effects are no longer depressed by the presence of two widely different macroscopic states, but become possible again as always happens with quantum variables at *low quantum numbers* (*). If, on the other hand, a counter is used with the purpose of detecting which of the two widely different values of the angular momentum the particle has acquired at a time $t < t_r$ as a consequence of its interaction with the spin, one should explicitly introduce into the picture the interaction between L_3 and a suitable counter leading to a correlation between the two states of the latter (neutral $|0\rangle$ or discharged $|D\rangle$) and the two widely different values l and $m \ll l$ of L_3.

The new density matrix W describing the total system spin + angular momentum + counter will, therefore, be given by

$$(73)\qquad W = |c_+|^2\, Y^{(3)}_{l\bar{m}}\, Y^{(3)}_{l\bar{m}} |D\rangle\langle D| u_+ u_+^\dagger + |c_-|^2\, Y^{(3)}_{ll}\, Y^{(3)*}_{ll} |0\rangle\langle 0| u_- u_-^\dagger + \\ + c_+ c_-^*\, Y^{(3)}_{l\bar{m}}\, Y^{(3)*}_{ll} |D\rangle\langle 0| u_+ u_-^\dagger + c_- c_+^*\, Y^{(3)}_{ll}\, Y^{(3)*}_{l\bar{m}} |0\rangle\langle D| u_- u_+^\dagger .$$

However, W will always coincide with its reduced form $\hat{W}$ because the recurrence time *of the counter* is practically infinite. Equation (72) is, therefore, nothing more than a shorthand notation for

$$(74)\qquad W(t_r) = \hat{W}(t_r) = Y^{(3)}_{ll}\, Y^{(3)*}_{ll} [|c_+|^2 u_+ u_+^\dagger |D\rangle\langle D| + |c_-|^2 u_- u_-^\dagger |0\rangle\langle 0|] .$$

Subsequent measurements of the microsystem's spin will, therefore, give the results one would have obtained had the wave function collapse have occurred.

The preceding developments clarify the relation between irreversibility and wave function collapse. The conventional assumption of validity of the projection postulate introduces a fundamental irreversibility at the microscopic

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(*) The interference experiments of a single particle with itself, following the recombination of beams splitted with suitable polarizers, confirm this expected behaviour [14,15].

(14) A. GOZZINI: private communication.

(15) H. RAUCH, U. BONSE and W. TREIMER: *Phys. Lett. A*, **47**, 369 (1974).

level which is sometimes invoked as the mechanism underlying the universal irreversible behaviour of the real world ([16]). In the present approach, which rejects the projection postulate, this connection is ruled out. This approach, however, differs also from the one of the pioneers ([4]) because it does not assume irreversibility as the cause of (pseudo) wave function collapse. We have shown that the latter follows in the first instance only from the macroscopic character of the measuring device. Of course, when the latter, as usually happens, is, in addition, a system with a very large number of degrees of freedom, then the process is also irreversible, in the usual approximate sense of statistical mechanics. In this case irreversibility reinforces the mechanism of (pseudo) collapse. Both phenomena are, therefore, a consequence of large numbers, but one can have the latter without the former, although not viceversa. One can say that irreversibility makes (pseudo) collapse irreversible.

The preceding discussion also helps in settling the old question of repeated measurements. As was remembered in the introduction, it was for a long time believed that the wave packet projection postulate was necessary to ensure that a measurement, repeated immediately after a preceding one, should give with certainty the result obtained the first time. With the present formulation this can immediately be seen to occur as a consequence of the two interactions of S with the counters M_1 and M_2 activated in succession.

The state vector Φ of the system $S + M_1 + M_2$ will become, after the two measurements have been performed (with the notation of eq. (73)),

$$\Phi = c_+ u_+ |D\rangle_1 \otimes |D\rangle_2 + c_- u_- |0\rangle_1 \otimes |0\rangle_2 . \tag{75}$$

Here again the reduced density matrix

$$\hat{W} = |c_+|^2 |D\rangle_{11}\langle D| \otimes |D\rangle_{22}\langle D| u_+ u_+^\dagger + |c_-|^2 |0\rangle_{11}\langle 0| \otimes |0\rangle_{22}\langle 0| u_- u_-^\dagger \tag{76}$$

gives the same answers as the exact one

$$W = \hat{W} + c_+ c_-^* |D\rangle_{11}\langle 0| \otimes |D\rangle_{22}\langle 0| u_+ u_-^\dagger + \text{c.c.} \tag{77}$$

This means that practically the second counter will always be discharged if the first one has been discharged and will remain neutral if the same thing has occurred to the first one. Both are strictly correlated with spin up and spin down, respectively.

This shows clearly not only that the projection postulate is unnecessary, but also that quantum mechanics without further additional postulates is capable

([16]) L. D. LANDAU, E. M. LIFSCHITZ: *Statistical Physics* (London, 1959), p. 31.

of reproducing correctly all the observed features of the interactions between microsystems and macroscopic objects.

6. – Final remarks and conclusions.

Against the elimination of the projection postulate, two further arguments have been raised repeatedly. The first one is based on the existence of the so-called « negative measurements » ([17,18]). It amounts to saying that sometimes, even by observing that a counter has *not* been discharged, one can deduce with certainty by inference the appropriate value of the microscopic variable. This would indicate, it is argued, that the wave packet collapse is a purely mental process produced by the observer's mind, because it has occurred even when nothing has happened to the counter. This conclusion is the result of a gross misunderstanding about the word « interaction ». In fact, one of the possible outcomes of the interaction *is* that the counter is not discharged. But this possibility is a consequence of the interaction on the same footing as the other one that the counter is discharged. The absence of a signal, *together with the knowledge that the particle has interacted*, has exactly the same information content as a positive signal. Only if it is not known whether the particle is present at all, the negative signal and the positive one are logically different. In other words the black fringes in an interference pattern have the same content as the bright ones: they are not the same blackness as a uniformly black screen.

The issue is perfectly clear in our formalism, because the two possible outcomes are both present in the density matrix $\hat{\varrho}$ with their appropriate probability.

The second argument is that, should one drop the projection postulate, it would become impossible to prepare a system in a well-defined state after having measured the relevant variable. However, the « preparation of a state » must *not* be confused with the measurement process. It is clearly true that, once a given result has been obtained, one can *assume* as initial state of the future development of a quantum system, the eigenvector of the measured observable corresponding to the eigenvalue actually found. But this has nothing to do with the problem of determining the probabilities of the different possible outcomes of a measurement. Here again the confusion arises from the wrong belief that quantum systems can be treated as isolated systems. If the macroscopic environment is properly taken into account, all the ambiguities disappear because the systems with which S interacts are *different* if the problem is to find out the results of a measurement of one of its ob-

([17]) J. M. JAUCH, E. P. WIGNER and M. M. YANASE: *Nuovo Cimento B*, **48**, 144 (1967).
([18]) The first refutation of this argument can be found in A. LOINGER: *Nucl. Phys. A*, **108**, 245 (1968).

servables, rather than to describe a new dynamical evolution of S by assuming that initially that observable has a well-defined value.

Once more the problem is reduced to classical probability theory. After all, any roulette player knows that, once the ball has stopped on a given number, the *a priori* probability that his (different) number had before this event has no cash value at all.

In conclusion, the main conceptual result of the present formulation of measurement in quantum mechanics is the elimination of the observer's role in determining the change of state of the observed system. A clear separation between the objective time evolution of the system $S + M_1 + M_2 + \dots$ and the subjective decision of the « observer » to look at the counter dials is established. The latter act does not in any way influence what happens to the microsystem as a consequence of its interactions with the macroscopic object which exists in its environment. Once this environment has been specified, the time evolution of the state vector proceeds without further interferences from the external world.

This shows that the widespread subjectivist view of reality according to which the latter is created by the act of observation is only the consequence of an incorrect physical assumption, namely the schematization of the microsystem as an isolated system.

As soon as one takes into account the physical fact that only the microsystem together with the apparatus can be correctly represented as an isolated system, the objectivity of reality is restored and the causal evolution of the state vector becomes, as should be, the source of a well-defined statistical information about the possible different outcomes of the interaction of the microsystem with the apparatus. Therefore, the decision of the « observer » to look or not to look at the pointer of an instrument exerts no more influence on the state of the microsystem than the decision to look or not to look at a tossed coin exerts on it being head or tails. In fact, one could very well describe the state of the coin by quantum mechanics (after all, the coin too is made of N atoms) as we have done for the measuring apparatus M by means of a superposition of two equiprobable states $|H\rangle$ and $|T\rangle$ which, according to our viewpoint, is completely equivalent to the mixture described by the corresponding statistical matrix

$$\hat{\varrho} = \tfrac{1}{2}[|H\rangle\langle H| + |T\rangle\langle T|] . \tag{78}$$

The identity between the description of the coin state by means of (78) and the description of the system $M + S$ by means of eq. (58) leaves, therefore, no doubt that no influence whatsoever is exerted by the observers on the results obtained in either case.

The inclusion of the macroscopic objects—and the measuring apparatus is only one of them—in the physical world of the microsystem thus eliminates that misterious influence of the observer's mind on the microsystem behaviour, which has been at the origin of so much frivolous nonsense by so many famous physicists ([19]).

APPENDIX ADDED IN PROOFS

The circulation of the preprint of this paper has raised some stimulating comments for which I am indebted to B. D'ESPAGNAT, A. RIMINI, A. LOINGER, L. LANZ.

These comments have convinced me that an effort might be useful in order to clarify further some of the statements presented above.

I am, therefore, grateful to the editor of this journal for having given to me the possibility of doing so.

Let me try first of all to reformulate as concisely as possible what my article proves.

It does prove that the postulate of wave packet collapse, introduced as an extra assumption in quantum mechanics in order to describe the change in the wave function of a quantum object occurring during the time interval which is necessary to perform the measurement of one of its physical variables, can be dropped and replaced by the Schrödinger time evolution of the state vector of the total system object + apparatus. Of course, this is proved, strictly speaking, only for the ideal measuring processes described by the interaction Hamiltonians introduced in the paper, and not for any conceivable measuring apparatus. But, within these limits, the elimination of the projection postulate from the conceptual foundations of quantum mechanics is shown to be consistent with the known features of any physical measurement.

It is on this basis that I argue that the projection postulate *should* be eliminated, for at least two good reasons.

The first one, physical and, therefore, more convincing, amounts to saying that by so doing a unified treatment of all physical phenomena is recovered. A measuring apparatus is no longer a nonphysical entity whose properties cannot be described by the same equations which represent the behaviour of ordinary matter, as implied by the adoption of the projection postulate. It becomes a physical object whose structure is such that it makes it suitable for the purpose of establishing a one-to-one correspondence between one of its physical variables and the physical variable of a quantum object which is brought in interaction with it. The conventional assumption on the absolute validity of the projection postulate leads instead to the absurdity that one would be allowed to treat the interaction of an electron with a given macroscopic object—say a container filled with gas in a strong electric field—by means of the Schrödinger equation, provided one does not know that the object is a counter, but would be forced to describe it as an intrinsically unanalysable event, not submitted to the laws of quantum mechanics, as soon as its nature of measuring apparatus is realized.

([19]) A brief survey of this nonsense is presented in sect. **13** of F. SELLERI and G. TAROZZI: *Riv. Nuovo Cimento*, **4**, 1 (1981).

From my point of view, on the contrary, the wave function collapse is only an *approximation* for describing what happens to the state of the microscopic object, an approximation whose validity can be estimated when the structure of a given apparatus is known.

Stated differently, a good measuring apparatus is a physical system which, when brought in interaction with a given quantum object, yields a statistical matrix ϱ practically indistinguishable from $\hat{\varrho}$.

It is likely that, as proved rigorously in the Coleman-Hepp model, in the limit $N \to \infty$ the collapse becomes exact. But, since any physical instrument is made of a finite number of particles, the collapse is only an approximate description of the actual time evolution of the system object + apparatus.

The whole point is that in nature there are no absolutely classical objects. A good measuring apparatus can be treated as classical with very high accuracy (namely completely characterized by an Abelian set of observables) and in this case the particle wave function collapse follows. But it cannot be assumed to be classical by *fiat*, as done, *e.g.* by JAUCH [20]. One has to *prove* that the assumption is a good approximation.

Now let me come to the second, metaphysical, reason for abandoning the projection postulate. The well-known terminal of the von Neumann chain is the observer's consciousness. In fact, the only way to avoid the contradiction mentioned above between the double nature of a measuring apparatus (physical and nonphysical) is to introduce the dichotomy between all physical entities, including the brain cells of the observer, which can be described by means of quantum mechanics, on the one hand, and the nonphysical entities (consciousness, mind, soul, God perhaps), on the other. It is among the latter that the collapse-producing agent is, therefore, identified. Now I am sure that many of our colleagues would agree that this is sheer nonsense. The only wayout is, therefore, to drop the projection postulate. If there is no collapse, there is no collapse-producing agent. Let it be clear that I do not have a mechanicistic view of reality. Reality is indeed a whole, which includes men (not Man, which is an abstraction), but certainly a very highly structured, multilevel whole. The identification of the different structures and levels is the result of human consciousness, more precisely of historically accumulated knowledge and experience, both social (tradition, culture, beliefs, needs, etc.) and individual (creativity, logical and analogical thought, etc.). Surely scientific theories are not a pure reflection of reality « as it is ». Surely empirical facts are not unanalysable data given once for all. They are both the product of a human social activity which gives a representation of a part of reality from the point of view of an historical given community [*]. But this does not mean that one should take human consciousness as an essential ingredient of a *physical* theory.

The physical level of reality has been identified so far as the one in which all events are the outcome of an evolution which occurs without the intervention of man. He can choose the objects, the conditions, the environment in which they are placed, but, once all this is prepared, he steps aside: what happens later is out of the range of his will. This of course does not mean that the result

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[20] J. M. JAUCH: *Helv. Phys. Acta*, **37**, 293 (1964).

[*] The interested reader may find more about this in [21,22].

[21] G. CICCOTTI, M. CINI, M. DE MARIA and G. JONA-LASINIO: *L'ape e l'architetto* (Milano, 1976); trad. française, *L'araignée et le tisserand* (Paris, 1981).

[22] D. MAZZONIS and M. CINI: *Il gioco delle regole* (Milano, 1981).

is determined. Identical (maybe only apparently) initial conditions may lead to different events. But they are nonetheless the result of something which happens independently of man. This is why any observer who looks at a counter gives the same answer. This is surely a property of counters, not of observers.

In this sense it is true that I want physics to describe reality without any reference to the community of observers. But what I mean by this is that it is not possible to describe reality without any reference to the collection of instruments and artifacts placed by the *experimenter* in order to detect the properties of the objects he is interested in. Not absolute properties, therefore, but properties which depend on the conditions he has chosen to operate with. I do not want to speak of observers because they do not have any role in the definition of physical reality. They do not interfere with reality when they look at a dial or a pointer (*).

Coming back to my paper, it should be now clear why the reference to the *practical* circumstances in which measurements are performed is coherent with my thesis that a good measuring apparatus should have ϱ indistinguishable from $\hat{\varrho}$. In order to be able to select it out of other physical systems which do not have this property, or have it with a lower degree of accuracy, it is, in fact, necessary to investigate when this is the case. The circumstance that papers proposing models which fail to have this property are not rare in the literature does prove that the understanding of the conditions which are necessary to ensure its occurrence is generally very poor. Only when measuring apparatuses are *defined* as abstract entities whose statistical matrix is *postulated* to be $\hat{\varrho}$, any reference to the practical circumstances becomes obviously unnecessary.

Let me now deal briefly with another point concerning the relationship between measurements performed by means of counters and measurements of the Stern-Gerlach (SG) type. Assume one measures the spin of a particle, for definiteness and simplicity. In my model of measurement processes the two measurements are mathematically equivalent. A polarized counter (**) is a quantum object with one degree of freedom (say the number of ionized particles N_1) whose macroscopic states are labelled by the two widely different quantum numbers: $N-\overline{n}(t)$ and zero. The pointer of the SG-type measurement is a quantum object with one degree of freedom (say the angular-momentum component L_3) whose macroscopic states are labelled by the two widely different quantum numbers $\overline{m}(t)$ and l. The reading device M_2 merely records the large difference between the quantum numbers, which are in a one-to-one correspondence with the two spin states, in both cases. In principle, statements about one type of measurement can be immediately translated into statements about the other one. There are, however, two physical differences which I want to discuss.

The first one has to do with the physical interpretation of the variables. There is no problem with those ones which label the macroscopic states, whose

(*) This is not true if one deals with other levels of reality. It is well known that in social sciences observation does change the piece of reality under investigation. But this is because the objects of investigation have a consciousness which electrons, as far as we know, do not have.

(**) The difference between a polarized counter and a detection counter is that the first one selects (say) the u^+ from the u^- component of the linear combination χ, and the second one detects the presence of the particle. Since one *cannot* form a linear combination χ of presence and absence (particles are conserved), the two counters are essentially different and cannot be conceptually identified.

correspondence is defined mathematically by eqs. (36)-(38). Clearly the semi-difference between the number of neutral and ionized particles for the counter corresponds to the third component of the angular momentum for the SG device. For the other ones (which do not commute with the preceding ones) the physical interpretation may not be so immediate. It is still easy to see that the couter time rate of discharge $\dot{N}_0$ corresponds to L_2 in the SG device (because they both are the result of a commutation with the Hamiltonian). But there is no straightforward meaning in the counter case for the variable corresponding to the angular space co-ordinate θ conjugate to L_3 of the SG device, even if one can see that formally

$$\exp[i\theta] \to a_0^* a_1 .$$

Obviously there is no instrument which can measure directly the counter « θ-variable ».

The second difference comes from real life. In fact, a real counter is a system with a very large number of degrees of freedom, while a SG device still uses as a pointer a one-degree-of-freedom system (the position of the particle).

In the first case, therefore, irreversibility plays a fundamental role. The lack of coherence between the two macroscopically different states is going to remain foreover. No matter what reading device M_2 is used to detect whether the counter has been discharged or not, it will always be either one way or the other, in strict correlation with the two spin states. The von Neumann chain is indeed broken between M_1 and M_2. The second case, however, maintains the features of the model. Even if a linear combination of two states labelled by macroscopically different values of the space co-ordinate is indeed indistinguishable from the corresponding mixture, this state of affairs may not last forever. It may happen that these states evolving reversibly recover a definite phase relationship. This happens, for instance, when the two beams are brought together again in the same space region. In this case the position of the particle ceases to play the role of a measuring-instrument pointer. The device has become an experiment proving the quantum-mechanical interference phenomenon of two polarized beams of particles. This is why, if one wants to make a spin measurement, he should use a reading device M_2 when the two beams are widely separated. In this case too the result will be either one way or the other, in strict correlation with the two spin states. Does this mean that it is M_2 which has produced the (pseudo) collapse, because, if it had not been there, the beams could have interfered again? In the light of the previous discussion about the approximate nature of the concept of collapse, I would prefer saying that $\hat{\varrho}$ is a good approximation for ϱ when the wave packets are widely separated and a very bad one when they overlap. The detector M_2 simply fixes irreversibly the (very good) approximate equivalence between ϱ and $\hat{\varrho}$ at the time when the two wave packets had no overlap. This equivalence, however, is the consequence of the fact that a local variable can never have nonvanishing matrix elements connecting two nonoverlapping wave packets. It is not a consequence of the presence of M_2. Only in this sense irreversibility plays a role also in the SG measuring devices.

A few more words may be useful to clarify the analysis performed in sect. 4 concerning the detectability of the difference between ϱ and $\hat{\varrho}$. The main point developed there is that not only has one to perform two successive measurements of two different spin components, but in addition one should detect, for the first instrument, the values of a variable which does not commute with the variable which labels its macroscopic states. It is for this reason that I have chosen to perform the first measurement with a SG device. In this case, in fact, the variable θ, conjugate to L_3, is, as we have seen, easily detectable and, therefore, its probability density distribution easily recorded. A simple reading device yielding the two macroscopic values $\overline{m}$ and l of L_3 would not do for our purposes: with this information one would find the subensembles corresponding to the two spin-up and -down states in the third direction, both split into two equally populated beams after the measurement of the spin in the second direction, namely the same result one would obtain had the collapse have occurred exactly. The recording of the θ-angle distribution instead allows, in principle, the detection of events which cannot be with certainty attributed either to $\overline{m}$ or to l, events not predicted by the projection postulate.

However, it is just the condition $l \gg 1$ which makes the probability distribution of θ concentrated practically in the two peaks $\theta = \pi/2$ and $\theta = \pi/2 + 2\alpha$, corresponding to $L_3 = l$ and $L_3 = \overline{m}$. This means that, even by detecting an observable which would allow us to discriminate between ϱ and $\hat{\varrho}$, no difference can be actually detected when l is macroscopically large. In principle, of course, one could set a counter between the two peaks at $\theta = \pi/2 + \alpha$ and wait.

It it would be hardly worthwhile sitting there for millions of years to check something which is anyway expected by any reasonable physicist, namely that ϱ (and not $\hat{\varrho}$) is the correct result.

In summary, we are free to choose whether we want to make a measurement, or an interference experiment. In the first case, quantum mechanics ensures us that a standard instrument will behave, as expected, in a classical way. In the second case, however, we need much more skill. The detection of a difference between ϱ and $\hat{\varrho}$ is not a disturbing possibility, but rather a difficult challenge. This is why I would not worry too much to understand why there are quantum systems which behave classically. I would rather concentrate on the problem of inventing ways of detecting quantum properties of systems which we expect to behave classically.

● RIASSUNTO

Si presenta una schematizzazione del processo di misura in meccanica quantistica che permette un trattamento unificato sia delle misure effettuate per mezzo di contatori polarizzati, sia di quelle compiute con dispositivi del tipo di Stern-Gerlach. Si dimostra in questo modo che il cosiddetto collasso della funzione d'onda non è un postulato di validità assoluta che deve essere aggiunto dall'esterno alle leggi della meccanica quantistica, ma piuttosto una conseguenza — non esatta ma valida a un grado di approssimazione elevatissimo — di queste stesse leggi. I limiti delle deviazioni da un processo di collasso rigoroso possono essere espressi in termini di quantità esplicitamente dipendenti dal carattere macroscopico del dispositivo sperimentale. Si discute inolter la relazione tra irreversibilità e (pseudo) collasso e si mostra che ambedue discendono dai grandi numeri connessi con questo carattere macroscopico. È possibile tuttavia avere collasso senza irreversibilità, ma non il viceversa. Si mostra così che tutti gli aspetti apparentemente paradossali del problema della misura nascono dalla confu-

sione fra il livello dei piccoli numeri quantici e quello dei numeri quantici elevati. Soltanto a quest'ultimo livello l'equivalenza fra il vettore di stato puro del sistema totale « apparato + oggetto » e la matrice statistica che rappresenta i possibili risultati della loro interazione garantisce che l'« osservatore » non ha alcun potere di « creare » la realtà, ma semplicemente ottiene da una rappresentazione oggettiva, anche se probabilistica, di questa realtà, tutte le informazioni statistiche possibili.

Резюме не получено.